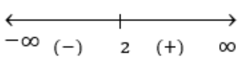
Marking Scheme

Mathematics (Term-I)

Class-XII (Code-041)

Q.N.	Correct Option	Hints / Solutions
1	d	$sin\left(\frac{\pi}{3}-\left(\frac{-\pi}{6}\right)\right)=\sin\left(\frac{\pi}{2}\right)=1$
2	b	$\lim\limits_{x\to o}\left(\frac{1-\cos kx}{x\sin x}\right)=\frac{1}{2}$ $\Rightarrow \lim\limits_{x\to o}\left(\frac{2sin^2\frac{kx}{2}}{x\sin x}\right)=\frac{1}{2}$ $\Rightarrow \lim\limits_{x\to o} 2\left(\frac{k}{2}\right)^2\left(\frac{sin\frac{kx}{2}}{\frac{kx}{2}}\right)^2\left(\frac{x}{sinx}\right)=\frac{1}{2}$ $\Rightarrow k^2=1 \Rightarrow \mathrm{k}=\pm 1$ $but\ k<0$ $\Rightarrow$ k= -1
3	d	$A^2=\begin{bmatrix}1&0\\0&1\end{bmatrix}$
4	c	As A is singular matrix $\Rightarrow \|A\|=0$ $\Rightarrow 2k^2-32=0 \Rightarrow \mathrm{k}=\pm 4$
5	b	$f(x)=x^2-4x+6$ $f'(x)=2x-4$ $let\ f'(x)=0 \Rightarrow \mathrm{x}=2$  as $f'(x)>0\ \ \forall\ \ x\in(2,\infty)$ $\Rightarrow f(x)$ is Strictly increasing in $(2,\infty)$
6	d	as $\|adj\ A\|=\|A\|^{n-1}$, where n is order of matrix A $=(-4)^2=16$
7	b	$(1,2)$
8	a	$\left.\begin{aligned}2a+b&=4\\a-2b&=-3\\5c-d&=11\\4c+3d&=24\end{aligned}\right\} \Rightarrow \begin{aligned}a&=1\\b&=2\\c&=3\\d&=4\end{aligned}$ $\therefore a+b-c+2d=8$
9	a	$f(x)=x+\frac{1}{x}, x>0 \Rightarrow f'(x)=1-\frac{1}{x^2}=\frac{x^2-1}{x^2}, x>0$ As normal to $f(x)$ is $\perp$ to given line $\Rightarrow\left(\frac{x^2}{1-x^2}\right)\times\frac{3}{4}=-1\ \ (m_1.m_2=-1)$ $\Rightarrow x^2=4 \Rightarrow \mathrm{x}=\pm 2$ But $x>0, \therefore x=2$ Therefore point=$\left(2,\ \frac{5}{2}\right)$
10	d	$\sin(\tan^{-1}x)=\sin\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)=\frac{x}{\sqrt{1+x^2}}$
11	a	$\{1,5,9\}$
12	c	$e^x+e^y=e^{x+y}$ $\Rightarrow e^{-y}+e^{-x}=1$ Differentiating *w.r.t.* x:

		$\Rightarrow\ -e^{-y}\frac{dy}{dx}-e^{-x}=0\Rightarrow\ \frac{\mathrm{dy}}{\mathrm{dx}}=-e^{y-x}$
13	**b**	3×5
14	**a**	$y=5\cos x-3\sin x\Rightarrow\frac{\mathrm{dy}}{\mathrm{dx}}=-5\sin x-3\cos x$ $\Rightarrow\ \frac{d^2y}{dx^2}=-5\cos x+3\sin x=\ -y$
15	**c**	adj A $=\begin{bmatrix}7 & -5\\ 11 & 2\end{bmatrix}\Rightarrow(adjA)'=\begin{bmatrix}7 & 11\\ -5 & 2\end{bmatrix}$
16	**c**	$\frac{x^2}{9}+\frac{y^2}{16}=1\Rightarrow\frac{2x}{9}+\frac{2y}{16}\frac{dy}{dx}=0$ $\Rightarrow slope\ of\ normal=\frac{-\mathrm{dx}}{\mathrm{dy}}=\frac{9y}{16x}$ As curve's tangent is parallel to *y*-axes $\Rightarrow\frac{9y}{16x}=0\Rightarrow y=0$ and $x=\pm 3$ $\therefore\ \ points=(\pm 3,0)$
17	**b**	$\lvert A\rvert=-7$ $\therefore\sum_{i=1}^{3}a_{i2}A_{i2}=a_{12}A_{12}+a_{22}A_{22}+a_{32}A_{32}=\lvert A\rvert=-7$
18	**d**	$y=\log(cos\ e^x)$ Differentiating wrt x: $\frac{dy}{dx}=\frac{1}{\cos(e^x)}.(-\sin e^x).\ e^x$ (chain rule) $\Rightarrow\frac{\mathrm{dy}}{\mathrm{dx}}=-e^x\tan e^x$
19	**d**	Z is maximum 180 at points *C (15,15)* and *D(0, 20).* ⇒ Z is maximum at every point on the line segment *CD*
20	**c**	$f(x)=2cosx+x\ ,\ \ x\in[0,\frac{\pi}{2}]$ $f'(x)=-2sinx+1$ Let$f'(x)=0\Rightarrow x=\frac{\pi}{6}\ \ \in[0,\frac{\pi}{2}]$ $f(0)=2$ $f\left(\frac{\pi}{6}\right)=\frac{\pi}{6}+\sqrt{3}$ $f\left(\frac{\pi}{2}\right)=\frac{\pi}{2}$ ⇒ least value of $f(x)$ is $\frac{\pi}{2}$ at $x=\frac{\pi}{2}$
		Section-B
21	**d**	$let\ f(x_1)=f(x_2)\ \forall x_1x_2\in R$ $\Rightarrow\ {x_1}^3={x_2}^3$ $\Rightarrow\ \ x_1=x_2$ ⇒ f is one - one $let\ f(x)=x^3=y\ \ \forall\ y\ \in R$ $\Rightarrow \mathrm{x}=y^{\frac{1}{3}}$ every image $y\ \in R$ has a unique pre image in R ⇒ f is onto $\therefore f$ is one-one and onto
22	**a**	$x=a\ sec\theta\Rightarrow\frac{dx}{d\theta}=a\tan\theta\sec\theta$ $y=b\tan\theta\ \Rightarrow\ \frac{dy}{d\theta}=b\ sec^2\theta$ $\therefore\frac{dy}{dx}=\frac{b}{a}cosec\theta$ $\Rightarrow\frac{d^2y}{dx^2}=\frac{-b}{a}\ cosec\theta.cot\theta.\frac{d\theta}{dx}=\frac{-b}{a^2}cot^3\theta$ $\therefore\ \frac{d^2y}{dx^2}\Big]_{\theta=\frac{\pi}{6}}=\frac{-3\sqrt{3}b}{a^2}$
23	**c**	Z is minimum -24 at (0, 8)
24	**a**	$let\ u=sin^{-1}(2x\sqrt{1-x^2})$

		and $v = sin^{-1}x,\ \frac{1}{\sqrt{2}} < x < 1 \ \Rightarrow \sin v = x$……(1) Using (1), we get : $= sin^{-1}(2\sin v\ \cos v)$ $\Rightarrow \mathrm{u} = 2\mathrm{v}$ Differentiating with respect to v, we get: $\frac{du}{dv} = 2$
25	**d**	$AB = 6I \ \Rightarrow \mathrm{B}^{-1} = \frac{1}{6}A$
26	**b**	$f'(x) = 6(x^2 - x - 6) \ = 6(x-3)(x+2)$ $\longleftrightarrow$ $-\infty\ (+)\quad -2\quad (-)\quad 3\quad (+)\quad \infty$ As $f'(x) < 0\ \forall\ x \in (-2,3)$ $\Rightarrow\ f(x)$ is strictly decreasing in $(-2,3)$
27	**a**	$tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-cosx}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)$ $= tan^{-1}\left(\frac{-\sqrt{2}\cos\frac{x}{2}+\sqrt{2}\sin\frac{x}{2}}{-\sqrt{2}\cos -\sqrt{2}\sin\frac{x}{2}}\right)\quad,\ \pi < x < \frac{3\pi}{2}$ $= tan^{-1}\left(\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\right)$ $= \frac{\pi}{4} - \frac{x}{2}$
28	**c**	$A^2 = 2A$ $\Rightarrow \lvert A^2\rvert = \lvert 2A\rvert$ $\Rightarrow \lvert A\rvert^2 = 2^3\lvert A\rvert$ as $\lvert kA\rvert = k^n\lvert A\rvert\ for\ a\ matrix\ of\ order\ n$ $\Rightarrow$ either $\lvert A\rvert = 0\ or\ \lvert A\rvert = 8$ But A is non-singular matrix $\therefore \lvert A\rvert = 8^2 = 64$
29	**b**	$f'(x) = 1 - sin\ x \Rightarrow f'(x) > 0\ \ \forall x \in R$ $\Rightarrow$ no value of b exists
30	**c**	$a = b - 2\ and\ b > 6$ $\Rightarrow (6,8) \in R$
31	**a**	$f(x) = \begin{cases} \frac{x}{-x} = -1\ , & x < 0 \\ -1\ , & x \geq 0 \end{cases}$ $\Rightarrow f(x) = -1\ \forall\ x \in R$ $\Rightarrow f(x) is\ continous\ \forall\ x \in R\ as\ it\ is\ a\ constant\ function$
32	**b**	$kA = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$ $\Rightarrow k = -6, a = -4\ and\ b = -9$
33	**d**	Corner points of feasible region — $Z = 30x + 50y$ (5,0) — 150 (9,0) — 270 (0,3) — 150 (0,6) — 300 Minimum value of Zoccurs at two points
34	**c**	$f'(x) = \frac{-2x^2-10x+100}{\sqrt{100-x^2}}$ $f'(x) = 0 \ \Rightarrow \mathrm{x} = -10$ or 5 , But $x > 0 \Rightarrow \mathrm{x} = 5$ $f''(x) = \frac{2x^3-300x-1000}{(100-x)^{\frac{3}{2}}} \Rightarrow f''(5) = \frac{-30}{\sqrt{75}} < 0$ $\Rightarrow$ Maximum area of trapezium is $75\sqrt{3}\ cm^2$ when x = 5
35	**d**	$(I - A)^3 - 7A = \ I + A + 3A + 3A - 7A \ = I$
36	**c**	$\frac{-\pi}{2} < y < \frac{\pi}{2}$

37	**b**	As every per-image $x \in$ A has a unique image y ∈ B $\Rightarrow f$ is injective function
38	**b**	$\lvert A\rvert = 7$, $adjA = \begin{bmatrix} 2 & -2 \\ 1 & 3 \end{bmatrix}$ $\therefore 14A^{-1} = \begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$
39	**b**	$y = x^3 - 11x + 5 \Rightarrow \frac{dy}{dx} = 3x^2 - 11$ Slope of line $y = x - 11$ is $1 \Rightarrow 3x^2 - 11 = 1 \Rightarrow x = \pm 2$ ∴ point is (2, -9) as (-2, 19) does not satisfy given line
40	**c**	$A^2 = 3I$ $\Rightarrow \begin{bmatrix} \alpha^2 + \beta r & 0 \\ 0 & \beta r + \alpha^2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \Rightarrow 3 - \alpha^2 - \beta r = 0$
		Section C
41	**a**	As Z is maximum at (30, 30) and (0, 40) $\Rightarrow 30a + 30b = 40b \Rightarrow b - 3a = 0$
42	**b**	$y = mx + 1 \ldots..(1)$ and $y^2 = 4x \ldots\ldots(2)$ Substituting (1) in (2) : $(mx + 1)^2 = 4x$ $\Rightarrow m^2x^2 + (2m - 4)x + 1 = 0 \ldots\ldots..(3)$ As line is tangent to the curve ⇒ line touches the curve at only one point $\Rightarrow (2m - 4)^2 - 4m^2 = 0 \Rightarrow m = 1$
43	**c**	Let $f(x) = [x(x-1) + 1]^{\frac{1}{3}}$, $0 \leq x \leq 1$ $f'(x) = \frac{2x-1}{3(x^2-x+1)^{\frac{2}{3}}}$ let $f'(x) = 0 \Rightarrow x = \frac{1}{2} \in [0,1]$ $f(o) = 1, f\left(\frac{1}{2}\right) = \left(\frac{3}{4}\right)^{\frac{1}{3}}$ and $f(1) = 1$ ∴ Maximum value of $f(x)$ is 1
44	**b**	Feasible region is bounded in the first quadrant
45	**d**	$\lvert A\rvert = 2 + 2sin^2\theta$ As $-1 \leq \sin\theta \leq 1, \forall\, 0 \leq \theta \leq 2\pi$ $\Rightarrow 2 \leq 2 + 2sin^2\theta \leq 4 \Rightarrow \lvert A\rvert \in [2,4]$
46	**d**	Fuel cost= $k(speed)^2$ $\Rightarrow 48 = k.16^2 \Rightarrow k = \frac{3}{16}$
47	**b**	Total cost of running train (let C) =$\frac{3}{16}v^2t + 1200t$ Distance covered = 500km $\Rightarrow$ time $= \frac{500}{v} hrs$ Total cost of running train 500 km= $\frac{3}{16}v^2\left(\frac{500}{v}\right) + 1200\left(\frac{500}{v}\right)$ $\Rightarrow C = \frac{375}{4}v + \frac{600000}{v}$
48	**c**	$\frac{dC}{dv} = \frac{375}{4} - \frac{600000}{v^2}$ Let $\frac{dC}{dv} = 0 \Rightarrow v = 80$ km/h
49	**c**	Fuel cost for running 500 km $\frac{375}{4}v = \frac{375}{4} \times 80 = Rs. 7500/-$
50	**d**	Total cost for running 500 km = $\frac{375}{4}v + \frac{600000}{v}$ $= \frac{375 \times 80}{4} + \frac{600000}{80} = Rs. 15000/-$