**Series : PTS/1**

**Code No. 13/9/1**

**Roll No.** | | | | | | | |

Candidates must write the Code on the title page of the answer-book.

- Please check that this question paper contains **3** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of answer-book by the candidate.
- Please check that this question paper contains **29** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 07.00 a.m. From 07.00 a.m. to 07.15 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

# PLEASURE TEST SERIES XII – 01

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**Time Allowed: 180 Minutes** **Max. Marks: 100**

***General Instructions :***

***(a)*** *All questions are compulsory.*

***(b)*** *The question paper consist of **29 questions** divided into **three sections A, B and C**. Section A comprises of **10 questions of one mark** each, section B comprises of **12 questions of four marks** each and section C comprises of **07 questions of six marks** each.*

***(c)*** *All questions in Section A are to be answered in one word, one sentence or as per **the exact requirement** of the question.*

***(d)*** *There is no overall choice. However, **internal choice** has been provided in **04 questions of four marks each** and **02 questions of six marks each**. You have to attempt only one of the alternatives in all such questions.*

***(e)*** *Use of calculators in not permitted. You may ask for logarithmic tables, if required.*

## SECTION A

*Question numbers 1 to 10 carry 1 mark each.*

Q01. A matrix A of order 3 has determinant value 7. What is the value of $|3A|$?

Q02. If $A=[a_{ij}]=\begin{pmatrix}2 & 3 & -5\\ 1 & 4 & 9\\ 0 & 7 & -2\end{pmatrix}$ and $B=[b_{ij}]=\begin{pmatrix}2 & 1 & -1\\ -3 & 4 & 4\\ 1 & 5 & 2\end{pmatrix}$ then, find the value of $a_{22}+b_{21}$.

Q03. Evaluate the determinant: $\begin{vmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{vmatrix}$.

Q04. What is the principal value of $\cos^{-1}\cos\dfrac{2\pi}{3}-\sin^{-1}\sin\dfrac{2\pi}{3}$?

Q05. Show that the points (2, 3, 4), (–1,–2, 1) and (5, 8, 7) are collinear.

Q06. Evaluate the integral of $\int \tan^2\dfrac{x}{2}dx$.

Q07. If $2\hat{i}+4\hat{j}-\hat{k}$ is perpendicular to $3\hat{i}-m\hat{j}-2\hat{k}$, then write the value of $m$.

Q08. If A = $[a_{ij}]$ be a square matrix of order 3 and $C_{ij}$ denotes the cofactor of $[a_{ij}]$ in A. If |A| = 5, then write the value of $a_{31}C_{31}+a_{32}C_{32}+a_{33}C_{33}$.

Q09. Radius of a circle is increasing at the rate of 0.7cm/s. Find the rate of increase of its circumference?

Q10. Find the value of $\mu$ where it is given that $\mu = \hat{i}.(\hat{j} \times \hat{k}) + \hat{j}.(\hat{k} \times \hat{i}) + \hat{k}.(\hat{i} \times \hat{j})$.

## SECTION B

*Question numbers 11 to 22 carry 4 marks each.*

Q11. Let $f(x) = |x + 2|$. Show that $f(x)$ is not differentiable at $x = -2$.

**OR**

Given that for the function $f(x) = x^3 - bx^2 + a\,x,\ x \in [1,3]$, Rolle's Theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$.

Find the values of a and b.

Q12. Write in the simplest form : $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right), x < \pi$.

Q13. If $y = x\log\left(\frac{x}{a+bx}\right)$ then, prove that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

**OR**

Find $\frac{dy}{dx}$ for $(\cos x)^y = (\cos y)^x$.

Q14. On the set R–{–1}, a binary operation is defined by a * b = a + b + ab for all a, b ∈ R–{–1}. Prove that * holds both commutative & associative properties on R–{–1}. Find the identity element and prove that every element of R–{–1}is invertible.

Q15. For any two vectors $\vec{a}$ and $\vec{b}$, show that: $(1+|\vec{a}|^2)(1+|\vec{b}|^2) = (1-\vec{a}.\vec{b})^2 + |\vec{a}+\vec{b}+\vec{a}\times\vec{b}|^2$.

**OR**

If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}+\vec{b}| = |\vec{a}|$, then prove that vector $2\vec{a}+\vec{b}$ is perpendicular to vector $\vec{b}$.

Q16. Evaluate : $\int_0^{\pi} \frac{1}{5+4\cos x}dx$.

**OR**

Evaluate : $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x}dx$.

Q17. Using properties of determinants, prove that: $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = -2(x^3+y^3)$.

Q18. Solve the differential equation : dy/dx + y cot x = 2x + $x^2$ cot x, x = $\pi$/4, y = 1.

Q19. Find the equation of the plane passing through the point (–1, 3, 2) and perpendicular to the each of the planes x + 2y + 3z = 5 and 3x + 3y + z = 7.

Q20. Evaluate : $\int \cos 2\theta \log\left(\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right)d\theta$.

Q21. Obtain the equation of tangent to the curves $x = a\sin^3\theta,\ y = b\cos^3\theta$ at $\theta = \frac{\pi}{4}$.

Q22. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective items?
**Naresh is knowingly producing defective items in his factory with an aim of earning more money. How would you stop him doing that by making him conscious of his wrong act?**

## SECTION C

*Question numbers 23 to 29 carry 6 marks each.*

Q23. One kind of cake requires 200 g of flour and 25 g of fat, and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.
Formulate the above as a linear programming problem and solve it graphically.
Explain the importance of balanced diet in food.

Q24. Using integration, find the area of the region : $\{(x, y) : 9x^2 + y^2 \leq 36,\ 3x + y \geq 6\}$.
What is the importance of integration in life.

Q25. Prove that the image of (3,–2, 1) in the plane 3x – y + 4z = 2 lies on the plane x + y + z + 4 = 0.

**OR**

Find the distance of the point (–1,–5,–10) from the point of intersection of the plane $\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 5$ and the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$.

Q26. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\dfrac{2R}{\sqrt{3}}$. Also find the maximum volume.

Q27. Evaluate : $\int_0^1 \sin^{-1}(x\sqrt{1-x} - \sqrt{x - x^3})\, dx$.

Q28. Assume that the chances of a patient having a heart-attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart-attack by 30% and prescription of certain drug reduces its chances by 25%. At a time, a patient can chose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart-attack. Find the probability that the patient followed a course a course of meditation and yoga.
**Interpret the result & state which of the above stated methods is more beneficial for the patient.**

**OR**

A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e., if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?
**Why do you think that these kind of medical tests must be more accurate?**

Q29. If $A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$ then, find the product AB.
Using this solve the following system of equations :
$3x - 4y + 2z = -1,\ 2x + 3y + 5z = 7,\ x + z = 2$.

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Q01. Use $|kA| = k^n A$, where n is order of A. So, $|3A| = 189$. Q02. 1 Q03. 1

Q04. $\frac{2\pi}{3}+\frac{\pi}{3}=\pi$ Q05. Let A(2, 3, 4), B(–1,–2, 1) & C(5, 8, 7). The d.r.'s of AB : 3, 5, 3; the d.r.'s of BC : 6, 10, 6. Since $\frac{3}{6}=\frac{5}{10}=\frac{3}{6}$, i.e., the d.r.'s of AB and BC are proportional so, AB and BC are parallel. But B is a common point so, A, B and C must be collinear. Q06. $2\tan\left(\frac{x}{2}\right)-x+k$

Q07. $m = 2$ Q08. 5 Q09. 4.4cm/s Q10. 3 Q11. OR $a = 11, b = 6$

Q12. $\frac{\pi}{4}-x$ Q13. $\frac{dy}{dx}=\frac{y(\cos x)^y \tan x+(\cos y)^x \log\cos y}{(\cos x)^y \log\cos x + x(\cos y)^x \tan y}$

OR $y = x\left[\log x-\log(a+bx)\right] \Rightarrow y'=\frac{a}{a+bx}+\frac{y}{x}$ ...(i) $\Rightarrow xy'=\frac{ax}{a+bx}+y$

$\Rightarrow xy''+y'=\frac{(a+bx).a-ax(b)}{(a+bx)^2}+y' \Rightarrow xy''=\frac{a^2}{(a+bx)^2}=\left(y'-\frac{y}{x}\right)^2$ [By (i)] $\therefore x^3y''=(xy'-y)^2$

Q14. Identity Element : 0 is the identity element for * defined on R–{–1}.

Also, tnverse of an element a is : $-\frac{a}{a+1} \in R-\{-1\}$ Q16. $\frac{\pi}{3}$ OR $\frac{\pi}{2}$

Q18. $y = x^2+\left(\frac{16-\pi^2}{16\sqrt{2}}\right)$ cosec x Q19. $7x - 8y + 3z + 25 = 0$

Q20. $\frac{\sin 2\theta}{2}\log\tan\left(\frac{\pi}{4}+\theta\right)-\frac{1}{2}\log\sec 2\theta+k$ Q21. $2\sqrt{2}x+2\sqrt{2}y = a+b$

Q22. Required probability = $\frac{29}{20}\times\left(\frac{19}{20}\right)^9$. I would tell Naresh that if he keeps on producing defective items then the good will of his company will suffer in long run which will result in heavy loss in future.

Q23. Maximum no. of cakes are: 30; 20 of kind one & 10 cakes of another kind. Q24. $3(\pi-2)$ sq.units

Q25. Find the image of given point in 1st plane and then show that it satisfies the 2nd plane.
OR The point of intersection is (2,–1, 2) so, required distance is 13units.

Q26. $\left(\frac{4\pi R^3}{3\sqrt{3}}\right)$cubic units Q27. $\frac{\pi}{4}-1$

Q28. Let A : the patient follows a course of meditation and yoga, B : he takes a certain drug.
Then, P(A) = 1/2, P(B) = 1/2. Let E : the patient suffers a heart-attack.
Also P(E|A) = $(70/100).(40/100)$, P(E|B) = $(75/100).(40/100)$.

By Bayes' Theorem, we get : $P(A|E)=\frac{P(E|A)P(A)}{P(E|A)P(A)+P(E|B)P(B)}=\frac{14}{29}$.

**Interpretation of result :** It is evident that if a patient follows a course of meditation and yoga, then he is less likely to get heart-attack. [Since P(B|E) = 15/29.] So, clearly a course of meditation and yoga is more beneficial as compared to the intake of drugs.

OR Let A : the person actually has the disease, $\bar{A}$ : the person doesn't have the disease. Then, P(A) = 0.1%, P($\bar{A}$) = 99.9%. Let E : blood test results in positive. ∴ P(E|A) = 99%, P(E|$\bar{A}$) = 0.5%.

By Bayes' Theorem, we get : $P(A|E)=\frac{P(E|A)P(A)}{P(E|A)P(A)+P(E|\bar{A})P(\bar{A})}=\frac{22}{133}$.

These medical tests must be accurate in order to avoid wrong prescription of medicines and also inaccurate tests may cause unnecessary tension. Q29. $-9I$; $x = 3, y = 2, z = -1$.