CLASS XII
SAMPLE PAPER
MATHS

DIFFERENCIATION-I

1-MARK QUESTIONS

1. If $x^y = y^x$ find $\frac{dy}{dx}$
2. Differentiate $\sin x^x$ w.r.t. $\log x$
3. Differentiate $\cos^{-1}\left(\frac{1+x}{1-x}\right)$
4. Differentiate: $\cot^{-1}\left(\frac{1+x}{1-x}\right)$
5. Differentiate: $\log[\tan(\frac{x}{2}+\frac{\pi}{4})]$
6. Differentiate: $\log_{\sin x}\cos x + \tan x^o$
7. If $y=\log(\log_7 x)$ find $\frac{dy}{dx}$

4-MARKS QUESTIONS

1. If $y = \sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$, find $\frac{dy}{dx}$.
2. If $y= e^{a\sin^{-1}x}$ prove that $(1-x^2)y_2 - xy_1 - a^2y=0$.
3. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for $-1<x<1$ show that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$
4. If $f(x) = \left(\frac{3+x}{1+x}\right)^{2+3x}$, find $f'(0)$
5. $y = x^{Sinx} + (Sinx)^x$, Find $\frac{dy}{dx}$
6. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ then prove that $\frac{dy}{dx} = \sqrt{\frac{1-x^2}{1-y^2}}$.
7. Differentiate $Tan^{-1}\frac{2x}{1-x^2}$ w.r.t $Sin^{-1}\frac{2x}{1+x^2}$

8. If $y = \frac{\log x}{x}$, Show that $\frac{d^2y}{dx^2} = \frac{2\log x - 3}{x^3}$

9. Differentiate w.r.t.x: $\tan^{-1}\left(\frac{2x}{1+15x^2}\right) + \tan^{-1}\left(\frac{5ax}{a^2-6x^2}\right)$

10. *Differentiate* $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ *w.r.to* $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

11. If y= $\left[\log\left(x+\sqrt{x^2+1}\right)\right]^2$ prove that $(1+x^2)\,y_2 + xy_1 = 2$.

12. If x = sin t, y = sin pt, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$

13. If $y = e^{ax}\sin bx$, then prove that $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2+b^2)y = 0$

14. Find $\frac{dy}{dx}$, *if* $x = \frac{1}{2}e^t(\cos t + \sin t)$, $y = \frac{1}{2}e^t(\cos t - \sin t)$

15. If $x = a\cos^3\theta$, $y = a\sin^3\theta$, find $\frac{d^2y}{dx^2}$

16. If $x = a(\theta - \sin\theta)$, $y = a(1+\cos\theta)$, find $\frac{d^2y}{d^2x}$ at $\theta = \frac{\pi}{2}$.

17. If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.

18. If $y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$, find $\frac{dy}{dx}$

19. If x = a (cost + t sint) and y = a (sint - t cost), find $\frac{d^2y}{dx^2}$

20. *If* $y = (\sin^{-1}x)^2$, *prove that* $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$

21. If $x^p y^q = (x+y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$ *and* $\frac{d^2y}{dx^2} = 0$

22. If $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$ prove that $\frac{dy}{dx} = -\frac{y}{x}$

23. If $y = (\tan^{-1}x)^2$, prove that $(1+x^2)y_2 + 2x(1+x^2)y_1 = 2$

24. If $x^y = e^{x-y}$ then show that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

25. Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$

26. Differentiate with respect to x: $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$

27. If Cos y = x cos (a + y), Show that $\frac{dy}{dx} = \frac{Cos^2(a+y)}{Sina}$

28. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ w.r.t. $Sin^{-1}\left(\frac{2x}{1+x^2}\right)$

29. Differentiate $Tan^{-1}\frac{2x}{1-x^2}$ *w.r.t* $Sin^{-1}\frac{2x}{1+x^2}$

30. If x=3sint-sin3t, y= 3cost-cos3t find $\frac{d^2y}{dx^2}$

31. If y=sin(m(sin^{-1}x)), prove that (1-x^2)y$_2$ – xy$_1$+m^{2}y=0.

32 .If x = a(cosθ + logtan$\frac{\theta}{2}$)and y = a sinθ then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

33. If y = sin^{-1}x , Prove that $\frac{d^2y}{dx^2} = \frac{x}{\left(1-x^2\right)^{\frac{3}{2}}}$

34.Differentiate: $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$

35. If x=2sint-sin2t, y= 2cost-cos2t find $\frac{d^2y}{dx^2}$

36. If $x = \frac{1+\log t}{t^2}$, $y = \frac{3+2\log t}{t}$, t > 0 prove that $yy_1 - 2xy_1^2 = 1$

37. Find $\frac{dy}{dx}$ if y = $\left(x+\frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

38. If y= $b\tan^{-1}\left(\frac{x}{a}+\tan^{-1}\frac{y}{x}\right)$, find $\frac{dy}{dx}$

39. If $y = x\log\left(\frac{x}{a+bx}\right)$ prove that $\frac{d^2y}{dx^2} = \frac{1}{x}\left(\frac{a}{a+bx}\right)^2$

40. If $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$, $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$, prove that $\frac{dy}{dx} = -\cot 3t$

41. If $y = \log\left(\frac{x}{a+bx}\right)^x$, then prove that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$

42. If (a+bx) $e^{\frac{y}{x}}$ = x then prove that $x^3\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^2$

43. If y= $\left[x+\sqrt{x^2+a^2}\right]^n$ then prove that $\frac{dy}{dx}=\frac{ny}{\sqrt{x^2+a^2}}$.

44. If $y\sqrt{x^2+1}=\log\left[\sqrt{x^2+1}-x\right]$, show that $\left(x^2+1\right)\frac{dy}{dx}+xy+1=0$

45. If f (1) = 4, f `(1) = 2, find the value of derivative of log $f(e^x)$ w.r.t x at the point x = 0.

46. *If* $x^2+y^2=t-\frac{1}{t}, x^4+y^4=t^2+\frac{1}{t^2}$, prove that $xyy_1=1$

47. If $\sqrt{1-x^6}+\sqrt{1-y^6}=a(x^3-y^3)$ prove that $\frac{dy}{dx}=\frac{x^2\sqrt{1-y^6}}{y^2\sqrt{1-x^6}}$

48 .If y = $\cos^{-1}\left(\frac{3\cos x-4\sin x}{5}\right)$, find $\frac{dy}{dx}$.

49. If siny = $\sqrt{x}\sqrt{1-x^4}-x^2\sqrt{1-x}$, find $\frac{dy}{dx}$.

50. If y= $\left[x+\sqrt{x^2+1}\right]^n$ then prove that $(1+x^2)y_2 + xy_1=n^2y$.

51. If x=asin2t(1+cos2t) and y= bcos2t(1-cos2t) , show that $\frac{dy}{dx}$ =b/a at t= $\pi/4$.

52. If x= $\sin\left(\frac{1}{a}\log y\right)$,show that $(1-x^2)\,y_2 - xy_1-a^2y=0$.

53. If y = $\tan^{-1}\left(\frac{1-x}{1+x}\right)-\tan^{-1}\left(\frac{x+2}{1-2x}\right)$, find $\frac{dy}{dx}$.

54. If $y = e^x\tan^{-1}x$, then prove that $(1+x^2)\frac{d^2y}{dx^2}-2(1-x+x^2)\frac{dy}{dx}+(1-x)^2y=0$

55. If y= $\frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right]$ prove that $\frac{dy}{dx}=\frac{1}{a+b\cos x}$,a>b>0