[SECTION – A]

Q01. Let * be a binary operation defined by a*b = 2a + b – 3, find the value of 3*4.

Q02. Write the value of : $\sin^{-1}\left(\sin\dfrac{3\pi}{5}\right)$.   **Q03.** What is the value of Δ , if $\Delta = \begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$?

Q04. Check if the function $-\dfrac{x^3}{3}+x^2-x+\dfrac{3}{2}$ is decreasing in R.

Q05. For what value of 'm' and 'p', is the matrix $\begin{pmatrix} 0 & 5 & -3 \\ -5 & m & 4 \\ p & -4 & 0 \end{pmatrix}$ skew-symmetric?

Q06. Show that a powerful bomb shot along the line of fire $\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-3}{4}$ will never hit a helicopter flying in the plane 2x + 4y – 4z + 11 = 0.

Q07. Write the number of binary operations that can be defined on the set {1, 2}.

Q08. Let $\vec{a}$ and $\vec{b}$ are non-collinear vectors. For what value of x, the vectors $\vec{c}=(x-2)\vec{a}+\vec{b}$ and $\vec{d}=(2x+1)\vec{a}-\vec{b}$ are collinear?

Q09. Write a unit vector perpendicular to the vectors $\vec{a}$ and $\vec{b}$ both, if it is given that $\vec{a}=3\hat{i}+2\hat{j}+6\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+2\hat{k}$.   **Q10.** Write the value of $\int\dfrac{3\cos x}{2\sin^2 x}dx$.

[SECTION – B]

Q11. Discuss the differentiability of $f(x)=\begin{cases} 1-x, & \text{if } x<1 \\ (1-x)(2-x), & \text{if } 1\le x\le 2 \\ 2-x, & \text{if } x>2 \end{cases}$ at x = 2.

(OR) A car driver is driving a car on the dangerous path given by

$$f(x)=\begin{cases} \dfrac{1-x^m}{1-x}, & \text{if } x\ne 1 \\ m-1, & \text{if } x=1 \end{cases}, \text{ m} \in \text{N}.$$

Find the dangerous point (point of discontinuity) on the path. Whether the driver should pass that point or not? Justify your answer.

Q12. Let $\vec{a}=2\hat{i}+\hat{k}, \vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=4\hat{i}-3\hat{j}+7\hat{k}$ be three vectors. Determine a vector $\vec{r}$ which satisfies the condition $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}.\vec{a}=0$.

Q13. Express $\cos^{-1}\sqrt{\dfrac{\sqrt{1+x^2}+1}{2\sqrt{1+x^2}}}$ in simplest form.   **(OR)** Solve : $\sec^2\tan^{-1}2+\csc^2\cot^{-1}3=x$.

Q14. Evaluate : $\int\limits_0^\pi \dfrac{x\sin x}{1+\cos^2 x}dx$.   **Q15.** Solve the differential equation : $(x-\tan^{-1}y)dy+(y^2+1)dx=0$.

Q16. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its volume.

(OR) Water is dripping out from a conical funnel at a uniform rate of 4cm³/s through a tiny hole at the vertex in the bottom. When the slant height of the water is 3cm, find the rate of decrease of the slant height of the water-cone. Given that the vertical angle of funnel is 120°.

Q17. A speaks truth in 60% of the cases, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact?

In the cases of contradiction do you think, the statement of B will carry more weight as he speaks truth in more number of cases than A?

Q18. Find the distance of the point (− 2, 4, − 5) from the line $\dfrac{x+3}{3} = \dfrac{y-4}{5} = \dfrac{z+8}{6}$.

Q19. Evaluate : $\displaystyle\int \dfrac{dx}{\sin(x-\alpha)\sin(x-\beta)}$. **(OR)** Evaluate : $\displaystyle\int \dfrac{(x+x^3)^{1/3}}{x^4}dx$.

Q20. Let f, g : R→R be defined as f (x) = |x| and g (x) = [x], where [x] denotes greatest integer less than or equal to x. Evaluate : $\dfrac{(gof)\left(-\dfrac{5}{3}\right)-(fog)\left(-\dfrac{5}{3}\right)}{(fo(gof))\left(-\dfrac{5}{3}\right)}$. **Q21.** If $y^2 = 4ax$, then evaluate : $\left(\dfrac{d^2y}{dx^2}\right)\cdot\left(\dfrac{d^2x}{dy^2}\right)$.

Q22. Using properties of determinants, prove that : $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = (a^3+b^3+c^3-3abc)^2$.

[SECTION – C]

Q23. Using integrals, find area of region bounded by the following curve after making a rough sketch: $y = 1+|x+1|$, $|x| = 3$, $y = 0$.

Q24. Three friends A, B and C visited a Super Market for purchasing fresh fruits. A purchased 1kg apples, 3kg grapes and 4kg oranges and paid ₹800. B purchased 2kg apples, 1kg grapes and 2kg oranges and paid ₹500. While C paid ₹700 for 5kg apples, 1kg grapes and 1kg oranges. Find the cost of each fruit per kg by using matrix method. Why are the fruits good for health?

  (OR) Using elementary operations, find the inverse of $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, if it exists.

Q25. A manufacturer has three machine operators A (skilled), B (semi-skilled) and C (non-skilled). The first operator A produces 1% defective items whereas the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of time, B in the job for 30% of the time and C is on the job for 20 % of the time. A defective item is produced, what is the probability that it was produced by B? What is the value of skill in industries?

Q26. A bird at A(7, 14, 5) in space wants to reach a point P on the plane 2x + 4y − z = 2 when AP is least. Find the position of P and also the distance AP travelled by the bird.

Q27. Evaluate $\displaystyle\int_0^{\pi/2} \log \cosec\, x\, dx$, by using properties of the definite integral.

Q28. If a class XII student aged 17 years, rides his motor cycle at 40km/hr, the petrol cost is ₹2 per km. If he rides at a speed of 70km/hr, the petrol cost increases to ₹7per km. He has ₹100 to spend on petrol and wishes to cover the maximum distance within one hour.
        (i) Express this as an L.P.P., and solve graphically.
        (ii) What is benefit of driving at an economical speed?
        (iii) Should a child below 18years be allowed to drive a motorcycle? Give reasons.

Q29. If PA and QB be two vertical poles of height 16m and 22m at points A and B respectively such that AB= 20m then, find the distance of a point R on AB from the point A such that RP^2+RQ^2 is minimum.

  (OR) A point P is given on the circumference of a circle of radius r. The chord QR is parallel to the tangent line at P. Find the maximum area of the triangle PQR.

HINTS & ANSWERS *for* PTS XII – 08 [2013-14]

Q01. 7   Q02. $\dfrac{2\pi}{5}$ Q03. $a^2 + b^2 + c^2 + d^2$

Q04. Decreasing function   Q05. 0, 3

Q06. Show that the line is parallel to the plane *i.e.,* the line is at right angle the normal vector of the plane.   Q07. $2^{2\times2} = 16$ Q08. 1/3

Q09. $\dfrac{4\hat{k} - 8\hat{i}}{4\sqrt{5}}$ Q10. $-\dfrac{3}{2}\operatorname{cosec} x + k$

Q11 Not differentiable as LHD = 1 but RHD = –1   OR   Point of discontinuity : x = 1. No, because life is precious so vehicles should be driven carefully.

Q12. $2\hat{k} - 8\hat{j} - \hat{i}$ Q13. $\dfrac{1}{2}\tan^{-1} x$ OR x = 15

Q14. $\left(\dfrac{\pi}{2}\right)^2$ Q15. $x = \tan^{-1} y + k\, e^{-\tan^{-1} y} - 1$ Q16. 9.72cm^3 OR $\dfrac{32}{27\pi}$ cm

Q17. 42%. Since no one trusts a liar, so the statement of B will carry more weight as he speaks truth in more number of cases than A.

Q18. $\sqrt{\dfrac{37}{10}}\,units$ Q19. $\dfrac{1}{\sin(\alpha-\beta)}\log\left|\dfrac{\sin(x-\alpha)}{\sin(x-\beta)}\right| + k$ OR $-\dfrac{3}{8}\left(\dfrac{1}{x^2}+1\right)^{4/3} + k$

Q20. –1   Q21. $-2a/y^3$ Q23. 16 sq.units

Q24. Let cost of each fruit be x, y, z respectively. Then solve the equations so formed by using matrix method. The inverse of matrix will be $\dfrac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$.

So, x = 100, y = 100, z = 100. Hence the cost of each fruit is ₹100 per kg.

Importance of fruits : Fruits contain nutrients and vitamins which help our body in its proper growth and maintenance.   OR $\begin{bmatrix} 1/3 & 1/3 & -2/3 \\ -2/3 & 1/3 & -2/3 \\ 1/3 & -1/6 & 5/6 \end{bmatrix}$ Q25. 15/34

Q26. P(1, 2, 8), AP = $\sqrt{189}$ units   Q27. $\dfrac{\pi}{2}\log 2$

Q28. Max. Z = x + y. Subject to constraints: $x/40 + y/70 \le 1$, $2x + 7y \le 100$; x, y ≥ 0. Here x & y represents the distance travelled by the boy at speed of 40km/hr & 70km/h respectively. (i) x = 1560/41km, y = 140/41km. (ii) It saves petrol. It saves money. (iii) No, because according to the law driving license is issued when a person is above the 18 years of age.

Q29 10m OR $\dfrac{3\sqrt{3}\,r^2}{4}$ sq.units .

Good Luck & God Bless You!!!