Series : PTS/19

Code No. 16/1/19

Candidates must write the Code on the title page of the answer-book.

Roll No. | | | | | | | |

PLEASURE TEST SERIES XII – 19

A Compilation By : O. P. Gupta [Call or WhatsApp @ +91-9650 350 480]

Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION – A

Q01. Write the number of binary operations that can be defined on the set {1, 2}.

Q02. Evaluate : $\tan 2\tan^{-1}(0.2)$.

Q03. If $y = \log_{\sqrt{e}} \sin x$, find $\frac{dy}{dx}$.

Q04. Check if the function $-2x^3 + 6x^2 - 6x + 9$ is decreasing in R.

Q05. If a, b, c are three non-zero real numbers, then find the inverse of $\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$.

Q06. Show that a powerful bomb shot along the line of fire $x = 2s+1, y = 3s+2, z = 4s+3$ will never hit a helicopter flying in the plane $2x + 4y - 4z + 11 = 0$.

SECTION – B

Q07. Using properties of determinants, prove that : $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = (a^3+b^3+c^3-3abc)^2$.

OR Using properties, prove that : $\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2-ac)(ax^2+2bxy+cy^2)$.

Q08. If $x = \frac{1}{z}$ and $y = f(x)$ then, prove that $\frac{d^2 f}{dx^2} = 2z^3\frac{dy}{dz} + z^4\frac{d^2y}{dz^2}$.

OR If $y^2 = 4ax$, then evaluate : $\left(\frac{d^2y}{dx^2}\right).\left(\frac{d^2x}{dy^2}\right)$.

Q09. Let f, g : R→R be defined as f (x) = |x| and g (x) = [x], where [x] denotes greatest integer less than or equal to x. Evaluate : $\frac{(gof)\left(-\frac{5}{3}\right)-(fog)\left(-\frac{5}{3}\right)}{(fo(gof))\left(-\frac{5}{3}\right)}$.

Q10. Discuss the differentiability of $f(x) = \begin{cases} 1-x, & \text{if } x<1 \\ (1-x)(2-x), & \text{if } 1 \le x \le 2 \\ 2-x, & \text{if } x>2 \end{cases}$ at x = 2.

Q11. Find the value of θ, satisfying $\begin{vmatrix} 1 & 1 & \sin 3\theta \\ -4 & 3 & \cos 2\theta \\ 7 & -7 & -2 \end{vmatrix} = 0$.

Q12. Evaluate $\int_0^{\pi/2} \log \operatorname{cosec} x \, dx$. **Q13.** Form the differential equation for $y = (\sin^{-1}x)^2 + A\cos^{-1}x + B$.

Q14. Express $\cos^{-1}\sqrt{\dfrac{\sqrt{1+x^2}+1}{2\sqrt{1+x^2}}}$ in simplest form. **OR** Solve : $\sec^2 \tan^{-1} 2 + \operatorname{cosec}^2 \cot^{-1} 3 = x$.

Q15. Solve the differential equation : $x^2\dfrac{dy}{dx} - xy = 1 + \cos\dfrac{y}{x}$, $x \neq 0$ and $x = 1$, $y = \dfrac{\pi}{2}$.

Q16. Find the values of a + 2 b if A = B, where $A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}$, $B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$.

Q17. A speaks truth in 60% of the cases, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact?
In the cases of contradiction do you think, the statement of B will carry more weight as he speaks truth in more number of cases than A?

Q18. Find the distance of the point (– 2, 4, – 5) from the line $\dfrac{x+3}{3} = \dfrac{y-4}{5} = \dfrac{z+8}{6}$.

Q19. Evaluate : $\displaystyle\int \frac{dx}{\sin(x-\alpha)\sin(x-\beta)}$. **OR** Evaluate : $\displaystyle\int \frac{(x+x^3)^{1/3}}{x^4} dx$.

SECTION – C

Q20. Let $\vec{a} = 2\hat{i} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = 4\hat{i} - 3\hat{j} + 7\hat{k}$ be three vectors. Determine a vector $\vec{r}$ which satisfies the condition $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$.

OR Show that : $[\vec{a} \;\; \vec{b} \;\; \vec{c}]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$.

Q21. A toy manufacturer produces two types of dolls; a basic version doll A and a deluxe version doll B. Each doll of type B takes twice as long to produce as one doll of type A. The company has time to make a maximum of 2000 dolls of type A per day. The supply of plastic is sufficient to produce 1500 dolls per day (both A and B combined). The deluxe version, *i.e.*, type B requires a fancy dress of which there are only 600 per day available. If the company makes a profit of ₹3 and ₹5 per doll respectively, on doll A and B; how many of each should be produced per day in order to maximize the profit? Solve it graphically.

Q22. If PA and QB be two vertical poles of height 16m and 22m at points A and B respectively such that AB = 20m then, find the distance of a point R on AB from the point A such that $RP^2 + RQ^2$ is minimum.

Q23. Find area of region bounded by $y = 1 + |x+1|$, $|x| = 3$ and, $y = 0$ after making a rough sketch.

Q24. (i) If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its volume.
(ii) A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2m/s. How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall?
OR (i) Water is dripping out from a conical funnel at a uniform rate of 4cm^3/s through a tiny hole at the vertex in the bottom. When the slant height of the water is 3cm, find the rate of decrease of the slant height of the water-cone. Given that the vertical angle of funnel is 120^o.
(ii) Use differentials to evaluate the approximate value of $\log_e(4.01)$, if $\log_e 4 = 1.3863$.

Q25. A manufacturer has three machine operators A (skilled), B (semi-skilled) and C (non-skilled). The first operator A produces 1% defective items whereas the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of time, B in the job for 30% of the time and C is on the job for 20 % of the time. A defective item is produced, what is the probability that it was produced by B?

Q26. A bird at A(7, 14, 5) in space wants to reach a point P on the plane 2x + 4y – z = 2 when AP is least. Find the position of P and also the distance AP travelled by the bird.

HINTS & ANSWERS *for* PTS XII – 19 [2015 - 16]

Q01. If set A has m elements, then number of binary operations on A is $m^{m\times m}$ so, we have $2^{2\times 2}=16$.

Q02. $\dfrac{5}{12}$

Q03. 2 cot x

Q04. Decreasing function as $f'(x)<0\ \forall\ x\in R$

Q05. $\begin{pmatrix} a^{-1} & 0 & 0 \\ 0 & b^{-1} & 0 \\ 0 & 0 & c^{-1} \end{pmatrix}$

Q06. Show that the line is parallel to the plane i.e., the line is at right angle with the normal vector of the plane.

Q07. Consider LHS : Let $\Delta=\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}$

$$\Rightarrow \quad =\begin{vmatrix} bc-a^2+bc & ab-ba+c^2 & ac-ac+b^2 \\ ab-ab+c^2 & ca-b^2+ca & bc-bc+a^2 \\ ac-ac+b^2 & bc-bc+a^2 & ab-c^2+ab \end{vmatrix}$$

$$\Rightarrow \quad =\begin{vmatrix} b & -a & c \\ c & -b & a \\ a & -c & b \end{vmatrix}\begin{vmatrix} c & a & b \\ a & b & c \\ b & c & a \end{vmatrix}$$ Taking -1 common from C_2 in det.(I), and $C_1\leftrightarrow C_2$ in det.(II)

$$\Rightarrow \Delta=(-1)\times\begin{vmatrix} b & a & c \\ c & b & a \\ a & c & b \end{vmatrix}(-1)\times\begin{vmatrix} a & c & b \\ b & a & c \\ c & b & a \end{vmatrix}$$ By $C_1\leftrightarrow C_2$ in det.(I) and $C_2\leftrightarrow C_3$ in det.(II)

$$\Rightarrow \Delta=(-1)\times(-1)\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}(-1)\times(-1)\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \qquad \Rightarrow \Delta=\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2=\left[\Delta'\right]^2$$

Now apply properties to evaluate the value of Δ'.

OR LHS : Let $\Delta=\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix}$ Apply $C_3\to C_3-xC_1-yC_2$

$$\Rightarrow \Delta=\begin{vmatrix} a & b & 0 \\ b & c & 0 \\ ax+by & bx+cy & -ax^2-2bxy-cy^2 \end{vmatrix}$$ Expanding along C_3

$$\Rightarrow \Delta=\left(-ax^2-2bxy-cy^2\right)\begin{vmatrix} a & b \\ b & c \end{vmatrix}=\left(-ax^2-2bxy-cy^2\right)\left(ac-b^2\right)$$

$$\therefore\ \Delta=\left(b^2-ac\right)\left(ax^2+2bxy+cy^2\right)=RHS$$

Q08. OR $-\dfrac{2a}{y^3}$

Q09. –1

Q10. Not differentiable as LHD = 1 but RHD = –1

Q11. $\theta=n\pi,\ n\pi+(-1)^n\dfrac{\pi}{6}, n\in Z$

Q12. $\dfrac{\pi}{2}\log 2$

Q13. $\left(1-x^2\right)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}-2=0$

Q14. $\dfrac{1}{2}\tan^{-1}x$ OR $x=15$

Q15. $\tan\left(\dfrac{y}{2x}\right)=\dfrac{3}{2}-\dfrac{1}{2x^2}$

Q16. $a+2b=2+2\times 2=6$

Q17. 42%. Since no one trusts a liar, so the statement of B will carry more weight as he speaks truth in more number of cases than A.

Q18. Foot of perpendicular : $\left(-\dfrac{21}{10},\dfrac{55}{10},-\dfrac{62}{10}\right)$, Required Distance : $\sqrt{\dfrac{37}{10}}$ units

Q19. $\dfrac{1}{\sin(\alpha-\beta)}\log\left|\dfrac{\sin(x-\alpha)}{\sin(x-\beta)}\right|+C$ OR $-\dfrac{3}{8}\left(\dfrac{1}{x^2}+1\right)^{4/3}+C$

Q20. Given that $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ $\Rightarrow(\vec{r}-\vec{c})\times\vec{b}=\vec{0}$ $\therefore(\vec{r}-\vec{c})\,||\,\vec{b}$. Therefore, $\vec{r}-\vec{c}=\lambda\vec{b}$ $\Rightarrow\vec{r}=\lambda\vec{b}+\vec{c}$

$\Rightarrow\vec{r}=\lambda(\hat{i}+\hat{j}+\hat{k})+(4\hat{i}-3\hat{j}+7\hat{k})$...(i)

$\because\vec{r}.\vec{a}=0$ $\therefore[\lambda(\hat{i}+\hat{j}+\hat{k})+(4\hat{i}-3\hat{j}+7\hat{k})].(2\hat{i}+\hat{k})=0$ $\Rightarrow\lambda=-5$

Replacing the value of $\lambda=-5$ in (i) we get : $\vec{r}=2\hat{k}-8\hat{j}-\hat{i}$.

OR See O.P. Gupta's Mathematicia Vol.2 Chapter 09 (Scalar Triple Product)

Q21. See O.P. Gupta's MATHEMATICIA Vol.1 Chapter 08

Q22. 10m

Q23. 16 sq.units

Q24. (i) 9.72cm^3 (ii) $\dfrac{8}{3}$m/s OR (i) $\dfrac{32}{27\pi}$cm/s (ii) 1.3888.

Q25. 15/34

Q26. P(1, 2, 8), AP = $3\sqrt{21}$ units.