Series : PTS/7

Code No. 13/11/1

Candidates must write the Code on the title page of the answer-book.

Roll No. | 2 | 1 | 3 | 1 | 2 | | |

PLEASURE TEST SERIES XII – 07

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Time Allowed: 180 Minutes **Max. Marks: 100**

SECTION – A

Q01. Write the value of x : $\sin\cot^{-1}(1+x)=\cos\tan^{-1}x$.

Q02. Evaluate : $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$.

Q03. If A = {1, 2, 3} and B = {a, b}, write the total number of function from A to B.

Q04. If x, y, z are in geometric progression, evaluate : $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix}$.

Q05. Evaluate : $\vec{A}.\left\{(\vec{B}+\vec{C})\times(\vec{A}+\vec{B}+\vec{C})\right\}$.

Q06. What are the points at which the function $f(x)=\left||x|-1\right|$ is not differentiable?

Q07. Determine the value of 'c' of Rolle's Theorem for the function $f(x)=x^{4/3}$ on $-1\le x\le 1$.

Q08. If D is the mid-point of side BC of a ΔABC, then prove that $\overrightarrow{AB}+\overrightarrow{AC}=2\overrightarrow{AD}$.

Q09. Find the value of k such that the line $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies in the plane $2x-4y+z=7$.

Q10. If $|adj A| = 36$ then, find $|3A^{-1}|$ if A is a square matrix of order 3.

SECTION – B

Q11. A function $f(x)$ is defined as follows :

$$f(x)=\frac{\sin x}{x},\ if\ x\ne 0$$
$$=2,\ if\ x=0$$

Is $f(x)$ continuous at $x=0$? If not, what should be the value of $f(x)$ at $x=0$ so that $f(x)$ becomes continuous at $x=0$?

Q12. Evaluate : $\int\frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x}dx$. **OR** Evaluate : $\int\frac{x^2}{(a+bx)^2}dx$.

Q13. A plane which is perpendicular to two planes $2x-2y+z=0$ and $x-y+2z=4$, passes through the point (1,–2, 1). Find the distance of the plane from the point (1, 2, 2).

Q14. If $x=\text{cosec}\,[\tan^{-1}\{\cos(\cot^{-1}\sec(\sin^{-1}a))\}]$ and $y=\sec\,[\cot^{-1}\{\sin(\tan^{-1}\text{cosec}(\cos^{-1}a))\}]$, then find a relation between x and y in terms of a.

OR Prove that : $\cot^{-1}\left[2\tan\left(\cos^{-1}\frac{8}{17}\right)\right]+\tan^{-1}\left[2\tan\left(\sin^{-1}\frac{8}{17}\right)\right]=\tan^{-1}\left(\frac{300}{161}\right)$.

Q15. Evaluate : $\int\frac{\cos 5x+\cos 4x}{1-2\cos 3x}dx$.

Q16. Evaluate : $\int_{1/e}^{e^2}\left|\frac{\log_e x}{x}\right|dx$.

Q17. Find the intervals in which $f(x)=xe^{x(1-x)}$ is **(i)** increasing, and **(ii)** decreasing.

Q18. Let A be the set of all students of class XII in a school and R be the relation having the same sex (*i.e.*, male or female) on set A, then prove that R is an equivalence relation. Do you think, co-education may be helpful in child development and why?

Q19. The probability of a man hitting a target is 1/4. How many times must he fire so that the probability of his hitting the target at least once is more than 2/3?
In recent past, it has been observed that India has done quite well (as compared to other sports) at various International Shooting Contests. What may be the reasons for this?

Q20. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{k} - \hat{i}$. If $\vec{d}$ is a unit vector such that $\vec{a}$ is perpendicular to $\vec{d}$ and $[\vec{b}\ \vec{c}\ \vec{d}] = 0$ then, find the vector $\vec{d}$.

OR Anisha walks 4km towards west, then 3km in a direction 60° east of north and then she stops. Determine her displacement with respect to the initial point of departure.

Q21. Using first principle of derivative, differentiate : log cot $2x$.

OR If $\sqrt{1+x^2} + \sqrt{1+y^2} = a(x-y)$ then, show that $\dfrac{dy}{dx} = \sqrt{\dfrac{1+y^2}{1+x^2}}$.

Q22. For positive numbers x, y and z, find the numerical value of : $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$.

SECTION – C

Q23. In a Legislative assembly election, a political party hired a public relation firm to promote its candidate in three ways : telephone, house calls and letters. The numbers of contacts of each type in three cities A, B & C are (500, 1000, 5000), (3000, 1000, 10000) and (2000, 1500, 4000), respectively. The party paid ₹3700, ₹7200, and ₹4300 in cities A, B & C respectively. Find the costs per contact using matrix method. Keeping in mind the economic condition of the country, which way of promotion is better in your view?

OR Using elementary column operations, find the inverse of matrix $\begin{bmatrix} 0 & 0 & -1 \\ 3 & 4 & 5 \\ -2 & -4 & -7 \end{bmatrix}$.

Q24. If the area enclosed between $y = mx^2$ and $x = my^2$, $(m > 0)$ is 1 sq. unit then, find the value of m.

Q25. By examining the chest X-ray, the probability that T.B. is detected when a person is actually suffering is 0.99. The probability that the doctor diagnosis incorrectly that a person has T.B. on the basis of X-ray is 0.001. In a certain city, 1 in 1000 suffers from T.B. A person is selected at random and is diagnosed to have T.B. What is the probability that he actually has T.B.?
'Tuberculosis (T.B.) is curable.' Comment in only one line.

Q26. For what value of 'a' the volume of parallelopiped formed by $\hat{i} + a\hat{j} + \hat{k}$, $\hat{j} + a\hat{k}$ and $a\hat{i} + \hat{k}$ is minimum? Also determine the volume.

OR Show that the condition that the curves $ax^2 + by^2 = 1$ and $mx^2 + ny^2 = 1$ should intersect orthogonally is given by: $\dfrac{1}{a} - \dfrac{1}{b} = \dfrac{1}{m} - \dfrac{1}{n}$.

Q27. Find the equation of the plane passing through (2, 1, 0), (4, 1, 1), (5, 0, 1). Find a point Q such that its distance from the plane obtained is equal to the distance of point P(2, 1, 6) from the plane and the line joining P and Q is perpendicular to the plane.

Q28. **A)** If $y(t)$ is a solution of $(1+t)dy = (1+ty)dt$ and $y(0) = -1$ then, what is the value of $y(1)$?

B) Write the degree of the differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where c is a positive parameter.

Q29. A farmer owns a field of area 1000m^2. He wants to plant fruit trees in it. He has sum of ₹2400 to purchase young trees. He has the choice of two types of trees. Type A requires 10m^2 of ground per tree and costs ₹30 per tree and, type B requires 20m^2 of ground per tree and costs ₹40 per tree. When full grown, a type A tree produces an average of 20kg of fruits which can be sold at a profit of ₹12 per kg and a type B tree produces an average of 35kg of fruits which can be sold at a profit of ₹10 per kg. How many of each type should be planted to achieve maximum profit when trees are fully grown? What is the maximum profit? 'India is a land of farmers.' Comment.

Q01. $\sin\sin^{-1}\dfrac{1}{\sqrt{2+x^2+2x}}=\cos\cos^{-1}\dfrac{1}{\sqrt{1+x^2}}\Rightarrow x=-\dfrac{1}{2}$

Q02. Let $\text{I}=\int\limits_{-\pi}^{\pi}\dfrac{\cos^2 x}{1+a^x}dx$ …(i). Use $\int\limits_a^b f(x)\,dx=\int\limits_a^b f(a+b-x)\,dx$ to get, $\text{I}=\int\limits_{-\pi}^{\pi}\dfrac{\cos^2(-x)}{1+a^{-x}}dx$ …(ii)

Adding (i) & (ii), we have : $\text{I}=\dfrac{1}{2}\int\limits_{-\pi}^{\pi}\cos^2 x\,dx=\dfrac{1}{2}\times 2\int\limits_0^{\pi}\cos^2 x\,dx=\int\limits_0^{\pi}\left[\dfrac{1+\cos 2x}{2}\right]dx=\dfrac{\pi}{2}$

Q03. Total number of function from A to B is $2^3 = 8$

Q04. As x, y, z are in GP, so $y^2 = xz$ ...(i) . Then apply $\text{C}_1\to\text{C}_1-p\text{C}_2\Rightarrow\text{C}_3\to\text{C}_3-\text{C}_1$. Then expand along C_3 and use (i) to get $\Delta=0$

Q05. $\vec{A}.\{\vec{B}\times\vec{A}+\vec{B}\times\vec{B}+\vec{B}\times\vec{C}+\vec{C}\times\vec{A}+\vec{C}\times\vec{B}+\vec{C}\times\vec{C}\}$

$=\vec{A}.(\vec{B}\times\vec{A})+\vec{A}.(\vec{0})+\vec{A}.(\vec{B}\times\vec{C})+\vec{A}.(\vec{C}\times\vec{A})+\vec{A}.(\vec{C}\times\vec{B})+\vec{A}.(\vec{0})$

$=[\vec{A}\ \vec{B}\ \vec{A}]+[\vec{A}\ \vec{B}\ \vec{C}]+[\vec{A}\ \vec{C}\ \vec{A}]+[\vec{A}\ \vec{C}\ \vec{B}]=0+[\vec{A}\ \vec{B}\ \vec{C}]+0-[\vec{A}\ \vec{B}\ \vec{C}]=0$

Q06. The function $f(x)=\left||x|-1\right|$ is not differentiable at $x = 0$. Also for $x\neq 0$, we have $f(x)=|x-1|$ if $x > 0$ and $f(x)=|-x-1|$ if $x < 0$ which reflects their nature of not being differentiable at $x = 1, -1$ respectively. So, the function $f(x)$ is not differentiable at $x = -1, 0, 1$.

Q07. $c=0$

Q08. Let $\overrightarrow{\text{OA}}=\vec{a}$, $\overrightarrow{\text{OB}}=\vec{b}$, $\overrightarrow{\text{OC}}=\vec{c}$. We have $\overrightarrow{\text{OD}}=\dfrac{\vec{b}+\vec{c}}{2}$.

Now LHS : $\overrightarrow{\text{AB}}+\overrightarrow{\text{AC}}=(\vec{b}-\vec{a})+(\vec{c}-\vec{a})=(\vec{b}+\vec{c}-2\vec{a})=2\left(\dfrac{\vec{b}+\vec{c}}{2}-\vec{a}\right)$

$=2\left(\overrightarrow{\text{OD}}-\overrightarrow{\text{OA}}\right)=2\overrightarrow{\text{AD}}$ = RHS.

Q09. Obtain the coordinates of random point M (*say*) on the given line then, M must satisfy the equation of plane $2x-4y+z=7$. So we get $k = 7$.

Q10. Use $|adj\text{A}| = |\text{A}|^{3-1}$ to find $|\text{A}| = \pm 6$ then $|\text{A}^{-1}| = \pm\dfrac{1}{6}$. So finally $|3\text{A}^{-1}| = 3^3|\text{A}^{-1}| = 27\left(\pm\dfrac{1}{6}\right)=\pm\dfrac{9}{2}$.

Q11. We have RHL = 1. Also $f(0) = 2$. Since RHL $\neq f(0)$ so, $f(x)$ is discontinuous at $x = 0$. In order to make it continuous, the value of $f(x)$ at $x = 0$ should be 1.

Q12. $\text{I}=\int\dfrac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^4 x}dx=\int\dfrac{(\cos^2 x+\cos^4 x)\cos x}{\sin^2 x+\sin^4 x}dx$. Put $\sin x=t\Rightarrow\cos x\,dx=dt$

$\Rightarrow\text{I}=\int\dfrac{[1-t^2+(1-t^2)^2]}{t^2+t^4}dt=\int\left[1+\dfrac{2-4t^2}{t^2+t^4}\right]dt=t+\int\dfrac{2-4t^2}{t^2(1+t^2)}dt$ …(i)

Consider $\dfrac{2-4t^2}{t^2(1+t^2)}=\dfrac{2-4y}{y(1+y)}=\dfrac{A}{y}+\dfrac{B}{1+y}$ where $y=t^2$ so, equation (i) becomes,

$\text{I}=t+\int\left(\dfrac{2}{t^2}-\dfrac{6}{1+t^2}\right)dt=t-\dfrac{2}{t}-6\tan^{-1}t+\text{C}\Rightarrow\text{I}=\sin x-2\operatorname{cosec}x-6\tan^{-1}\sin x+\text{C}$.

OR Put $a+bx=t\Rightarrow x=\dfrac{t-a}{b}\Rightarrow dx=\dfrac{1}{b}dt$. So, $\text{I}=\int\left(\dfrac{t-a}{b}\right)^2\dfrac{1}{t^2}\dfrac{1}{b}dt=\dfrac{1}{b^3}\int\left(1-\dfrac{2a}{t}+\dfrac{a^2}{t^2}\right)dt$

$\Rightarrow\text{I}=\dfrac{1}{b^3}\left[(a+bx)-2a\log(a+bx)-\dfrac{a^2}{a+bx}\right]+\text{C}$

$\Rightarrow\text{I}=\dfrac{x}{b^2}-\dfrac{2a}{b^3}\log|a+bx|-\dfrac{a^2}{b^3(a+bx)}+\text{k}$, where $\text{k}=\text{C}+\dfrac{a}{b^3}$.

Q13. Let the d.r.'s of required plane be A, B, C. Since required plane is perpendicular to the given planes so, $2A-2B+C=0$ and $A-B+2C=0 \Rightarrow \frac{A}{-3}=\frac{B}{-3}=\frac{C}{0}$. So the required equation of plane is : $-3(x-1)-3(y+2)+0(z-1)=0$ i.e., $x+y+1=0$. And its distance from (1, 2, 2) is $2\sqrt{2}$ units.

Q14. $x=y=\sqrt{3-a^2}$ **OR** OPG Vol.1 Q No.08 (*l*)

Q15. See C-30 on Indefinite Integrals Q No.25. Download it from www.theOPGupta.com/ in the section Class XII Advanced Level Questions.

Q16. $I=\int_{1/e}^{e^2}\left|\frac{\log_e x}{x}\right|dx=\int_{1/e}^{1}-\frac{\log_e x}{x}dx+\int_{1}^{e^2}\frac{\log_e x}{x}dx$ ...(i). Consider $\int\frac{\log_e x}{x}dx=\frac{(\log x)^2}{2}$. So by (i), $I=\frac{5}{2}$.

Q17. $f'(x)=e^{x(1-x)}[1+x-2x^2] \Rightarrow x=-\frac{1}{2}, 1.$

$-\infty$ to $-1/2$	$-1/2$ to 1	1 to $+\infty$
–ve	+ ve	–ve

Q18. The relation R is reflexive, symmetric and transitive. Co-education is very helpful because it leads to the balanced development of the children and in future they become good citizens.

Q19. Let p = probability of hitting the target = 1/4. So q = 1 – p = 3/4. Let the man fires 'n' times. According to question, $P(r\ge 1)=1-P(r<1)>\frac{2}{3}\Rightarrow 1-P(0)>\frac{2}{3}\Rightarrow P(0)<\frac{1}{3}$

i.e., ${}^nC_0(1/4)^0(3/4)^{n-0}<1/3\Rightarrow(3/4)^n<1/3$. That is, the least value of n is 4. Better coaching, training and more exposure to the shooters along with good quality equipments are responsible for good show of shooters at international level.

Q20. Let $\vec{d}=x\hat{i}+y\hat{j}+z\hat{k}\Rightarrow x^2+y^2+z^2=1$ …(i). As $\vec{a}\perp\vec{d}\Rightarrow\vec{a}.\vec{d}=0\Rightarrow x=y$ …(ii).
Also $[\vec{b}\ \vec{c}\ \vec{d}]=0\Rightarrow x+y+z=0$ …(iii).

Solving (i), (ii) & (iii), we get : $\vec{d}=\pm\left(\frac{2}{\sqrt{6}}\hat{k}-\frac{1}{\sqrt{6}}\hat{i}-\frac{1}{\sqrt{6}}\hat{j}\right)$.

OR Similar question on Page 03 of OPG Vol.2 Q No.02

Q21. Let $f(x)=\log\cot 2x$. So, $f'(x)=\lim_{h\to 0}\frac{\log\cot 2(x+h)-\log\cot 2x}{h}=-\frac{2\operatorname{cosec}^2 2x}{\cot 2x}$

OR OPG Vol.1 Page 50 Q No. 60

Q22. **M 1 :** $\Delta=\begin{vmatrix}1 & \log_x y & \log_x z\\ \log_y x & 1 & \log_y z\\ \log_z x & \log_z y & 1\end{vmatrix}=\begin{vmatrix}\frac{\log x}{\log x} & \frac{\log y}{\log x} & \frac{\log z}{\log x}\\ \frac{\log x}{\log y} & \frac{\log y}{\log y} & \frac{\log z}{\log y}\\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & \frac{\log z}{\log z}\end{vmatrix}$ [Using $\frac{\log_b p}{\log_b a}=\log_a p$

Take $\log x$, $\log y$ and $\log z$ common from C_1, C_2 and C_3 respectively. We have :

$\Delta=\log x\log y\log z\begin{vmatrix}\frac{1}{\log x} & \frac{1}{\log x} & \frac{1}{\log x}\\ \frac{1}{\log y} & \frac{1}{\log y} & \frac{1}{\log y}\\ \frac{1}{\log z} & \frac{1}{\log z} & \frac{1}{\log z}\end{vmatrix}$ Again take $\frac{1}{\log x}$, $\frac{1}{\log y}$ and $\frac{1}{\log z}$ common from R_1, R_2 and R_3 respectively. We have : $\Delta=\frac{\log x\log y\log z}{\log x\log y\log z}\begin{vmatrix}1&1&1\\1&1&1\\1&1&1\end{vmatrix}=0.$

M 2 : $\Delta=1(1-\log_z y\log_y z)-\log_x y(\log_y x-\log_z x\log_y z)+\log_x z(\log_y x\log_z y-\log_z x)$
$=(1-1)-\log_x y\log_y x+\log_z x\log_x y\log_y z+\log_y x\log_z y\log_x z-\log_x z\log_z x$

$$= -\log_x y\left(\frac{1}{\log_x y}\right) + \log_z x\left(\frac{\log_y z}{\log_y x}\right) + \log_z y\left(\frac{\log_x z}{\log_x y}\right) - \log_x z\left(\frac{1}{\log_x z}\right)$$

$$= -1 + \log_z x \log_x z + \log_z y \log_y z - 1 \Rightarrow \Delta = 0 \quad [\because \log_a b = \frac{1}{\log_b a}, \frac{\log_b p}{\log_b a} = \log_a p].$$

Q23. Cost per Contact : Telephone = ₹0.40, House calls = ₹1.00, Letters = ₹0.50.
Telephone is better medium for promotion as it is cheap.

OR Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 3 & 4 & 5 \\ -2 & -4 & -7 \end{bmatrix}$.

Since A = A I (Using column operations), we have : $\begin{bmatrix} 0 & 0 & -1 \\ 3 & 4 & 5 \\ -2 & -4 & -7 \end{bmatrix} = A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Follow the following steps of properties :

I : $C_1 \to C_1 - C_3$ **II** : $C_2 \to C_2 - C_3$ **III** : $C_2 \to C_2 - C_1$ **IV** : $C_3 \to C_3 + C_1$

V : $C_1 \to C_1 + C_3 - C_2$ **VI** : $C_3 \to C_3 - 3C_2$ **VII** : $C_1 \to C_1 - C_3$ **VIII** : $C_1 \to C_1 - \frac{1}{4}C_3$

IX : $C_3 \to \left(\frac{1}{4}\right)C_3$ **X** : $C_2 \to C_2 + 2C_3$.

Now since $AA^{-1} = I$ so, $A^{-1} = \begin{bmatrix} -2 & 1 & 1 \\ 11/4 & -1/2 & -3/4 \\ -1 & 0 & 0 \end{bmatrix}$.

Q24. On solving given eqs., we have : $x = 1/m$, 0. Required Area $= 1 = \int_0^{1/m} \sqrt{\frac{x}{m}}dx - \int_0^{1/m} mx^2 dx \Rightarrow m = \frac{1}{\sqrt{3}}$.

Q25. Let E : A person is diagnosed to have T.B., A : The person actually has T.B.
So, P(A) = 1/1000, P($\bar{A}$) = 999/1000, P(E|A) = 990/1000, P(E|$\bar{A}$) = 1/1000.
By Bayes' Theorem, $P(A|E) = \frac{P(E|A)P(A)}{P(E|A)P(A) + P(E|\bar{A})P(\bar{A})} = \frac{110}{221}$. Although T.B. is a dangerous disease still it can be cured with proper medicines (DOTS) under the supervision of medical expert.

Q26. Use Scalar Triple Product of vectors to obtain the volume. Volume, $V = a^3 - a + 1 \Rightarrow a = \frac{1}{\sqrt{3}}$. Also the minimum volume is $V = 1 - \frac{2}{3\sqrt{3}}$ cubic units.

OR OPG Vol.1 Page 61 Q No. 10

Q27. Equation of plane : $x + y - 2z = 3$. Note that Q is the Image of point P in the plane. So Q(6, 5, –2).

Q28. **A)** $\frac{dy}{dt} + \frac{-t}{(1+t)}y = \frac{1}{(1+t)}$. I.F. $= (1+t)e^{-t}$ so, solution is $e^{-t}(1+t)y = -e^{-t} + C$. Use $y(0) = -1$ to get : $y = -\frac{1}{(1+t)}$ and then, $y(1) = -\frac{1}{2}$.

B) $y^2 = 2cx + 2c^{3/2}$ …(i). On differentiating we get : $c = yy'$. Put value of c in (i), we have :
$y^2 = 2(yy')x + 2(yy')^{3/2} \Rightarrow \left(\frac{y^2 - 2xyy'}{2}\right)^2 = (yy')^3$. It is clear that degree is 3.

Q29. Z = ₹(240x + 350y). Also $x + 2y \le 100$; $3x + 4y \le 240$; $x, y \ge 0$. Max.Z = ₹20100 at (40, 30).