PLEASURE TEST SERIES XI – 01

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Time Allowed : 180 Minutes **(For 2014 - 15)** **Max. Marks : 100**

General Instructions :

(i) The question paper consists of three parts A, B and C. Each question of each part is compulsory.
(ii) Section A (Very Short Answer Type) consists of 06 questions of 1 mark each.
(iii) Section B (Short Answer Type) consists of 13 questions of 4 marks each.
(iv) Section C (Long Answer Type) consists of 7 questions of 6 marks each.
(v) There is no overall choice. However internal choices in **four questions** of section B and **two questions** of section C have been provided.
(vi) Use of calculator is not permitted.

SECTION – A

(This section contains 06 questions of **one mark** each.)

Q01. If $\tan\theta = \frac{1}{2}$ and $\tan\varphi = \frac{1}{3}$, then what is the value of $(\theta+\varphi)$?

Q02. Three identical dice are rolled. What is the probability that the same number will appear on each of them?

Q03. The intercept of the line 2x + 3y – k = 0 on the x-axis is the value of $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$. What is the value of k?

In Questions 4 and 5, choose the correct option(s) from the given four options :

Q04. The solution of the equation $\cos^2\theta+\sin\theta+1=0$ lies in the interval :
(A) $\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$ (B) $\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$ (C) $\left(\frac{3\pi}{4},\frac{5\pi}{4}\right)$ (D) $\left(\frac{5\pi}{4},\frac{7\pi}{4}\right)$

Q05. For a complex number z, the value of arg.z + arg.$\bar{z}$, $z\neq 0$ is :
(A) 0 (B) π (C) $-\pi$ (D) ∞

Q06. What is the contra-positive of the following statement :
"If a number is divisible by 6, then it is divisible by 3"?

SECTION – B

(This section contains 13 questions of **four marks** each.)

Q07. If $A'\cup B = U$, show by using laws of algebra of sets that $A\subset B$, where A' denotes the compliment of A and U is the universal set.

Q08. Solve : cos A + cos B = 0 = sin A + sin B, then prove that cos 2A + cos 2B = – 2 cos (A + B).

OR If $\cos x = \frac{1}{7}$ and $\cos y = \frac{13}{14}$, x, y being acute angles, prove that x – y = 60°.

Q09. Using principle of mathematical induction, show that $2^{3n}-1$ is divisible by 7 for all $n\in N$.

Q10. Simplify $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)$ and then write its conjugate as well.

OR Write $z=-4+i4\sqrt{3}$ in the polar form.

Q11. Solve the system of linear inequations and represent the solution on the number line :

$$3x-7>2(x-6) \text{ and } 6-x>11-2x.$$

Q12. If $a+b+c\neq 0$ and $\frac{b+c}{a},\frac{c+a}{b},\frac{a+b}{c}$ are in AP, prove that $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are also in AP.

Q13. A school decided to award its students who were 100% regular in the 'attendance'. Find the number of permutations of all letters of word 'attendance' such that all the vowels are never together. Do you think being regular in school helps in overall development of the student? Justify your answer.

OR A team of 4 students is to be sent for a competition. Twelve students confirmed their availability for the same. But from the past experiences, it was observed that five students, who had confirmed their availability, were not true to their work, they would always be late in doing the work assigned to them. So it was decided to not include them. In how many ways can the team of students be selected for the competition? Which value system these five students need to acquire?

Q14. A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point (1, 5). Find the equation of this line.

Q15. Find the coordinates of the point R which divides the join of the points P(0, 0, 0) and $Q(4, -1, -2)$ in the ratio 1:2 externally and verify that P is the mid-point of RQ.

Q16. The function f is defined by $f(x) = \dfrac{3-x}{3+4x}$. Differentiate $f(x)$ with respect to x by first principle.

Q17. Find the length of the major and minor axes, the coordinates of foci, the vertices, the eccentricity and the length of the latus-rectum of the ellipse $\dfrac{x^2}{169} + \dfrac{y^2}{144} = 1$.

OR A beam is supported at its ends by supports which are 12metres apart. Since the load is concentrated at its centre, there is a deflection of 3cm at the centre and the deflected beam is in the shape of a parabola. How far from the centre is the deflection 1cm?

Q18. What is the probability that

(i) a non-leap year have 53 Sundays (ii) a leap year have 53 Fridays

(iii) a leap year have 53 Sundays and 53 Mondays.

Q19. Let $f(x) = x^2$ and $g(x) = \sqrt{x}$ be two functions defined over the set of non-negative real numbers. Find the followings :

(i) $(f+g)(4)$ (ii) $(f-g)(9)$ (iii) $(f.g)(4)$ (iv) $\left(\dfrac{f}{g}\right)(9)$.

SECTION – C

(This section contains 7 questions of **six marks** each.**)**

Q20. Prove that : $\sin A \sin(60^\circ - A)\sin(60^\circ + A) = \dfrac{1}{4}\sin 3A$.

Q21. Find the ratio of the fourth term from the beginning to the fifth term from the end in the binomial expansion of $\left(\dfrac{x^3}{3} - \dfrac{3}{x^2}\right)^{10}$.

Q22. Find the mean, variance and standard deviation for the following data:

Class-interval	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

Q23. In a school examination, 80% students passed in Mathematics, 72% passed in Physics and 13% failed in both the subjects despite that they used unfair means in the exam. If 312 students passed in both the subjects, find the total number of students who appeared in the examination.

Q24. First term of an AP is 'a' and S_p is zero, show that the sum of its next q terms is $\dfrac{a(p+q)q}{1-p}$.

OR Find the sum of the series upto n terms : $5 + 11 + 19 + 29 + \ldots + a_{n-1} + a_n$.

Q25. A furniture dealer deals in only two kind of items : tables and stools. He has ₹1500 to invest and a space to store at most 60 pieces. A table costs him ₹750 and a stool costs him ₹150. Formulate this information in the form of inequalities and draw a graph representing the solution of these inequations.

Q26. For the function $f(x) = \begin{cases} a + bx, & \text{if } x < 1 \\ 4, & \text{if } x = 1 \\ b - ax, & \text{if } x > 1 \end{cases}$, it is given that $\lim\limits_{x \to 1} f(x) = f(1)$. Find the possible values of 'a'. **OR** Evaluate : $\lim\limits_{x \to 0} \dfrac{\sin x - 2\sin 3x + \sin 5x}{\log(1 + \tan^3 x)}$.

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Q01. $\pi/4$ **Q02.** 1/36 **Q03.** $k=1$ **Q04.** D **Q05.** A

Q06. If a number is not divisible by 3, then it is not divisible by 6.

Q07. $B = B\cup\varphi = B\cup(A\cap A') = (B\cup A)\cap(B\cup A')$

$= (B\cup A)\cap(A'\cup B) = (B\cup A)\cap U$ [Given $(A'\cup B) = U$

$= (B\cup A) \quad \Rightarrow A\subset B$.

Q08. Given cos A + cos B = 0 and sin A + sin B = 0 $\Rightarrow$ $(\cos A + \cos B)^2 - (\sin A + \sin B)^2 = 0$
Now simplify to obtain the desired result.

OR $\cos x = \dfrac{1}{7} \Rightarrow \sin x = \sqrt{1-\cos^2 x} = \dfrac{4\sqrt{3}}{7}$ and $\cos y = \dfrac{13}{14} \Rightarrow \sin y = \sqrt{1-\cos^2 y} = \dfrac{3\sqrt{3}}{14}$

So, $\cos(x-y) = \cos x\cos y + \sin x\sin y = \left(\dfrac{1}{7}\right)\left(\dfrac{13}{14}\right) + \left(\dfrac{4\sqrt{3}}{7}\right)\left(\dfrac{3\sqrt{3}}{14}\right) = \dfrac{1}{2} \quad \Rightarrow x - y = \pi/3$.

Q09. Let P(n) : $2^{3n} - 1$ is divisible by 7.
P(1) : $2^3 - 1 = 7$ is divisible by 7 $\Rightarrow$ P(1) is true.
Let P(k) be true i.e., '$2^{3k} - 1$ is divisible by 7' then, $2^{3k} - 1 = 7m$, $m\in N$ …(i)
We have $2^{3(k+1)} - 1 = 2^{3k}.2^3 - 1$

$= (2^{3k}-1).8 + 7 = (7m).8 + 7 = 7(8m+1)$ [By (i)

$\Rightarrow$ P(k+1) is true, hence p(n) is true for $n\in N$.

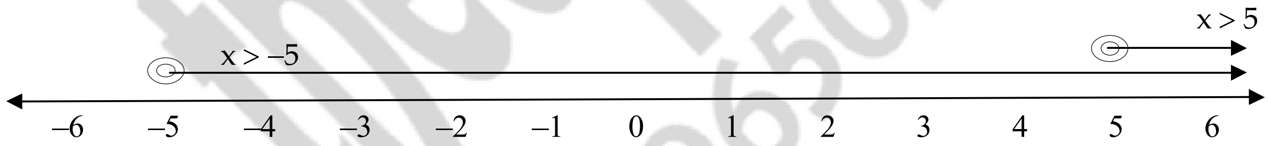
Q10. Let $z = \left(\dfrac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta}\right) \Rightarrow z = \left(\dfrac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta}\right)\times\left(\dfrac{1+\sin\theta + i\cos\theta}{1+\sin\theta + i\cos\theta}\right)$

$\therefore z = \sin\theta + i\cos\theta$ and $\bar{z} = \sin\theta - i\cos\theta$

OR Let $z = -4 + i4\sqrt{3} = r(\cos\theta + i\sin\theta) \quad \Rightarrow r\cos\theta = -4,\ r\sin\theta = 4\sqrt{3} \quad \Rightarrow r^2 = 16 + 48 \Rightarrow r = 8$

And $\tan\theta = -\sqrt{3} \Rightarrow \theta = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$ $\quad\therefore\quad z = -4 + i4\sqrt{3} = 8\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right)$.

Q11. The given inequations are :
$3x - 7 > 2(x-6)$ …(i) and $6 - x > 11 - 2x$ …(ii)
$3x - 2x > -12 + 7 \Rightarrow x > -5$ ...(A) and $-x + 2x > 11 - 6 \Rightarrow x > 5$ ...(B)
From (A) and (B), the solutions of the given system are : $x > 5$.



Q12. Given that $\dfrac{b+c}{a}, \dfrac{c+a}{b}, \dfrac{a+b}{c}$ are in AP.

$\therefore\ 1+\dfrac{b+c}{a},\ 1+\dfrac{c+a}{b},\ 1+\dfrac{a+b}{c}$ will also be in AP.

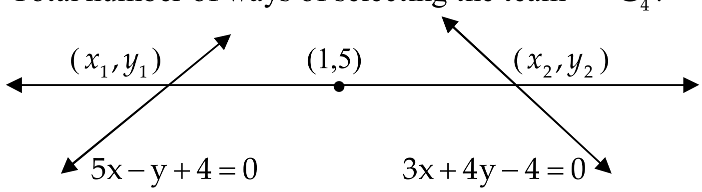
i.e., $\dfrac{a+b+c}{a}, \dfrac{a+b+c}{b}, \dfrac{a+b+c}{c}$ will be in AP.

Since $a+b+c \neq 0$ so, $\dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ will also be in AP.

Q13. Required number of words = $\dfrac{10!}{2!2!2!2!} - \dfrac{7!4!}{2!2!2!2!}$.

OR Total number of ways of selecting the team = ${}^7C_4$.

Q14.



Let the required line intersects $5x - y + 4 = 0$ at (x_1, y_1) and $3x + 4y - 4 = 0$ at (x_2, y_2).

$\therefore\ 5x_1 - y_1 + 4 = 0 \Rightarrow y_1 = 5x_1 + 4$ and $y_2 = \dfrac{4 - 3x_2}{4}$.

So, point of intersection are $(x_1, 5x_1 + 4)$ and $\left(x_2, \dfrac{4 - 3x_2}{4}\right)$.

$\therefore\ \dfrac{x_1 + x_2}{2} = 1$ and $\dfrac{\dfrac{4 - 3x_2}{4} + 5x_1 + 4}{2} = 5$

$\Rightarrow x_1 + x_2 = 2$ and $20x_1 - 3x_2 = 20$.

Solving these equations to get $x_1 = \dfrac{26}{23}, x_2 = \dfrac{20}{23}, y_1 = \dfrac{222}{23}, y_2 = \dfrac{8}{23}$.

So, equation of line is, $y - 5 = \left(\dfrac{\dfrac{222}{23} - 5}{\dfrac{26}{23} - 1}\right)(x - 1) \Rightarrow 107x - 3y = 92$.

Q15. Let the coordinate of R be (x, y, z)

$\therefore\ x = \dfrac{1(4) - 2(0)}{1 - 2} = -4$, $y = \dfrac{1(-1) - 2(0)}{1 - 2} = 1$, $z = \dfrac{1(-2) - 2(0)}{1 - 2} = 2$ $\quad \Rightarrow R(-4, 1, 2)$

Mid-point of QR is $\left(\dfrac{-4 + 4}{2}, \dfrac{1 - 1}{2}, \dfrac{2 - 2}{2}\right)$ i.e., (0, 0, 0). Hence verified.

Q16. $f(x) = \dfrac{3 - x}{3 + 4x} \Rightarrow f(x + \Delta x) = \dfrac{3 - (x + \Delta x)}{3 + 4(x + \Delta x)}$

$\therefore\ f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{\dfrac{3 - (x + \Delta x)}{3 + 4(x + \Delta x)} - \dfrac{3 - x}{3 + 4x}}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{[3 - (x + \Delta x)].[3 + 4x] - [3 - x].[3 + 4(x + \Delta x)]}{\Delta x[3 + 4(x + \Delta x)].[3 + 4x]}$

$\Rightarrow f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{-15\Delta x}{\Delta x[3 + 4(x + \Delta x)].[3 + 4x]} = -\dfrac{15}{(3 + 4x)^2}$.

Q17. Here $a^2 = 169$ and $b^2 = 144 \Rightarrow a = 13, b = 12$.

$\therefore$ Length of major axis = 26, length of minor axis = 24.

Since $e^2 = 1 - \dfrac{b^2}{a^2} = 1 - \dfrac{144}{169} \Rightarrow e = \dfrac{5}{13}$.

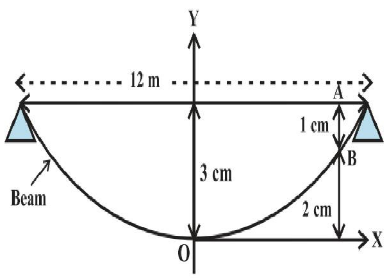
Foci are $(\pm 5, 0)$, vertices are $(\pm 13, 0)$. Also latus-rectum = $\dfrac{2b^2}{a} = \dfrac{288}{13}$.

OR Let the vertex be at the lowest point and the axis vertical. Let the coordinate axis be chosen as shown in Figure. The equation of the parabola takes the form $x^2 = 4ay$. Since it passes through the point (6, 3/100) we have, $6^2 = 4a(3/100) \Rightarrow a = 300$.

Let AB is the deflection of the beam which is 1/100m.

Coordinates of B are (x, 2/100).

Therefore $x^2 = 4(300)(2/100) = 24 \Rightarrow x = 2\sqrt{6}$ m.



Q18. (i) Total number of days in a non leap year = 365 i.e., 52 weeks and 1 day.

$\therefore$ P(53 Sundays) = 1/7

(ii) Total number of days in a leap year = 366 i.e., 52 weeks and 2 days.

So, these two days can be (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday) and (Sunday, Monday).

$\therefore$ P(53 Fridays) = 2/7

(iii) P(53 Sundays and 53 Mondays) = 1/7.

Q19. (i) $(f + g)(4) = f(4) + g(4) = 4^2 + \sqrt{4} = 18$ $\qquad$ (ii) $(f - g)(9) = f(9) - g(9) = 9^2 - \sqrt{9} = 78$

(iii) $(f.g)(4) = f(4).g(4) = 4^2 \times \sqrt{4} = 32$ (iv) $\left(\frac{f}{g}\right)(9) = \frac{f(9)}{g(9)} = \frac{9^2}{\sqrt{9}} = 27$.

Q20. Use sin(A + B) sin(A – B) = $\sin^2 A - \sin^2 B$ in LHS

Q21. Using $T_{r+1} = {}^nC_r x^{n-r} y^r$ for $(x + y)^n$

We have $T_4 = {}^{10}C_3 \left(\frac{x^3}{3}\right)^{10-3} \left(-\frac{3}{x^2}\right)^3 = -\frac{10\times 9\times 8}{3\times 2\times 1} \times \left(\frac{1}{3^4}\right) x^{15} = -\frac{40}{27} x^{15}$

Now 5^{th} term from end = (11 – 5 + 1) = 7^{th} term from beginning.

$T_7 = {}^{10}C_6 \left(\frac{x^3}{3}\right)^{10-6} \left(-\frac{3}{x^2}\right)^6 = \frac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1} \times \frac{3^2}{1} = 1890$.

Q22.

Classes	30-40	40-50	50-60	60-70	70-80	80-90	90-100	
f_i	3	7	12	15	8	3	2	$\sum f_i = 50$
x_i	35	45	55	65	75	85	95	
$d_i = \frac{x_i - 65}{10}$	–3	–2	–1	0	1	2	3	
$f_i d_i$	–9	–14	–12	0	8	6	6	$\sum f_i d_i = -15$
$f_i d_i^2$	27	28	12	0	8	12	18	$\sum f_i d_i^2 = 105$

Mean, $\bar{x} = 65 - \frac{15}{50} \times 10 = 65 - 3 = 62$.

Variance, $\sigma^2 = \left[\frac{105}{50} - \left(-\frac{15}{50}\right)^2\right] \times 10^2 = 201$.

S.D., $\sigma = \sqrt{201} = 14.17$.

Q23. Let M and P denote the number of students passing in Mathematics and Physics respectively. $\therefore$n(M) = 80%, n(P) = 72%. Also number of students passing in at least one subjects, n(M $\cup$ P) = 87%. So, n(M $\cap$ P) = 65% = number of students passing in both the subjects.
Let total number of students be x. According to question, 65% of x = 312 $\Rightarrow$ x = 480.

Q24. **OR** $\frac{n(n+2)(n+4)}{3}$

Q25. $5x + y \le 10$, $x + y \le 60$, $x \ge 0$, $y \ge 0$ where x : no. of tables and y : no. of stools.

Q26. a = 0, b = 4 **OR** –12.