

CLASS XII

SAMPLE PAPER-01

MATHEMATICS

Time : 3 Hours

Maximum Marks : 100

GENERAL INSTRUCTIONS : The question paper consists of 29 questions divided into three sections A, B & C. Section A comprises of 10 questions of 1 mark each, section B comprises of 12 questions of 4 marks each & section C comprises of 7 questions of 6 marks each.

SECTION – A

1. If $f(x) = x^2 + 2$ and $g(x) = \frac{x}{x+1}$, find $(g \circ f)(5)$.
2. What is the principal value of $\sin^{-1}\left(\sin \frac{3\pi}{5}\right)$?
3. If the position vector of the points A and B are $-3\hat{i} + 3\hat{j}$ and $4\hat{i} + 2\hat{k}$ respectively, then find a unit vector in the direction \overrightarrow{AB} .
4. If $\vec{a} = 3\hat{i} + 2\hat{j}$ and $\vec{b} = 2\hat{i} + 3\hat{j}$. Is $|\vec{a}| = |\vec{b}|$? Is $\vec{a} = \vec{b}$?
5. If $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \\ -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -2 & 3 \\ 7 & 5 & -3 \end{bmatrix}$, then what are the orders of AB and BA.
6. If $A = \begin{bmatrix} 5 & -1 \\ -2 & 6 \end{bmatrix}$, determine whether $A - A'$ is symmetric or skew symmetric.
7. If $\begin{vmatrix} 3x & 1 \\ 5 & x \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ -6 & 2 \end{vmatrix}$, then find the value of x.
8. Evaluate : $\int \frac{x^{e-1} + e^{x-1}}{x^e + e^x} dx$
9. Find the value of $\int_{-5}^5 x^{18} \sin^{195} x dx$

10. If a line makes α, β, γ with the x-axis, y-axis and z-axis respectively, then find the value of $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$.

SECTION – B

11. Let R be the relation defined in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b) : \text{both } a \text{ and } b \text{ are either odd or even}\}$. Show that R is an equivalence relation. Further, show that all the elements of the subset $\{1, 3, 5, 7\}$ are related to each other and all the elements of the subset $\{2, 4, 6\}$ are related to each other, but no element of the subset $\{1, 3, 5, 7\}$ is related to any element of the subset $\{2, 4, 6\}$.

12. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x.

OR

If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$.

13. Find the equation of the tangent to the curve $x^2 + 3y = 3$, which is parallel to the line $y - 4x + 5 = 0$.
14. Solve the differential equation: $y + x \sin\left(\frac{y}{x}\right) = x \frac{dy}{dx}$.
15. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution number of tails.
16. Solve the differential equation: $(1 + x^2)\frac{dy}{dx} + y = \tan^{-1} x$.
17. Express the vector $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$ as sum of two vectors such that one is parallel to $\vec{b} = 3\hat{i} + \hat{k}$ and the other is perpendicular to \vec{b} .

18. If a, b, c are in A.P., then prove that
$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} = 0$$

OR

Express the matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix.

19. Evaluate : $\int \frac{x^2 + 1}{(x+1)^2} e^x dx$ **OR** Evaluate : $\int \frac{1}{\sqrt{(2-x)^2 + 1}} dx$

20. Prove that : $\frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3} = \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3}$.

OR

Prove that : $\sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5} = \tan^{-1} \frac{63}{16}$.

21. Find the shortest distance between the lines whose vector equations are :

$$\vec{r} = (1 - \lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}, \text{ and } \vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$$

22. Find the value of k for which $f(x) = \begin{cases} \frac{\log(1+ax) - \log(1-bx)}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$.

SECTION – C

23. Find the area bounded by the curve $x^2 = 4y$ and the line $x + 2 = 4y$.

24. Evaluate : $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$ OR Evaluate : $\int_0^{\pi/2} \frac{1}{2 \cos x + 4 \sin x} dx$

25. Find the vector equation of the plane passing through the points $2\hat{i} + \hat{j} - \hat{k}$ and $-\hat{i} + 3\hat{j} + 4\hat{k}$ and perpendicular to the plane $\vec{r} \cdot (\hat{i} - 2\hat{j} + 4\hat{k}) = 10$.

26. Suppose that the reliability of a HIV test is specified as follows: Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV–ive but 1% are diagnosed as showing HIV+ive. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ive. What is the probability that the person actually has HIV?

27. If $A = \begin{bmatrix} 4 & -5 & -11 \\ 1 & -3 & 1 \\ 2 & 3 & -7 \end{bmatrix}$, find A^{-1} . Using A^{-1} , solve the following system of linear equations :

$$4x - 5y - 11z = 12, \quad x - 3y + z = 1, \quad 2x + 3y - 7z = 2.$$

28. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

OR

A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the flops to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum? Also find the maximum volume.

29. A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour

hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week?