CODE: 201701

CLASS XII
MATHS

TIME 3:00 HRS **MM: 100**

Date: 21.02.17

Note: Q. No. 1 to 4 carry 1 marks each, Q. No. 5 to 12 carry 2 marks each, Q. No. 13 to 23 carry 4 marks each, Q. No. 24 to 29 carry 6 marks each.

Section-A

Q1. If a line makes angles α, β and γ with the axes respectively, then find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$.

Q2. If $A(adjA) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then find the value of $|A|$.

Q3. If $\sin^{-1} x - \cos^{1} x = \frac{\pi}{6}$, then find the value of *x*.

Q4. Let * be a binary operation on *N* defined by $a * b = a + b + 10$ for all $a, b \in N$. Find the identity element for * in *N*.

Section-B

Q5. Find *X*, if *f* is invertible where $f : [2, \infty) \to X$ and $f(x) = 4x - x^2$.

Q6. If $|\vec{a} \times \vec{b}| = 4$ and $|\vec{a}.\vec{b}| = 2$, then find the value of $|\vec{a}|^2 |\vec{b}|^2$.

Q7. Find the direction ratios of the line which is perpendicular to the lines with direction ratios as $1, -2, -2$ and $0, 2, 1$.

Q8. Form the differential equation of the family of curves $y = a\sin(bx + c)$, a and c being parameters.

Q9. The slope of tangent to the curve at any point is twice the ordinate at that point. The curve passes through the point (4, 3). Determine the equation of the curve.

Q10. Find the maximum value of $z = 10x + 6y$, subject to the constraints $x \geq 0, y \geq 0, x + y \leq 12, 2x + y \leq 20$.

Q11. Find the approximate change in the volume V of a cube of side *x* meters caused by increasing the side by 2%.

Q12. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$. Prove that A & B are independent events.

Section-C

Q13. Find the interval in the function $f(x) = 2x^2 - \log x,\ x \neq 0$ is increasing.

Q14. If $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$, then prove that

$$y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0.$$

OR

Show that the function $f(x) = |x-2|, x \in R$ is continuous but not differentiable at $x = 2$.

Q15. Find the area of the region bounded by the curves $x - y + 2 = 0$ & $x = \sqrt{y}$.

Q16. Evaluate $\int \frac{\tan x}{\sqrt{\sin^4 x + \cos^4 x}}dx.$

OR

Evaluate $\int_0^{\pi/4} \frac{\sin x + \cos x}{\cos^2 x + \sin^4 x}dx.$

Q17. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually perpendicular, then find the values of λ and μ.

OR

Let $\hat{a}$ and $\hat{b}$ are two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then find the angle between $\hat{a}$ and $\hat{b}$.

Q18. Solve the differential equation $x\cos x\left(\frac{dy}{dx}\right) + y(x\sin x + \cos x) = 1$.

Q19. If the line $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z-3}{3}$ intersects the curve $xy = c^2, z = 0$, find the value of c.

Q20. Each of the n urns contains 4 white and 6 black balls. The $(n + 1)^{th}$ urn contains 5 white and 5 black balls. One of the $(n + 1)$ urns is chosen at random and two balls are drawn from it without replacement. Both the balls turn out to be black. If the probability that the $(n + 1)^{th}$ urn was chosen to draw the balls is 1/16. Find the value of n.

Q21. A coin is tossed n times. If the probability of getting head atleast once is greater than 0.8, then find the value of n.

Q22. Two tailors, A and B expense Rs. 15 and Rs. 20 per day respectively. A can stitch 6 shirts and 4 pants while B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to produce (at least) 60 shirts and 32 pants at a minimum expense?

Q23. Show that $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2.$

Section-D

Q24. Evaluate $\int \frac{xe^x}{\sqrt{1+e^x}}dx.$

OR

Evaluate $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}dx,\ a>0.$

Q25. If $\sqrt{x^2+y^2} = ae^{\tan^{-1}\frac{y}{x}},\ a>0$, then find $y''(0)$.

Q26. Solve for x, $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$.

Q27. If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

OR

Find the matrix A satisfying the matrix equation

$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Q28. If the lines $\frac{x-1}{-3} = \frac{y-2}{-2k} = \frac{z-3}{2}$ and $\frac{x-1}{k} = \frac{y-2}{1} = \frac{z-3}{5}$ are perpendicular, find the value of k and hence find the equation of plane containing these lines.

Q29. Find the maximum area of the rectangle whose sides pass through the angular points of a given rectangle of sides a and b.

ANSWERS:

A1. – 1

A2. 10

A3. $\frac{\sqrt{3}}{2}$

A4. – 10

A5. $(-\infty, 4]$

A6. 20

A7. 2, –1, 2

A8. $\frac{d^2y}{dx^2} + b^2 y = 0.$

A9. $y = 3e^{2x-8}$

A10. 104

A11. $0.06\ x^3\ \text{m}^3$

A13. $\left(-\frac{1}{2},0\right)\cup\left(\frac{1}{2},\infty\right)$

A15. $\frac{10}{3}$

A16. $\frac{1}{2}\log\left|\tan^2 x+\sqrt{1+\tan^4 x}\right|+C$ OR $\frac{\pi}{4}-\frac{1}{2\sqrt{3}}\log\left|2-\sqrt{3}\right|$

A17. $\lambda = -3, \mu = 2$ OR $\frac{\pi}{3}$

A18. $xy=\sin x+C\cos x$

A19. $\pm 2\sqrt{2}$

A20. 10

A21. 3

A22. A : 5 days, B: 3 days

A24. $(2x-4)\sqrt{1+e^x}-2\log\left|\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right|+C$ OR $\frac{\pi}{2}$

A25. $-\frac{2}{a}e^{-\pi/2}$

A26. – 1

A27. OR $\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$

A28. $2, -22x+19y+5z=31$

A29. $\frac{1}{2}(a+b)^2$

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