## THRAT MATHENELILSS The Excellence Key...

## CODE:1401-AG-1-FC-TS-22-23

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## General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks,

2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E

## EXAMINATION 2022-23



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| Q. 4 | The angle between the lines whose direction cosines are connected by the relations $l+m+n=0$ and $2 l m+2 n l-m n=0$, is <br> (a) $\frac{\pi}{3}$ <br> (b) $\frac{2 \pi}{3}$ <br> (c) $\pi$ <br> (d)None of these | 1 |
| :---: | :---: | :---: |
| Q. 5 | The value of $\int \sec ^{3} x d x$ will be <br> (a) $\frac{1}{2}[\sec x \tan x+\log (\sec x+\tan x)]$ <br> (b) $\frac{1}{3}[\sec x \tan x+\log (\sec x+\tan x)]$ <br> (c) $\frac{1}{4}[\sec x \tan x+\log (\sec x+\tan x)]\left(\right.$ d) $\frac{1}{8}[\sec x \tan x+\log (\sec x+\tan x)]$ | 1 |
| Q. 6 | An integrating factor of the differential equation $\left(1-x^{2}\right) \frac{d y}{d x}-x y=1$, is <br> (a) $-x$ <br> (b) $-\frac{x}{\left(1-x^{2}\right)}$ <br> (c) $\sqrt{\left(1-x^{2}\right)}$ <br> (d) $\frac{1}{2} \log \left(1-x^{2}\right)$ | 1 |
| Q. 7 | The constraints $-\mathrm{x}_{1}+\mathrm{x}_{2}+\leq 1 ;-\mathrm{x}_{1}+3 \mathrm{x}_{2} \leq 9 ; \mathrm{x}_{1}, \mathrm{x}_{2} \geq 0$ define <br> (a) Bounded feasible space <br> (b) Unbounded feasible space <br> (c) Both bounded and unbounded feasible space <br> (d) None of these | 1 |
| Q. 8 | If the position vectors of $A, B, C, D$ are $2 \mathbf{i}+\mathbf{j}, \mathbf{i}-3 \mathbf{j}, 3 \mathbf{i}+2 \mathbf{j}$ and $\mathbf{i}+\lambda \mathbf{j}$ respectively and $\overrightarrow{A B} \\| \overrightarrow{C D}$, then $\lambda$ will be <br> (a) -8 <br> (b) -6 <br> (c) 8 <br> (d) 6 | 1 |
| Q. 9 | $\int_{0}^{1.5}\left[x^{2}\right] d x$, where $[$.$] denotes the greatest integer function, equals$ <br> (a) $2+\sqrt{2}$ <br> (b) $2-\sqrt{2}$ <br> (c) $-2+\sqrt{2}$ <br> (d) $-2-\sqrt{2}$ | 1 |
| Q. 10 | The matrix $\left[\begin{array}{ccc}2 & \lambda & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3\end{array}\right]$ is nonsingular, if <br> (a) $\lambda \neq-2$ <br> (b) $\lambda \neq 2$ <br> (c) $\lambda \neq 3$ <br> (d) $\lambda \neq-3$ | 1 |
| Q. 11 | For the following shaded area, the linear constraints except $x \geq 0$ and $y \geq 0$, are <br> (a) $2 x+y \leq 2, x-y \leq 1, x+2 y \leq 8$ <br> (b) $2 x+y \geq 2, x-y \leq 1, x+2 y \leq 8$ <br> (c) $2 x+y \geq 2, x-y \geq 1, x+2 y \leq 8$ <br> (d) $2 x+y \geq 2, x-y \geq 1, x+2 y \geq 8$ | 1 |

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| Q. 12 | If $A=\left[\begin{array}{ll}\alpha & 2 \\ 2 & \alpha\end{array}\right]$ and $\left\|A^{3}\right\|=125$, then $\alpha=$ <br> (a) $\pm 3$ <br> (b) $\pm 2$ <br> (c) $\pm 5$ <br> (d) 0 | 1 |
| :---: | :---: | :---: |
| Q. 13 | If a matrix $A$ of order $3 \times 3$ has determinant 2, then the value of $\|\mathrm{A}(8 \mathrm{I})\|=$ <br> (a) 1024 <br> (b) 512 <br> (c) 256 <br> (d) none | 1 |
| Q. 14 | A man and his wife appear for an interview for two posts. The probability of the husband's selection is $\frac{1}{7}$ and that of the wife's selection is $\frac{1}{5}$. What is the probability that only one of them will be selected <br> (a) $\frac{1}{7}$ <br> (b) $\frac{2}{7}$ <br> (c) $\frac{3}{7}$ <br> (d)None of these | 1 |
| Q. 15 | The solution of the differential equation $x \cos y d y=\left(x e^{x} \log x+e^{x}\right) d x$ is <br> (a) $\sin y=\frac{1}{x} e^{x}+c$ <br> (b) $\sin y+e^{x} \log x+c=0$ <br> (c) $\sin y=e^{x} \log x+c$ <br> (d)None of these | 1 |
| Q. 16 | If $y=\cot ^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$, then $\frac{d y}{d x}=$ <br> (a) $\frac{1}{2}$ <br> (b) $\frac{2}{3}$ <br> (c) 3 <br> (d) 1 | 1 |
| Q. 17 | $\begin{aligned} & (\vec{a} \times \vec{b})^{2}+(\vec{a} \cdot \vec{b})^{2}= \\ & (\mathrm{a})^{(\mathbf{a} \times \mathbf{a}) \cdot(\mathbf{b} \times \mathbf{b})}(\mathrm{b})^{(\mathbf{a} \cdot \mathbf{a})(\mathbf{b} \cdot \mathbf{b})}(\mathrm{c})^{\|(\mathbf{a} \times \mathbf{b})\|(\mathbf{a} \cdot \mathbf{b})}(\mathrm{d})^{2(\mathbf{a} \cdot \mathbf{b})(\mathbf{a} \cdot \mathbf{b})} \end{aligned}$ | 1 |
| Q. 18 | The direction ratios of the line perpendicular to the lines $\frac{x-7}{2}=\frac{y+17}{-3}=\frac{z-6}{1}$ and $\frac{x+5}{1}=\frac{y+3}{2}=\frac{z-4}{-2}$ are proportional to <br> a) $4,5,7$ <br> b) $-4,5,7$ <br> c) $4,-5,-7$ <br> d) $4,-5,7$ |  |
|  | ASSERTION-REASON BASED QUESTIONS <br> In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both $A$ and $R$ are true and $R$ is the correct explanation of $A$. (b) Both $A$ and $R$ are true but $R$ is not the correct explanation of $A$. (c) $A$ is true but $R$ is false. (d) A is false but $R$ is true. |  |
| Q. 19 | Assertion (A) : The domain of the function $\sec ^{-1} 2 x$ is $\left(-\infty,-\frac{1}{2}\right] \cup\left[\frac{1}{2}, \infty\right)$ $\boldsymbol{\operatorname { R e a s o n }}(\mathbf{R}): \sec ^{-1}(2)=-\frac{\pi}{4}$ | 1 |
| Q. 20 | $f(x)=[x-1]+\|x-2\|$, where [.] denotes the greatest integer function. | 1 |

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|  | Assertion (A) : $f(x)$ is discontinuous at $\mathbf{x}=2$. <br> Reason (R): $f(x)$ is non derivable at $\mathbf{x}=2$. |  |
| :---: | :---: | :---: |
|  | SECTION - B <br> This section comprises of very short answer type-questions (VSA) of 2 marks each |  |
| Q. 21 | Sand is pouring from a pipe at the rate of $12 \mathrm{~cm}^{3 /} \mathrm{sec}$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one - sixth of the radius of the base. How fast is the height of the sand - cone increasing when the height is 4 cm ? | 2 |
| Q. 22 | Given that vectors $\vec{a}, \vec{b}, \vec{c}$ from a triangle such that $\vec{a}=\vec{b}+\vec{c}$. Find $p, q, r, s$ such that area of triangle is $5 \sqrt{6}$ where $\vec{a}=p \hat{i}+q \hat{j}+r \hat{k}, \quad \vec{b}=s \hat{i}+3 \hat{j}+4 \hat{k}$ and $\vec{c}=3 \hat{i}+\hat{j}-2 \hat{k}$. | 2 |
| Q. 23 | Prove that: $\tan ^{-1} \frac{1}{4}+\tan ^{-1} \frac{2}{9}=\frac{1}{2} \cos ^{-1}\left(\frac{3}{5}\right)$. <br> OR <br> Find the total number of bijective function from set $A$ to $A$ if $A=\{1,2,3,4\}$ | 2 |
| Q. 24 | If $y=\left(\operatorname{Tan}^{-1} \mathrm{x}\right)^{2}$, Prove that $\left(\mathrm{x}^{2}+1\right)^{2} \mathrm{~d}^{2} \mathrm{y} / \mathrm{dx}^{2}+2 \mathrm{x}\left(\mathrm{x}^{2}+1\right) \mathrm{dy} / \mathrm{dx}-2=0$. | 2 |
| Q. 25 | Find the value of p so that the lines $\frac{1-x}{3}=\frac{7 y-14}{2 p}=\frac{z-3}{2}$ and $\frac{7-7 x}{3 p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angle. <br> OR <br> Show that the following two lines are intersect $\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a-d}{\alpha+\delta} \text { and } \frac{x-b+c}{\beta-\gamma}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+\gamma} .$ | 2 |
|  | SECTION - C <br> (This section comprises of short answer type questions (SA) of 3 marks each) |  |
| Q. 26 | A die is thrown three times. Events A and B are defined as below : <br> $A: 5$ on the first and 6 on the second throw. <br> B:3 or 4 on the third throw. Find the probability of B, given that A has already occurred. <br> OR <br> A problem of mathematics is given to three students, whose chances of solving it are $\frac{1}{3}, \frac{1}{4}, \frac{1}{6}$ What is the probability that exactly one of them solve the problem. | 3 |

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| Q. 27 | Evaluate: $\int \frac{2 x+3}{(x-1)(x-2)(x-3)} d x$. | 3 |
| :---: | :---: | :---: |
| Q. 28 | Evaluate: $\int_{0}^{\pi} \frac{x \sin x}{1+\sin x} d x$. <br> OR <br> Evaluate: $\int_{0}^{\pi / 4} \log (1+\tan \theta) d \theta$. | 3 |
| Q. 29 | Evaluate: $\int \frac{d x}{x^{4}-5 x^{2}+16}$. | 3 |
| Q. 30 | Minimise and Maximise $\mathrm{Z}=3 x+9 y$ subject to the constraints: $x+3 y \leq 60$ $; x+y \geq 10 ; x \leq y ; x \geq 0, y \geq 0$. | 3 |
| Q. 31 | Solve the following differential equation, given that $\mathrm{y}=0$, when $\mathrm{x}=\frac{\pi}{4}$ : $\sin 2 x \frac{d y}{d x}-y=\tan x$ <br> OR <br> Solve the differential equation $\left\{x \cos \left(\frac{y}{x}\right)+y \sin \left(\frac{y}{x}\right)\right\} y d x=\left\{y \sin \left(\frac{y}{x}\right)-x \cos \left(\frac{y}{x}\right)\right\} x d y .$ | 3 |
|  | SECTION - D <br> (This section comprises of long answer-type questions (LA) of 5 marks each) |  |
| Q. 32 | Show that each of the relation R in the set $\mathrm{A}=\{x \in Z: 0 \leq x \leq 12\}$, given by $\mathrm{R}=\{(\mathrm{a}, \mathrm{b}):\|a-b\|$ is a multiple of 4$\}$. Find the set of all elements related to 1 . <br> OR <br> Let $A=R-\{3\}, B=R-\{1\}$. Let $f: A \rightarrow B$ be defined by $f(x)=\frac{x-2}{x-3}, \forall x \in A$. Show that $f$ is bijective. Also, find $x$, if $f^{-1}(x)=4$ and $f^{-1}(7)$. | 5 |
| Q. 33 | Find the area of the region $\left\{(\mathrm{x}, \mathrm{y}): \mathrm{y}^{2} \geq 6 \mathrm{x}, \mathrm{x}^{2}+\mathrm{y}^{2} \leq 16\right\}$. | 5 |
| Q. 34 | Show that the lines $\frac{x-1}{3}=\frac{y-1}{-1}, z+1=0$ and $\frac{x-4}{2}=\frac{z+1}{3}, y=0$ intersect each other. Also find their point of intersection. <br> OR <br> Find the shortest distance between the lines $\vec{r}=(\hat{i}+2 \hat{j}+3 \hat{k})+\lambda(2 \hat{i}+3 \hat{j}+4 \hat{k})$ $; \vec{r}=(2 \hat{i}+4 \hat{j}+5 \hat{k})+\mu(4 \hat{i}+6 \hat{j}+8 \hat{k})$. | 5 |
| Q. 35 | Note :- Attempt any two . <br> (i) Find the interval in which $f(x)=\frac{4 \sin x-2 x-x \cos x}{2+\cos x}$ on $(0,2 \pi)$ is (i) | 5 |

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|  | increasing (ii) decreasing. <br> (ii)Determine the intervals in which the function f given by $f(x)=\sin x-\cos x, 0 \leq x \leq 2 \pi$ is increasing or decreasing. <br> (iii)Determine the intervals in which the function $f$ given by $f(x)=\log (1+x)-\frac{2 x}{2+x}, x \neq-2$ Is increasing or decreasing. |  |
| :---: | :---: | :---: |
|  | SECTION - E <br> (This section comprises of 3 case study / passage - based questions of 4 marks each with two sub parts (i),(ii),(iii) of marks 1, 1, 2 respectively.The third case study question has two sub - parts of 2 marks each.) |  |
| Q. 36 | Case Study based-1 <br> Consider 2 families A and B. Suppose there are 4 men, 4 women and 4 children in family A and 2 men, 2 women and 2 children in family B. The recommended daily amount of calories is 2400 for a man, 1900 for a women, 1800 for children and 45 grams of proteins for a man, 55 grams for a woman and 33 grams for children. |  |
| i. | Represent the requirement of calories and proteins for each person in matrix form. | 1 |
| ii. | Find the requirement of calories of family $A$ and requirement of proteins of family B. | 1 |
| iii. | Represent the requirement of calories and proteins. If each person increases the protein intake by $5 \%$ and decrease the calories by $5 \%$ in matrix form. <br> OR <br> If $A$ and $B$ are two matrices such that $A B=B$ and $B A=A$, then find $A^{2}+B^{2}$ in terms of A and B . | 2 |
| Q. 37 | CASE STUDY-2 <br> A telephone company in a town has 500 subcribers on its list and collects fixed charges of 300 per subscriber per year. The company proposes to increase the annual subscription kand it is believed that for ecvery increase of 1 one subscriber will discontinue the sesrvice. |  |

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| i. | If $x$ be the annual subscription then the total revenue of the company after increment will be: <br> a. $R(x)=-x^{2}+200 x+150000$ <br> b. $R(x)=x^{2}-200 x-140000$ <br> c. $R(x)=200 x^{2}+x+150000$ <br> d. $R(x)=-x^{2}+100 x+100000$ | 1 |
| :---: | :---: | :---: |
| ii. | How much fee the company should increase to have maximum profit? <br> a. Rs. 150 b. Rs. 100 c. Rs. 200 d. Rs. 250 | 1 |
| iii. | Find the maximum profit that the company can make if the profit function is given by $P(x)=41+24 x-18 x^{2}$ <br> a. 25 <br> b. 44 <br> c. 45 <br> d. 49 <br> OR <br> Find both the maximum and minimum value respectively of $3 x^{4}-8 x^{3}+$ $48 \mathrm{x}+1$ on the interval $[1,4]$. <br> a. $-63,257$ <br> b. 258, -63 <br> c. $257,-63$ <br> d. $-63,-257$ | 2 |
| Q. 38 | Case Study based-3 <br> In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process $50 \%$ of the forms, Priyanka processes $20 \%$ and Tahseen the remaining $30 \%$ of the forms. Govind has an error rate of 0.06 , priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03 . |  |
| i. | The manager of the compay wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the forms selected at random has an error, find the probability that the form is NOT processed by Govind. | 2 |
| ii. | Find the probability that Priyanka processed the form and committed an error. | 2 |
|  | $* * * * * * * * * * * * * * * * * *$ |  |
|  | $\begin{aligned} & \text { अपने महनन लक्यों को तय कीजिये } \\ & \text { और तब तक नहीं रूके तब तक पा न लें/ / } \end{aligned}$ |  |

