Series : PTS/17

Code No. 13/1/2

Candidates must write the Code on the title page of the answer-book.

Roll No. | 2 | 1 | 3 | 1 | 2 | | |

PLEASURE TEST SERIES XII – 17

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Time Allowed : 180 Minutes **Max. Marks : 100**

General Instructions :

(a) *All questions are compulsory.*

(b) *The question paper consist of* ***29 questions*** *divided into* ***three sections A, B and C****. Section A comprises of* ***10 questions of one mark*** *each, section B comprises of* ***12 questions of four marks*** *each and section C comprises of* ***07 questions of six marks*** *each.*

(c) *All questions in Section A are to be answered in one word, one sentence or as per* ***the exact requirement*** *of the question.*

(d) *There is no overall choice. However,* ***internal choice*** *has been provided in* ***04 questions of four marks each*** *and* ***02 questions of six marks each****. You have to attempt only one of the alternatives in all such questions.*

(e) *Use of calculators in not permitted. You may ask for logarithmic tables, if required.*

SECTION – A

Question numbers 01 to 10 carry 1 mark each.

Q01. Evaluate : $\tan^{-1}\left(\frac{1}{2}\tan 2\theta\right)+\tan^{-1}(\cot\theta)+\tan^{-1}(\cot^3\theta),\ \frac{\pi}{4}\le\theta\le\frac{\pi}{2}$.

Q02. Find the value of $\int\frac{\sec x\tan x+\sec^2 x}{(\sec x+\tan x)^n}dx$.

Q03. Find the value of x, if $\begin{pmatrix}5x+y & -y\\ 2y-x & 3\end{pmatrix}=\begin{pmatrix}4 & 1\\ -3 & 3\end{pmatrix}$.

Q04. Write the direction cosines of the line equally inclined to the three coordinate axes.

Q05. If $\vec{a}$ is a unit vector and $(\vec{b}-\vec{a}).(\vec{b}+\vec{a})=80$ then, find $|\vec{b}|$.

Q06. Let * be a binary operation on N given by a*b = HCF(a, b); a, b$\in$N. Write the value of 16*4.

Q07. Write the value of the determinant : $\begin{vmatrix}a-b & b-c & c-a\\ b-c & c-a & a-b\\ c-a & a-b & b-c\end{vmatrix}$.

Q08. For what value of p, the line : $\frac{x-2}{9}=\frac{y-1}{p}=\frac{z-3}{6}$ is perpendicular to $3x-y-2z=7$.

Q09. Write the value of $\int_0^{\frac{1}{\sqrt{2}}}\frac{1}{\sqrt{1-x^2}}dx$.

Q10. If A is a matrix of order '2 by 3' and B is of '3 by 5', what is the order of $(AB)^T$.

SECTION – B

Question numbers 11 to 22 carry 4 marks each.

Q11. Everyone wants to be a perfect ideal human being. Let us assume that dishonesty is one of the factors that affects our perfectness and perfectness has an inverse square relationship with dishonesty. For any value x of level of dishonesty, we have a unique value y of perfection.
(i) Write down the equation that relates y with x.

(ii) Does this relationship from $x \in (0,\infty)$ to $y \in (0,\infty)$, form a function?

(iii) For what level of dishonesty one can achieve $\left(\frac{1}{4}\right)^{th}$ level of perfection?

(iv) What will be the change in level of perfection when the level of dishonesty changes from 4 to 2?

(v) **What are the values depicted in this question? What are their importance in life?**

Q12. The probability of a student A passing an examination is 3/5 and of student B passing is 4/5. Find the probability of passing the examination by (i) both the students A and B (ii) at least one of the students A and B. **What ideal conditions a student should keep in mind while appearing in an examination?**

OR A can solve 80% of the problems and B can solve 60% of the problems given in the **MathsMania**, a question bank for CBSE Exams. They decide to help each other in solving the remaining sums. If a problem is selected at random, what is the probability that at least one of them will solve the problem? **What value(s) has/ve been indicated in this question?**

Q13. If $f(x) = \begin{cases} \dfrac{e^{ax} - e^{bx}}{x}, x > 0 \\ 4, x = 0 \\ \dfrac{a \log x}{1 - x}, x < 0 \end{cases}$ is continuous function at x = 0 then, find value of 'a' and 'b'.

Q14. Find the intervals in which the function f given by f (x) = sin x + cos x, $0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

OR Show that the curves $2x = y^2$ and $2xy = k$ cut at right angle if $k^2 = 8$.

Q15. Using properties, prove that : $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$.

Q16. The scalar product of $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

Q17. Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ intersect each other. Also find their point of intersection.

Q18. Prove that $\cot^{-1}\left(\dfrac{\sqrt{1+\sin\theta} + \sqrt{1-\sin\theta}}{\sqrt{1+\sin\theta} - \sqrt{1-\sin\theta}}\right) = \dfrac{\theta}{2}; \theta \in \left(0, \dfrac{\pi}{4}\right)$.

OR Simplify : $\sin\left[\tan^{-1}\left(\dfrac{1-x^2}{2x}\right) + \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)\right]$.

Q19. Evaluate : $\displaystyle\int_0^{\pi/2} \frac{\sec^2 x}{(\sec x + \tan x)^n} dx$.

Q20. Evaluate : $\int x \sin^{-1} x \, dx$.

Q21. If $y = \dfrac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1-x^2)\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} - y = 0$. **OR** Find $\dfrac{dy}{dx}$, if $(\cos x)^y = (\sin y)^x$.

Q22. Solve the differential equation : $x\dfrac{dy}{dx} = y - x\tan\left(\dfrac{y}{x}\right)$.

SECTION – C

Question numbers 23 to 29 carry 6 marks each.

Q23. A poor deceased farmer had agriculture land bounded by the curve y = cos x, between x = 0 and the line x = 2π. He had two sons. Now they want to distribute this land in two parts as decided by their deceased father, such that both of them have equal share of land. Find the area of each part. **Do you think that the decision taken by the farmer before his death was good? Justify your answer.**

OR Using integration, find the area of the region : $\{(x, y) : x^2 + y^2 \le 1 \le x + y\}$.

Q24. If a young man rides his motorcycle at 25kmph, he has to spend ₹2 per km on fuel. If he rides it at a faster speed of 40kmph, the fuel cost increases to ₹5 per km. He has ₹100 to spend and wishes to cover the maximum distance within an hour time. Express this as an LPP and solve. **'Speed thrills but kills'. Comment.**

Q25. Coloured balls are distributed in three bags as shown in the following table :

Bag	Colour of the ball		
	Black	White	Red
Bag I	1	2	3
Bag II	2	4	1
Bag III	4	5	3

A bag is selected at random and then two balls are randomly drawn from the selected bag. They happened to be black and red. What is the probability that they came from the bag I?

Q26. Using elementary operations, find the inverse of : $\begin{pmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{pmatrix}$.

Q27. Evaluate $\int_1^3 (3^x - x^2 + 4)dx$, using first principle of integrals.

Q28. If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between then is $\pi/3$.

Q29. Determine the equation of the plane determined by the points A(3,–1, 2), B(5, 2, 4) and C(–1,–1, 6). Also find the distance of P(6, 5, 9) from the plane.

OR Find the image of the point P(1, 2, 3) in the plane containing the points A(1, 1, 0), B(1, 2, 1) and C(–2, 2,–1).