PLAY WITH MATH

TEST NO-04 TIME:-3Hrs.

F.M:-100

instructions:-

1. All questions are compulsory.
2. This question paper contains 29 questions.
3. Questions 1-4 in section A are short-answer type questions carrying 1 mark each.
4. Questions 5-12 in section B are short-answer type questions carrying 2 marks each.
5. Questions 13-23 in section C are long-answer type questions carrying 4 marks each.
6. Questions 24-29 in section D are long-answer type questions carrying 6 marks each.

Section-A

1.If A and B are invertible matrices of order 3, | A | = 2 and | $(AB)^{-1}$ | = - $\frac{1}{6}$ Find |B|.

2.Differentiate $\sin^2(x^2)$ w.r.t x^2.

3.Write the order of the differential equation:

$$\log\left(\frac{d^2y}{dx^2}\right) = \left(\frac{dy}{dx}\right)^3 + x$$

4.Find the acute angle which the line with direction cosines $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}$, n makes with positive direction of z-axis.

OR

Find the direction cosines of the line: $\frac{x-1}{2} = -y = \frac{z+1}{2}$

Section B

5. Let A = Z × Z and * be a binary operation on A defined by (a, b)*(c, d)= (ad+ bc, bd).

Find the identity element for * in the set A.

6.If A= $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,find k so that A^2 =5A+kI.

7. Find $\int \frac{(x^2+\sin^2 x)\sec^2 x}{1+x^2}\, dx$

8. Find $\int \frac{e^x(x-3)}{(x-1)^3} dx$

OR

Find $\int \frac{(x^4-x)^{1/4}}{x^5}\, dx$

9. Form the differential equation of all circles which touch the x-axis at the origin.

10. Find the area of the parallelogram whose diagonals are represented by the vectors $\vec{a}=2\hat{\imath} - 3\hat{\jmath}+4\hat{k}$ and $\vec{b}=2\hat{\imath} - \hat{\jmath}+2\hat{k}$

OR

Find the angle between the vector $\vec{a} = \hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{b}=\hat{\imath} - \hat{\jmath}+\hat{k}$

11. If A and B are two independent events, prove that Aand B are also independent.

12. One bag contains 3 red and 5 black balls. Another bag contains 6 red and 4 black balls. A ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is red.

OR

If P(A) = 0.6, P(B) = 0.5 and P(A|B) = 0.3, then find P(A∪B).

SECTION –C

13.Prove that the function f:[0,∞) →R given by f(x) = $9x^2 + 6x- 5$ is not invertible. Modify the codomain of the function f to make it invertible, and hence find f^{-1}.

OR

Check whether the relation R in the set R of real numbers, defined by
R = {(a, b) : 1 + ab > 0}, is reflexive, symmetric or transitive.

14. Find the value of : $\sin(2\tan^{-1}\frac{1}{4})+\cos(\tan^{-1} 2\sqrt{2})$.

15. Using properties of determinants, prove that:

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$$

16.If $y= x^{\sin x}+(\sin x)^x$, find $\frac{dy}{dx}$

OR

If $y = \log(1+2t^2+t^4)$,$x= \tan^{-1}t$, find $\frac{d^2y}{dx^2}$.

17.If $y= \cos(m\cos^{-1}x)$

Show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$

18. Find the equations of the normal to the curve $y = 4x^3 -3x+5$ which are perpendicular to the line $9x- y+5 = 0$.

19. find $\int \frac{x^4+1}{x(x^2+1)^2}\, dx$

20.Evlaute $\int_{-1}^{1} \frac{x+|x|+1}{x^2+2|x|+1}\, dx$

21. Find the particular solution of the following differential equation.

$\cos y\, dx +(1+2e^{-x})\sin y\, dy= 0$; $y(0)=\frac{\pi}{4}$

OR

Find the general solution of the differential equation:

$$\frac{dx}{dy} = \frac{y\tan y - x\tan y - xy}{y\tan y}$$

22. If $\vec{p} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{q} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$, find a vector of magnitude $5\sqrt{3}$ units perpendicular to the vector $\vec{q}$ and coplanar with vectors $\vec{p}$ and $\vec{q}$.

23. Find the vector equation of the line joining (1, 2, 3) and (-3, 4, 3) and show that it is perpendicular to the z-axis.

Section D

24. If A = $\begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$, find A^{-1}.

Hence, solve the system of equations:

3x + 3y+ 2z = 1

x + 2y = 4

2x- 3y- z = 5

OR

Find the inverse of the following matrix using elementary transformations.

$$\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$$

25. A cuboidal shaped godown with square base is to be constructed. Three times as much cost per square meter is incurred for constructing the roof as compared to the walls. Find the dimensions of the godown if it is to enclose a given volume and minimize the cost of constructing the roof and the walls.

26. Find the area bounded by the curves y= $\sqrt{x}$, 2y+3= x and x axis.

OR

Find the area of the region.{(x, y) : $x^2 + y^2 \leq 8$, $x^2 \leq 2y$}

27. Find the equation of the plane through the line $\frac{x-1}{3} = \frac{y-4}{2} = \frac{z-4}{-2}$ and parallel to the Line $\frac{x+1}{2} = \frac{1-y}{4} = \frac{z+2}{1}$ Hence, find the shortest distance between the lines.

OR

Show that the line of intersection of the planes x + 2y + 3z = 8 and 2x + 3y + 4z = 11 is coplanar with the line $\frac{x+1}{1} = \frac{y+1}{2} = \frac{z+1}{3}$. Also find the equation of the plane containing them.

28. The members of a consulting firm rent cars from three rental agencies: 50% from agency X, 30% from agency Y and 20% from agency Z. From past experience it is known that 9% of the cars from agency X need a service and tuning before renting, 12% of the cars from agency Y need a service and tuning before renting and 10% of the cars from agency Z need a service and tuning before renting. If the rental car delivered to the firm needs service and tuning, find the probability that agency Z is not to be blamed.

29. A manufacturer makes two types of toys A and B. Three machine are needed for this purpose and the time (in minutes) required for each toy on the machines is given below:

Types of toys	Machines		
	I	II	III
A	20	10	10
B	10	20	30

The machines I, II and III are available for a maximum of 3 hours, 2 hours and 2 hours 30 minutes respectively. The profit on each toy of type A is 50 and that of type B is 60. Formulate the above problem as a L.P.P and solve it graphically to maximize profit.