Series : PTS/3

Code No. 13/10/1

Roll No.	2	1	3	1	2		

Candidates must write the Code on the title page of the answer-book.

PLEASURE TEST SERIES XII – 03

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Time Allowed: 180 Minutes **Max. Marks: 100**

SECTION A [Question numbers 1 to 10 carry 1 mark each.]

Q01. If $y = \log_5(\log x)$ then, write the value of dy/dx.

Q02. If the order of two matrices A and B are 3×4 and 4×3 respectively then, write the order of $(AB)^T$.

Q03. If A is a square matrix of order 3 and $|A^{-1}| = 9$ then what is the value of $|A|$?

Q04. Let M = {1, 2, 3, 4} and P = {1, 2}. Find the number of onto functions from M to P?

Q05. Find the direction ratios of the line $5x - 3 = 15y + 7 = 2 - 6z$.

Q06. Evaluate : $\int \frac{1}{e^x + e^{-x}} dx$.

Q07. Find the value of $\sin 2\tan^{-1}\frac{1}{3}$.

Q08. How are the two non-zero vectors $\vec{a}$ and $\vec{b}$ related, if $\vec{a}\times\vec{b} = \vec{0}$?

Q09. What is the approximate change in the volume V of a cube of side x metres caused by increasing the side by 2%?

Q10. If $|\vec{a}+\vec{b}| = |\vec{a}| + |\vec{b}|$ then, what is the angle between $\vec{a}$ and $\vec{b}$?

SECTION B [Question numbers 11 to 22 carry 4 marks each.]

Q11. Find the value of k so that the following function is continuous at x = 2 :

$$f(x) = \begin{cases} \frac{x^3 + x^2 - 16x + 20}{(x-2)^2}; & x \neq 2 \\ k; & x = 2 \end{cases}.$$

OR If $\sqrt{1+x^2} + \sqrt{1+y^2} = a(x-y)$ then, show that $\frac{dy}{dx} = \sqrt{\frac{1+y^2}{1+x^2}}$.

Q12. Show that : $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right) = \cos^{-1}\left(\frac{a\cos\theta + b}{a + b\cos\theta}\right)$.

Q13. Evaluate : $\int_0^1 \frac{x\tan^{-1}x}{(1+x^2)^{3/2}} dx$.

Q14. Show that $f : N \to N$, defined by $f(x) = \begin{cases} x+1; & \text{if x is odd} \\ x-1; & \text{if x is even} \end{cases}$ is a bijective function.

Q15. If $\hat{a}$ and $\hat{b}$ are unit vectors and θ is the angle between them, then show that : $\tan\left(\frac{\theta}{2}\right) = \frac{|\hat{a}-\hat{b}|}{|\hat{a}+\hat{b}|}$.

OR Establish the relationship between scalar product and vector of two non-zero vectors.

Q16. Evaluate : $\int x^2 \tan^{-1}x\, dx$.

OR Evaluate : $\int \frac{\sin^{-1}x}{x^2} dx$.

Q17. If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ then prove that $A^n = \begin{pmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{pmatrix}$ for all natural numbers n.

Q18. Form the differential equation corresponding to the family of curves $(x-a)^2 + (y-b)^2 = 36$.

Q19. Find the equation of line of shortest distance between the lines whose equations are given as below :

$\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + \lambda(3\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = 3\hat{i} - 7\hat{j} + 6\hat{k} + \mu(2\hat{j} - 3\hat{i} + 4\hat{k})$.

Q20. Verify Rolle's theorem for the function $f(x) = x(x-3)^2$, $x \in [0, 3]$.

Q21. Using approximation, evaluate $(3.968)^{3/2}$. If $1000(3.968)^{3/2}$ is the number of people who took part in a blood donation camp organized by an NGO, find the approximate number of these people.

What is the importance of blood donation?

OR A ladder 20m long has one end on the ground and the other end in contact with a vertical wall. The lower end slips along the ground. Show that when the lower end of the ladder is 16m away from the wall, the upper end is moving $\frac{4}{3}$ times as fast as the lower end.

Q22. A mathematics teacher in a school has a question bank consisting of 300 easy True / False questions, 200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

SECTION C [Question numbers 23 to 29 carry 6 marks each.]

Q23. Two store-rooms A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to three ration shops D, E and F whose requirements are 60, 50 and 40 quintals respectively. Cost of transportation per quintal from the store-rooms to the shops are listed in the following table:

Transportation Cost Per Quintal (in ₹)		
From / To	**A**	**B**
D	6	4
E	3	2
F	2.50	3

How should the supplies be managed in order to minimize the transportation cost? Also find what is the minimum cost? **While transportation of grains from store-room A to shop D, five packs of grains got stolen. You know the thief, who is a poor friend of yours. How would you inform the store room owner of the theft at the same time saving your friend?**

Q24. Using integrals, find the area of the triangle formed by positive *x*-axis, and the tangent and the normal to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$.

Q25. Let the point P(5, 9, 3) lies on the top of Qutub Minar, Delhi. Find image of the point P(5, 9, 3) on the line : $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. **"Conservation of monuments is important." Why?**

OR Considering the earth as a plane having equation $5x + 9y - 10z + 153 = 0$, a monument is standing vertically such that its peak is at the point (1, 2,–3). Find image of this point. Also find height of the monument.

How can we save our monument? Mention any two points.

Q26. An expensive square piece of golden color board of side 24cm is to be made into a box without top by cutting a square from each corner and folding the flaps to form a box. What should be the side of the square piece to be cut from each corner of the board to hold maximum volume and minimize the wastage?

What is the importance of minimizing the wastage in utilizing the resources? How can a student utilise the resources?

Q27. Evaluate $\int_0^2 (x^2 - 6x + e^x)\,dx$ as a limit of sum.

Approximately 30 times the absolute value of this definite integral is the number of trees planted by **Palash**, a local NGO, in a locality. How would you stress this as a welcome step for the society?

Mention the ways in which trees help us. **OR** Evaluate : $\int \sin^3 \sqrt{x}\,dx$.

Q28. In a group of students from a certain school, 200 students attend coaching classes, 400 students attend school regularly and 600 students study themselves with help of peers (self study). The probability that a student will succeed in life who attend coaching classes, attend school regularly and study themselves with the help of peers (self study) are 0.1, 0.2 and 0.5 respectively. One student is selected who succeeded in life, what is the probability that he studied himself with the help of peers?

What type of study can be considered for the success in life and why?

Q29. If a, b and c are positive and unequal then, show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} < 0$.

Q01. $\dfrac{1}{x(\log_e 5)(\log x)}$ **Q02.** 3×3 **Q03.** $|A| = 1/9$

Q04. ${}^4P_2 = 12$. See OPG Vol. 2 P62 Q. No. 51(e) for explanation.

Q05. 6, 2, –5 **Q06.** $\tan^{-1} e^x + k$ **Q07.** 3/5 **Q08.** $\vec{a} \parallel \vec{b}$

Q09. $0.6x^3$ cubic metres **Q10.** 0^o **Q11.** k = 7 **Q13.** $\dfrac{1}{\sqrt{2}}\left(1-\dfrac{\pi}{4}\right)$

Q14. See OPG Vol. 2 P61 Q. No. 20

Q15. See OPG Vol. 2 P09 Q. No. 30 **OR** See OPG Vol. 2 P06 Point No. 02

Q16. $\dfrac{x^3\tan^{-1}x}{3} - \dfrac{x^2}{6} + \dfrac{1}{6}\log|1+x^2| + C$ **OR** $\dfrac{1}{2}\log\left|\dfrac{1-\sqrt{1-x^2}}{1+\sqrt{1-x^2}}\right| - \dfrac{\sin^{-1}x}{x} + C$

Q18. $[1+(y')^2]^3 = 36(y'')^2$ **Q19.** $\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + \lambda(6\hat{i} + 15\hat{j} - 3\hat{k})$

Q20. c = 1 **Q21.** 7.968; No. of people is 7968 **Q22.** 5/9

Q23. From A : 10, 50, 40 units; From B : 50, 0, 0 units to D, E and F respectively. Minimum cost = ₹510.
[For solution, visit **www.theOPGupta.com** & download NCERT Solutions Miscell. Ex. Q. No. 6]

Q24. $2\sqrt{3}$ sq.units **Q25.** Foot of perpendicular : (3, 5, 7) and Image : (1, 1, 11) **OR**
Foot of perpendicular : (–4, –7, 7) and Image : (–9, –16, 17) **Q26.** 4cm

Q27. $e^2 - \dfrac{31}{3}$ **OR** Put $\sqrt{x} = t \Rightarrow x = t^2 \Rightarrow dx = 2t\,dt$.

So, $I = \int \sin^3 t \,.\, (2t)\,dt$. Integrate using By Parts.

$I = 2t\int \sin^3 t\, dt - \int\left[\dfrac{d}{dt}(2t)\int \sin^3 t\, dt\right]dt$ ...(i) Consider $I_1 = \int \sin^3 t\, dt = \int (1-\cos^2 t)\sin t\, dt$

Put $\cos t = u \Rightarrow \sin t\, dt = -du \Rightarrow I_1 = -\int (1-u^2)\, du = \dfrac{1}{3}\cos^3 t - \cos t$

By (i), $I = 2t\left(\dfrac{1}{3}\cos^3 t - \cos t\right) - \dfrac{2}{3}\int \cos^3 t\, dt + 2\int \cos t\, dt$ ...(ii)

Consider $I_2 = \int \cos^3 t\, dt = \int (1-\sin^2 t)\cos t\, dt$

Put $\sin t = v \Rightarrow \cos t\, dt = dv \Rightarrow I_2 = \int (1-v^2)\, dv = \sin t - \dfrac{1}{3}\sin^3 t$

By (ii), $I = 2t\left(\dfrac{1}{3}\cos^3 t - \cos t\right) - \dfrac{2}{3}\left(\sin t - \dfrac{1}{3}\sin^3 t\right) + 2\sin t + C$

$\Rightarrow I = \left(\dfrac{2}{3}\sqrt{x}\cos^3\sqrt{x} - 2\sqrt{x}\cos\sqrt{x}\right) - \dfrac{2}{3}\sin\sqrt{x} + \dfrac{2}{9}\sin^3\sqrt{x} + 2\sin\sqrt{x} + C$

i.e., $I = \dfrac{2}{9}\sin^3\sqrt{x} + \dfrac{2}{3}\sqrt{x}\cos^3\sqrt{x} - 2\sqrt{x}\cos\sqrt{x} + \dfrac{4}{3}\sin\sqrt{x} + C$

Q28. 3/4. For explanation, see OPG Vol. 2 P105 Q. No. 16

Q29. See OPG Vol. 1 P18 Q. No. 09

Good Luck & God Bless You!!!