Marking Scheme

Applied Mathematics
Term - II
Code-241

Q.N.	Hints/Solutions	Marks
	Section – A	
1	Given, $MR = 9 + 2x - 6x^2$ $TR = \int (9 + 2x - 6x^2) dx$ $TR = 9x + x^2 - 2x^3 + k$ When $x = 0$, $TR = 0$, so $k = 0$ $TR = 9x + x^2 - 2x^3$ $\Rightarrow \quad px = 9x + x^2 - 2x^3$ $\Rightarrow \quad p = 9 + x - 2x^2$ which is the demand function **OR** $TC = \int (50 + \frac{300}{x+1}) dx$ $TC = 50x + 300 \log\|x + 1\| + k$ If $x = 0, TC = ₹2000$ So $\quad 2000 = 300(\log 1) + k \quad \Rightarrow k = 2000$ So $\quad TC = 50x + 300 \log(x + 1) + 2000$	 1 1 1 1
2	$R = ₹600$ $i = \frac{0.08}{4} = 0.02$ Present value of perpetuity $= P = \frac{R}{i}$ $\Longrightarrow P = \frac{600}{0.02} = ₹30{,}000$	 1 1
3	$r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$ $= \left(1 + \frac{0.08}{4}\right)^4 - 1$ $= (1.02)^4 - 1 = 0.0824 \quad or \quad 8.24\%$ So effective rate is 8.24% compounded annually. **OR** Present value of ordinary annuity $= R\left(\frac{1-(1+r)^{-n}}{r}\right)$ $= 1000\left(\frac{1-(1.06)^{-5}}{0.06}\right)$ $= 1000\left(\frac{1-0.7473}{0.06}\right) = ₹4211.67$	1 1 1 1
4	$\boldsymbol{E}(\bar{X}) = 60kg$	1

	Standard deviation of $\bar{X} = SE(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{9}{6} = 1.5\ kg$	1

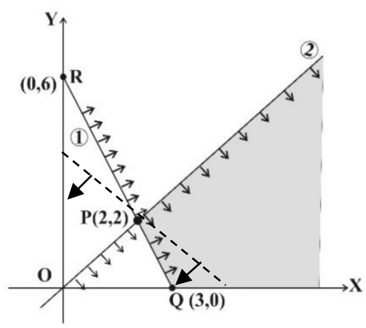
5

Year	Y	3 yearly moving total	3 yearly moving average(Trend) (in ₹ lakh)
2016	25	---	---
2017	30	87	29
2018	32	102	34
2019	40	117	39
2020	45	135	45
2021	50	---	---

1M for 3-yearly moving totals

1M for 3-yearly moving average

6



Corner Point	Z=3x+2y
P (2, 2)	10
Q (3, 0)	9

1

The smallest value of Z is 9. Since the feasible region is unbounded, we draw the graph of $3x + 2y < 9$. The resulting open half plane has points common with feasible region, therefore Z = 9 is not the minimum value of Z. Hence the optimal solution does not exist. 1

Section –B

7

Substituting, $p_0 = ₹48$ in $p = x^2 + 4x + 3$

We get $x_0 = 5$ 1

$PS = p_0 x_0 - \int_0^{x_0} g(x)dx$

$= 48 \times 5 - \int_0^5 (x^2 + 4x + 3)dx$ 1

$= 240 - \left[\frac{x^3}{3} + 2x^2 + 3x\right]_0^5 = ₹133.33$ 1

8

<table>
<tr><th>Year</th><th>Quarters</th><th>Y</th><th>4-Quarterly Moving Total</th><th>4 Quarterly Moving average (Centered) (in ₹crore)</th></tr>
<tr><td rowspan="4">2018</td><td>Q_1</td><td>12</td><td rowspan="12">64
70
72
74
76
85
93
103
117</td><td>--</td></tr>
<tr><td>Q_2</td><td>14</td><td>--</td></tr>
<tr><td>Q_3</td><td>18</td><td>16.75</td></tr>
<tr><td>Q_4</td><td>20</td><td>17.75</td></tr>
<tr><td rowspan="4">2019</td><td>Q_1</td><td>18</td><td>18.25</td></tr>
<tr><td>Q_2</td><td>16</td><td>18.75</td></tr>
<tr><td>Q_3</td><td>20</td><td>20.125</td></tr>
<tr><td>Q_4</td><td>22</td><td>22.25</td></tr>
<tr><td rowspan="4">2020</td><td>Q_1</td><td>27</td><td>24.50</td></tr>
<tr><td>Q_2</td><td>24</td><td>27.5</td></tr>
<tr><td>Q_3</td><td>30</td><td>--</td></tr>
<tr><td>Q_4</td><td>36</td><td>--</td></tr>
</table>

$1\frac{1}{2}$ for 4 quarterly moving totals

$1\frac{1}{2}$ for 4 Quarterly moving average (Centered)

The trend value are given by 4 quarterly centered moving average.

OR

$Year$	Y	$X = Year - 2017$	X^2	XY
2014	26	-3	9	-78
2015	26	-2	4	-52
2016	44	-1	1	-44
2017	42	0	0	0
2018	108	1	1	108
2019	120	2	4	240
2020	166	3	9	498
$\sum Y = 532$			$\sum X^2 = 28$	$\sum XY = 672$

1

$a = \frac{\sum Y}{n} = \frac{532}{7} = 76,$ $\qquad b = \frac{\sum XY}{\sum X^2} = \frac{672}{28} = 24$

1

$Y_c = a + bX,\ Y_c = 76 + 24X$

1

Estimated sales = Y_c for 2023 = 76 + 24 × 6 = ₹220 lacs

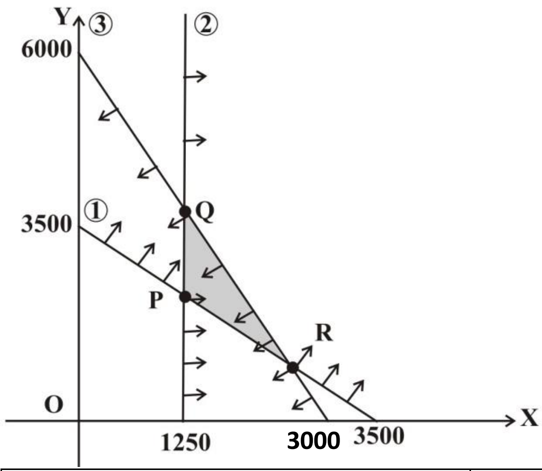
9

Define Null hypothesis H_0 and alternate hypothesis H_1 as follows:

$H_0: \mu = 0.50\ mm$

$H_1: \mu \neq 0.50\ mm$

1

	Thus a two-tailed test is applied under hypothesis H_0, we have $t = \frac{\bar{X}-\mu}{S}\sqrt{n-1} = \frac{0.53-0.50}{0.03} \times 3 = 3$ Since the calculated value of $t = 3$ does not lie in the internal $-t_{0.025}$ to $t_{0.025}$ i.e., -2.262 to 2.262 for 10-1= 9 degree of freedom So we Reject H_0 at 0.05 level. Hence we conclude that machine is not working properly.	1 1
10	We know CAGR=$\left[\left(\frac{FV}{IV}\right)^{\frac{1}{n}} - 1\right] \times 100$, where, IV= Initial value of investment FV=Final value of investment $\Rightarrow 8.88 = \left[\left(\frac{25000}{15000}\right)^{\frac{1}{n}} - 1\right] \times 100 \quad \Rightarrow 0.0888 = \left(\frac{5}{3}\right)^{\frac{1}{n}} - 1$ $\Rightarrow 1.089 = (1.667)^{\frac{1}{n}}$ $\Rightarrow \frac{1}{n}\log(1.667) = \log(1.089) \quad \Rightarrow n(0.037) = 0.2219$ $\Rightarrow n = 5.99 \approx 6 \; years$	1 1 1
Section –C		
11	Let the company produces x and y gallons of alkaline solution and base oil respectively, also let C be the production cost. Min $C = 200x + 300y$ subject to constraints: $x + y \geq 3500 \ldots.(1)$ $x \geq 1250 \quad \ldots.(2)$ $2x + y \leq 6000\ldots(3)$ $x, y \geq 0$ 	$1\frac{1}{2}$ $1\frac{1}{2}$

Corner Points	$C = 200x + 300y$
P(1250, 2250)	₹9,25,000

	<table><tr><td>Q(1250, 3500)</td><td>₹13,00,000</td></tr><tr><td>R(2500, 1000)</td><td>₹8,00,000</td></tr></table> Minimum cost is 8,00,000 when 2500 gallons of alkaline solutions & 1000 gallons of base oil are manufactured.	1
12	The amount of sinking fund S at any time is given by $S = R\left[\frac{(1+i)^n-1}{i}\right]$ Where $R = Periodic\ payment$, $i = Interest\ per\ period$, $n = number\ of\ payments$ S= Cost of machine – Salvage value = 50,000-5000 = ₹45,000 $i = \frac{8\%}{4} = 0.02$ $\Longrightarrow \quad 45000 = R\left[\frac{(1+.02)^{40}-1}{0.02}\right]$ $\Longrightarrow \quad 45000 = R\left[\frac{2.208-1}{0.02}\right]$ $\Longrightarrow \ R = \frac{900}{1.208} \Longrightarrow R =$ ₹745.03	1 $1\frac{1}{2}$ $1\frac{1}{2}$
13.	Amortized Amount i.e., P= Cost of house-Cash down payment P= 15,00,000 – 4,00,000 = ₹11,00,000 $i = \frac{0.09}{12} = 0.0075$ $n = 10 \times 12 = 120$ EMI = $R = \frac{P}{a_{n\neg i}}$, $R = \frac{P \times i}{1-(1+i)^{-n}}$ $= \frac{11,00,000 \times 0.0075}{1-(1.0075)^{-120}} = \frac{8250}{1-0.4079}$ $= \frac{8250}{0.5921} =$ ₹13933.5 Total interest paid $= nR - R = 13933.5 \times 120 - 11,00,000$ =₹5,72,020 **OR** Face value of bond, F = ₹2000 Redemption value C = 1.05 × 2000 = ₹2100 Nominal rate =8% $R = C \times i_d = 2000 \times 0.08 =$ ₹160 Number of periods before redemption i.e., n = 10 Annual yield rate, i =10% or 0.1 Purchase price $V = R\left[\frac{1-(1+i)^{-n}}{i}\right] + C(1+i)^{-n}$ $= 160\left[\frac{1-(1+0.1)^{-10}}{0.1}\right] + 2100(1+0.1)^{-10}$ $= 160 \times 6.14 + 2100 \times 0.3855$ $= 982.4 + 809.6 = 1792$	 1 1 1 1 $1\frac{1}{2}$ $1\frac{1}{2}$

	Thus present value of the bond is ₹1792.	1
14 a) b)	**Case Study** $\because \dfrac{dx}{dt} \propto x, \therefore \dfrac{dx}{dt} = -kx$ $\Rightarrow \int \frac{dx}{x} = \int -k\,dt \quad \Rightarrow \log x = -kt + c$ $\Rightarrow x = e^{-kt+C} \quad \Rightarrow x = \lambda e^{-kt}$ Let $x = x_0\ at\ t = 0$ $\because\ x_0 = \lambda \quad \Rightarrow x = x_0 e^{-kt}$ where $x_0 = \text{original quantity}$ $x = x_0\, e^{-kt} \quad .....(1)$ Now, $\frac{x_0}{2} = x_0 e^{-5k}$ ($\because$ half life = 5 hours) $\Rightarrow e^{-5k} = \frac{1}{2} \quad \Rightarrow e^k = 2^{\frac{1}{5}}$ The quantity of propofol needed in a 50 Kg adult at the end of 2 hours = 50 × 3 = 150 mg $\Rightarrow\ 150 = x_0 e^{-2k}$ [using… (1)] $\Rightarrow x_0 = 150\ e^{2k} \ \Rightarrow\ x_0 = 150\,(e^k)^2$ $\Rightarrow x_0 = 150(2^{\frac{1}{5}})^2$ = 150 × 1.3195 = 197.93 mg	 1 1 1 1