Series : PTS/4

Code No. 13/10/2

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Candidates must write the Code on the title page of the answer-book.

PLEASURE TEST SERIES XII – 04

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Time Allowed: 180 Minutes **Max. Marks: 100**

SECTION – A

Q01. Find $|A'|$, if $|adjA| = 225$ *s.t.* $A = [a_{ij}]_{3\times3}$. **Q02.** Find $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}^{-1}$.

Q03. Write the value of $x+y+z$ if $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

Q04. Evaluate $2\cos^{-1}\frac{1}{2} + 3\sin^{-1}\frac{1}{2}$. **Q05.** Evaluate: $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$.

Q06. The contentment obtained after eating x-units of a new dish at a trial function is given by the function $C(x) = x^3 + 6x^2 + 5x + 3$. If the marginal contentment is defined as rate of change of C(x) with respect to the number of units consumed at an instant, then find the marginal contentment when three units of dish are consumed. **Q07.** Write the degree of $\left(\frac{d^2y}{dx^2}\right)^2 - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 1 = 0$.

Q08. If $\vec{a}$ and $\vec{b}$ are two vectors of magnitude 3 and $\frac{2}{3}$ respectively such that $\vec{a}\times\vec{b}$ is a unit vector, write the angle between $\vec{a}$ and $\vec{b}$. **Q09.** Find the projection of $7\hat{i}+\hat{j}-4\hat{k}$ on $2\hat{i}+6\hat{j}+3\hat{k}$.

Q10. Write the distance between the parallel planes $2x - y + 3z = 4$ and $2x - y + 3z = 18$.

SECTION – B

Q11. Show that : $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}] = \sqrt{\frac{x^2+1}{x^2+2}}$.

OR Solve for x : $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$.

Q12. Prove that the function $f : N \to N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

Q13. Two schools A and B decided to award prizes to their students for three values honesty (x), punctuality (y) and obedience (z). School A decided to award a total of ₹11000 for the three values to 5, 4 and 3 students respectively while school B decided to award ₹10700 for the three values to 4, 3 and 5 students respectively. If all the three prizes together amount to ₹2700, then :

i. Represent this situation by matrix equation and form linear equations using matrix multiplication.

ii. Is it possible to solve the system of equations so obtained using matrices?

iii. Which **value** you prefer to be rewarded most and why?

Q14. If $x = a(\theta - \sin\theta)$ and $y = a(1-\cos\theta)$, find $\frac{d^2y}{dx^2}$.

Q15. If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$.

Q16. For what values of a and b, the function $f(x)=\begin{cases} x^2+ax+b, & 0\le x<2 \\ 3x+2, & 2\le x\le 4 \\ 2ax+5b, & 4<x\le 8 \end{cases}$ is continuous on [0, 8]?

OR Differentiate $\tan^{-1}\left[\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ with respect to $\cos^{-1}x^2$.

Q17. Evaluate : $\int \dfrac{x^3+x+1}{x^2-1}dx$. **OR** Evaluate : $\int e^x\left(\dfrac{1-\sin x}{1-\cos x}\right)dx$.

Q18. Evaluate : $\int \dfrac{2x}{(x^2+1)(x^2+2)}dx$. **Q19.** Evaluate : $\int_0^{\pi/4}\log(1+\tan x)\,dx$.

Q20. Let $\vec{a}=4\hat{i}+5\hat{j}-\hat{k}$, $\vec{b}=\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}-\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and satisfying $\vec{d}.\vec{c}=21$.

Q21. Find the distance between the point P(6, 5, 9) and the plane determined by the points A(3,–1, 2), B(5, 2, 4), and C(–1,–1, 6).

OR Find the equation of the perpendicular drawn from the point P(2, 4,–1) to the line $\dfrac{x+5}{1}=\dfrac{y+3}{4}=\dfrac{z-6}{-9}$. Also, write down the coordinates of foot of the perpendicular from P to the line.

Q22. There is a group of 50 people who are patriotic out of which 20 believe in non-violence. Two persons are selected at random out of them, write the probability distribution for the selected persons who are non-violent. Also find the mean of the distribution. **Explain the importance of Non-violence in patriotism.**

SECTION – C

Q23. If $A=\begin{bmatrix}1 & 2 & -3\\ 2 & 3 & 2\\ 3 & -3 & -4\end{bmatrix}$, find A^{-1}. Hence solve : $x+2y-3z=-4$, $2x+3y+2z=2$, $3x-3y-4z=11$.

Q24. Find the equations of tangent & normal to the curve $y=\dfrac{x-7}{(x-2)(x-3)}$ at the point where it cuts *x*-axis.

OR Prove that the radius of the base of right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half that of the cone.

Q25. Find the area of the region enclosed between the two circles $x^2+y^2=1$ and $(x-1)^2+y^2=1$.

Q26. Find the particular solution of : $(x-\sin y)dy+(\tan y)dx=0$; given that $y(0)=0$.

Q27. Find the vector and Cartesian equations of the plane containing the two lines whose vector equations are given as $\vec{r}=2\hat{i}+\hat{j}-3\hat{k}+\lambda(\hat{i}+2\hat{j}+5\hat{k})$ and $\vec{r}=3\hat{i}+3\hat{j}+2\hat{k}+\mu(3\hat{i}-2\hat{j}+5\hat{k})$.

Q28. A cooperative society of farmers has 50 hectares of land to grow two crops A and B. The profit from crops A and B per hectares are estimated as ₹10500 and ₹9000 respectively. To control weeds, a liquid herbicide has to be used for crops A and B at the rate of 20 litres and 10 litres per hectare, respectively. Further not more than 800 litres of herbicide should be used in order to protect fish and wildlife using a pond which collects drainage from this land. Keeping in mind that the protection of fish and other wildlife is more important than earning profit, how much land should be allocated to each crop so as to maximize the total profit? Form an LPP from the above and solve it graphically.

Do you agree with the message that the protection of wildlife is utmost necessary to preserve the balance in environment?

Q29. In a game of gambling, a card from a pack of 52 cards is lost. From the remaining cards, two cards are drawn at random and are found to be hearts. Find the probability of the missing card to be a heart. **Why do you think that gambling is a curse on society?**

OR In a class, 50% of the boys and 10% of the girls have an I.Q. of more than 150. In the same class 60% of the students are boys. If a student is chosen at random and found to have an I.Q. of more than 150, find the probability that the student is a boy. **What measures would you suggest to those having low I.Q.s so that they also are competent enough to succeed in life?**

Q01. $|A'| = \pm 15$ **Q02.** $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ **Q03.** $x + y + z = 1 + (-1) + 0 = 0$ **Q04.** $\frac{7\pi}{6}$

Q05. $\frac{\pi}{4}$ **Q06.** 68 units **Q07.** Degree = 2 **Q08.** $\frac{\pi}{6}$

Q09. $\frac{8}{7}$ **Q10.** $\sqrt{14}$ units **Q11.** **OR** $x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

Q12. See OPG Vol. 2 Solved Mathematics **Sample Question Paper**

Q13. **i.** $\begin{pmatrix} 5 & 4 & 3 \\ 4 & 3 & 5 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11000 \\ 10700 \\ 2700 \end{pmatrix} \Rightarrow \begin{matrix} 5x + 4y + 3z = 11000, \\ 4x + 3y + 5z = 10700, \\ x + y + z = 2700. \end{matrix}$

ii. Since $|A| = -3 \neq 0$ i.e., so A^{-1} exists and the equations have a unique solution.

iii. **Any answer** of the three values with **proper reasoning** will be considered correct.

Q14. $-\frac{1}{4a}\text{cosec}^4\frac{\theta}{2}$ **Q16.** $a = 3$ and $b = -2$ **OR** $-\frac{1}{2}$

Q17. $\frac{x^2}{2} + \frac{3}{2}\log|x-1| + + \frac{1}{2}\log|x+1| + k$ **OR** $-e^x \cot\frac{x}{2} + k$

Q18. $\log|x^2+1| - \log|x^2+2| + k$ **Q19.** $\left(\frac{\pi}{8}\right)\log 2$ **Q20.** $\vec{d} = 7\hat{i} - 7\hat{j} - 7\hat{k}$

Q21. $\frac{6}{\sqrt{34}}$ units **OR** Foot of perpendicular drawn from the point P on the line L is Q(–4, 1,–3).

And, the equation of PQ is : $\frac{x-2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$ **Q22.** Mean = $\frac{196}{245}$

Q23. $A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$; $x = 3, y = -2, z = 1$

Q24. Tangent : $x - 20y - 7 = 0$ and Normal : $20x + y - 140 = 0$ **Q25.** $\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$ sq.units

Q26. $2x = \sin y$

Q27. Vector equation : $\vec{r}.(10\hat{i} + 5\hat{j} - 4\hat{k}) = 37$. And Cartesian equation : $10x + 5y - 4z = 37$

Q28. Max.Z = ₹495000 at (30, 20) **Q29.** $\frac{11}{50}$ **OR** $\frac{15}{17}$.

Good Luck & God Bless You!!!