## CBSE Board

## Class X Mathematics

Time: 3 hrs

## General Instructions:

1. All questions are compulsory.
2. The question paper consists of $\mathbf{3 0}$ questions divided into four sections $A, B, C$, and $D$. Section A comprises of $\mathbf{6}$ questions of 1 mark each, Section B comprises of $\mathbf{6}$ questions of 2 marks each, Section C comprises of $\mathbf{1 0}$ questions of 3 marks each and Section D comprises of $\mathbf{8}$ questions of 4 marks each.
3. Use of calculator is not permitted.

## Section A <br> (Questions 1 to 6 carry 1 mark each)

1. What is the probability of getting a prime number when a die is thrown once?

OR
Find the probability that a number selected at random from the numbers $3,4,5, \ldots . . .25$ is prime.
2. The ratio of the length of a pole and its shadow is $\sqrt{3}: 1$. Find the angle of elevation of the Sun.
3. In the adjoining figure, DE is parallel to BC . If $\mathrm{AD}=\mathrm{x}, \mathrm{DB}=\mathrm{x}-2, \mathrm{AE}=\mathrm{x}+2$ and $\mathrm{EC}=\mathrm{x}-$ 1 , find the value of $x$.

4. If $\alpha$ and $\beta$ are the zeroes of the quadratic polynomial $f(x)=x^{2}+2 x+1$, find the value of $\frac{1}{\alpha}+\frac{1}{\beta}$.

## OR

If $p(x)=x^{2}-2 x-3$ then find $p(3)$.
5. Is 0.101100101010 an irrational number? Justify your answer.
6. In figure, $P Q$ and $P R$ are two tangents to a circle with centre 0 . If $\angle Q P R=80^{\circ}$, then find $\angle Q O R$.


## Section B

(Questions 7 to 12 carry 2 marks each)
7. If the zeros of the polynomial $f(x)=x^{3}-3 x^{2}+x+1$ are $a-b, a, a+b$, find $a$ and $b$.
8. Find the H.C.F of 455 and 84 using the division algorithm.

OR
Show that any number of the form $4^{\mathrm{n}}, \mathrm{n} \in \mathrm{N}$ can never end with the digit 0 .
9. In the given figure, $X P$ and $X Q$ are tangents from $X$ to the circle. $R$ is a point on the circle. Prove that $X A+A R=X B+B R$.

10. A bicycle wheel makes 5000 revolutions in moving 11 km . Find the diameter of the wheel.
11. A bridge across a river makes an angle of $45^{\circ}$ with the river bank as shown in the figure. If the length of the bridge across the river is 150 m , what is the width of the river?

12. If $7 \sin ^{2} \theta+3 \cos ^{2} \theta=4$, then find $\theta$ and hence prove that $\sec \theta+\operatorname{cosec} \theta=2+\frac{2}{\sqrt{3}}$

OR
If $\sin 3 A=\cos \left(A-10^{\circ}\right)$ where $3 A$ is acute angle, then find the value of $A$.

## Section C

(Questions 13 to 22 carry 3 marks each)
13. Prove that: $\frac{\sec A+\tan A}{\sec A-\tan A}=\left(\frac{1+\sin A}{\cos A}\right)^{2}$

## OR

Without using tables evaluate:

$$
\left(\frac{\sin 47^{\circ}}{\cos 43^{\circ}}\right)^{2}+\left(\frac{\cos 43^{\circ}}{\sin 47^{\circ}}\right)^{2}-4 \cos ^{2} 45^{\circ}
$$

14. Find the area of the quadrilateral $A B C D$ whose vertices are $A(1,1), B(7,-3), C(12,2)$ and $D(7,21)$ respectively.
15. Solve: $\frac{1}{\mathrm{a}}+\frac{1}{\mathrm{~b}}+\frac{1}{\mathrm{x}}=\frac{1}{\mathrm{a}+\mathrm{b}+\mathrm{x}}$
16. Solve for $x$ and $y$ :

$$
\frac{x}{a}+\frac{y}{b}=2 ; \quad a x-b y=a^{2}-b^{2}
$$

17. Prove that $\frac{3}{2 \sqrt{5}}$ is an irrational number.

## OR

Find the HCF of 96 and 404 by prime factorisation method. Hence, find their LCM.
18. Prema invests a certain sum at the rate of $10 \%$ per annum of interest and another sum at the rate of $8 \%$ per annum to get a yield of Rs. 1640 in one year's time. Next year she interchanges the rates and gets a yield of Rs. 40 less than the previous year. How much did she invest in each type in the first year?
19. If the point $(x, y)$ is equidistant from the points $(a+b, b-a)$ and $(a-b, a+b)$, then prove that $b x=a y$.
20. Find the mean of following distribution by the step deviation method.

| Daily Expenditure: | $100-150$ | $150-200$ | $200-250$ | $250-300$ | $300-350$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. of householders: | 4 | 5 | 12 | 2 | 2 |

## OR

Cards numbered from 1 to 18 are put in a box and mixed thoroughly. One card is drawn at a random. Find the probability that the card drawn bears:
i. a prime number
ii. a factor of 18
iii. a number divisible by 2 and 3
21. In the figure, sides $X Y$ and $Y Z$ and median $X A$ of a triangle $X Y Z$ are proportional to sides $\mathrm{DE}, \mathrm{EF}$ and median DB of $\triangle \mathrm{DEF}$. Show that $\triangle \mathrm{XYZ} \sim \Delta \mathrm{DEF}$.

22. One card is drawn from a pack of 52 cards, each of which is equally likely to be drawn.

Find the probability that the card drawn is
i. either red or king
ii. a red face card
iii. '10' of a black suit

Find the mean for the following distribution:

| Class <br> Interval | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 10 | 6 | 8 | 12 | 5 |

## Section D

(Questions 23 to 30 carry 4 marks each)
23. Construct a triangle similar to $\triangle \mathrm{ABC}$ in which $\mathrm{AB}=4.6 \mathrm{~cm}, \mathrm{BC}=5.1 \mathrm{~cm}, \mathrm{~m} \angle \mathrm{~A}=60^{\circ}$ with scale factor 4 : 5 .
24. In a school, students thought of planting trees in an around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class in which they are studying, e.g., a section of class-I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students? What value can you infer from the planting the trees?
25. In the given figure, $\triangle P Q R$ is right-angled triangle right-angled at $Q$. $D E \perp P R$. Prove $\Delta \mathrm{PQR} \sim \Delta \mathrm{PED}$ and find the lengths of PE and DE if $\mathrm{PD}=3 \mathrm{~cm}, \mathrm{QD}=2 \mathrm{~cm}$ and $\mathrm{QR}=12$ cm .

26. Form a pair of linear equations for the following problem, and find the solution graphically.
"10 students of Class $X$ took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz."

## OR

The co-ordinates of the mid-points of the sides of a triangle are $(1,2),(0,-1),(2,-1)$. Find the co-ordinates of the vertices of the triangle.
27. From a window of a house in a street, $h$ metres above the ground, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are $\alpha$ and $\beta$ respectively. Show that the height of the opposite house is $h(1+\tan \alpha \cdot \cot \beta)$ metres.

A man in a boat rowing away from a light house 100 m high, takes 2 minutes to change the angle of elevation of the top of the light house from $60^{\circ}$ to $45^{\circ}$. Show that the speed of the boat is $50\left(\frac{3-\sqrt{3}}{3}\right) \mathrm{m} / \mathrm{min}$.
28. A tent is of the shape of a right circular cylinder upto a height of 3 metres and conical above it. The total height of the tent is 13.5 metres above the ground. Calculate the cost of painting the inner side of the tent at the rate of Rs. 2 per square metre, if the radius of the base is 14 metres.
29. In the given figure, $\mathrm{AC}=\mathrm{BD}=7 \mathrm{~cm}$ and $\mathrm{AB}=\mathrm{CD}=1.75 \mathrm{~cm}$. Semicircles are drawn as shown in the figure. Find the area of the shaded region. [Take $\left.\pi=\frac{22}{7}\right]$

30. The following table gives production yield per hectare of wheat of 100 farms of a village.

| Production yield | $50-55$ | $55-60$ | $60-65$ | $65-70$ | $70-75$ | $75-80$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Number of farms | 2 | 8 | 12 | 24 | 38 | 16 |

Change the distribution to a 'more than' type distribution and draw ogive.

## OR

An integer is chosen at random from 1 to 50 . Find the probability that the number is:
(i) divisible by 5
(ii) a perfect cube
(iii) a prime number

