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SQP 5

Target Mathematics by Dr. Agyat Gupta

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	3(3)	–	1(3)	–	**4(6)**
2.	Inverse Trigonometric Functions	–	1(2)	–	–	**1(2)**
3.	Matrices	2(2)	–	–	–	**2(2)**
4.	Determinants	1(1)*	1(2)	–	1(5)*	**3(8)**
5.	Continuity and Differentiability	–	1(2)	2(6)#	–	**3(8)**
6.	Application of Derivatives	1(4)	1(2)	1(3)*	–	**3(9)**
7.	Integrals	2(2)#	1(2)	1(3)	–	**4(7)**
8.	Application of Integrals	–	1(2)*	1(3)	–	**2(5)**
9.	Differential Equations	1(1)*	1(2)	1(3)	–	**3(6)**
10.	Vector Algebra	1(1) + 1(4)	–	–	–	**2(5)**
11.	Three Dimensional Geometry	2(2)#	1(2)*	–	1(5)*	**4(9)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	4(4)#	2(4)#	–	–	**6(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.
#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : **041**

SQP-5

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. Solve the differential equation $\frac{dy}{dx} = 1 - x + y - xy$.

OR

What is the degree of the differential equation $5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$?

2. If $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$, then find the values of x and y.

3. The random variable X has a probability distribution $P(X)$ of the following form, where 'k' is some number,

$$P(X=x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of 'k'.

OR

Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

4. Check whether the relation R on the set $A = \{1, 2, 3\}$ defined as $R = \{(1, 1), (1, 2), (2, 1), (3, 3)\}$ is reflexive, symmetric and transitive.

5. Evaluate : $\int \frac{\sqrt{x}}{\sqrt{x^2 - x}} dx$

OR

Evaluate : $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$

6. If lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are mutually perpendicular, then find the value of k.

7. Find the vector equation of a plane which is at a distance of 6 units from the origin and which has $\hat{k}$ as the unit vector normal to it.

OR

Find the vector equation of the plane which is at a distance of $\frac{6}{\sqrt{29}}$ units from the origin and its normal vector from the origin is $2\hat{i} - 3\hat{j} + 4\hat{k}$.

8. Prove that for any square matrix A, AA^T is a symmetric matrix.

9. If $\Delta(x) = \begin{vmatrix} f(x) & g(x) \\ a & b \end{vmatrix}$, then prove that $\int \Delta(x) dx = \begin{vmatrix} \int f(x)dx & \int g(x)dx \\ a & b \end{vmatrix}$.

OR

If A is invertible matrix of order 3×3, then prove that $|A^{-1}| = |A|^{-1}$.

10. Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is neither 9 nor 11?

11. Let $f: [2, \infty) \to R$ be the function defined by $f(x) = x^2 - 4x + 5$, then find the range of f.

12. If A and B are two events such that $P(A) = 0.2$, $P(B) = 0.4$ and $P(A \cup B) = 0.5$, then find the value of $P(A/B)$.

13. Evaluate : $\int_{\pi/6}^{\pi/4} (\sec^2 x + \operatorname{cosec}^2 x)\, dx$

14. A bag contains 3 white and 6 black balls while another bag contains 6 white and 3 black balls. A bag is selected at random and a ball is drawn. Find the probability that the ball drawn is of white colour.

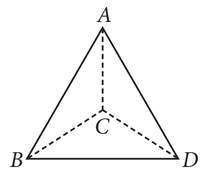
15. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of points A, B, C respectively such that $5\vec{a} - 3\vec{b} - 2\vec{c} = \vec{0}$, then find the ratio in which C divides AB externally.

16. For real numbers x and y, we write $xRy \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Prove that the relation R is not transitive.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. A building of a multinational company is to be constructed in the form of a triangular pyramid, $ABCD$ as shown in the figure.



Let its angular points are $A(3, 0, 1)$, $B(-1, 4, 1)$, $C(5, 2, 3)$ and $D(0, -5, 4)$ and G be the point of intersection of the medians of ΔBCD.

Based on the above answer the following.

(i) The coordinates of points G are

(a) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ (b) $\left(0, \frac{1}{2}, \frac{1}{3}\right)$ (c) $\left(\frac{4}{3}, \frac{1}{3}, \frac{8}{3}\right)$ (d) $\left(\frac{4}{3}, \frac{8}{3}, \frac{1}{3}\right)$

(ii) The length of vector $\overrightarrow{AG}$ is

(a) $\sqrt{17}$ units (b) $\frac{\sqrt{51}}{3}$ units (c) $\frac{3}{\sqrt{6}}$ units (d) $\frac{\sqrt{59}}{4}$ units

(iii) Area of triangle ABC (in sq. units) is

(a) 24 (b) $8\sqrt{6}$ (c) $4\sqrt{6}$ (d) $5\sqrt{6}$

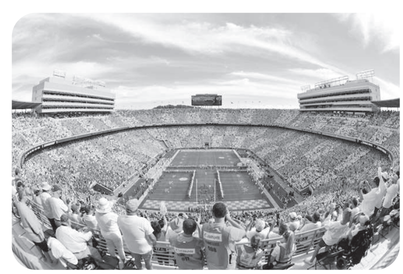
(iv) The sum of lengths of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is

(a) 4 units (b) 9.1 units (c) 8.7 units (d) 6 units

(v) The length of the perpendicular from the vertex D on the opposite face is

(a) $\frac{14}{\sqrt{6}}$ units (b) $\frac{2}{\sqrt{6}}$ units (c) $\frac{3}{\sqrt{6}}$ units (d) $8\sqrt{6}$ units

18. A concert is organised every year in the stadium that can hold 42000 spectators. With ticket price of ₹ 10, the average attendance has been 27000. Some financial expert estimated that price of a ticket should be determined by the function $p(x) = 19 - \frac{x}{3000}$, where x is the number of tickets sold.



Based on the above information, answer the following questions.

(i) The revenue, R as a function of x can be represented as

(a) $19x - \frac{x^2}{3000}$ (b) $19 - \frac{x^2}{3000}$ (c) $19x - \frac{1}{30000}$ (d) $19x - \frac{x}{3000}$

(ii) The range of x is

(a) [27000, 42000] (b) [0, 27000]

(c) [0, 42000] (d) none of these

(iii) The value of x for which revenue is maximum, is

(a) 20000 (b) 27000 (c) 28500 (d) 28000

(iv) When the revenue is maximum, the price of the ticket is

(a) ₹ 8 (b) ₹ 5 (c) ₹ 9 (d) ₹ 9.5

(v) How many spectators should be present to maximize the revenue?

(a) 25000 (b) 27000 (c) 22000 (d) 28500

PART - B

Section - III

19. Solve the differential equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$.

20. Find the area of the larger part bounded by $y = \cos x$, $y = x + 1$ and $y = 0$.

OR

Find the area enclosed by the lines $y = 0$, $y = x$, $x = 1$, $x = 2$.

21. Consider $f(x) = \begin{cases} 3x - 8, & \text{if} \quad x \le 5 \\ 2k, & \text{if} \quad x > 5 \end{cases}$

Find the value of k, if $f(x)$ is continuous at $x = 5$.

22. A random variable X has the following distribution.

X	1	2	3	4	5	6	7	8
$P(X)$	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the event $E = \{X \text{ is prime number}\}$ and $F = \{X < 4\}$, find $P(E \cup F)$.

23. Find the direction cosines of the line passing through the two points $(-2, 4, -5)$ and $(1, 2, 3)$.

OR

If O be the origin and the coordinates of P be $(1, 2, -3)$, then find the equation of the plane passing through P and perpendicular to OP.

24. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then for what value of α, A is an identity matrix?

25. Evaluate : $\int_a^b \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a+b-x}}\, dx$

26. Show, that the function $f(x) = x^9 + 4x^7 + 11$ is increasing on R.

27. Find the number of triplets (x, y, z) satisfying the equation $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$.

28. Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Find p if A and B are (i) mutually exclusive (ii) independent.

OR

A bag contains 12 white pearls and 18 black pearls. Two pearls are drawn in succession without replacement. Find the probability that the first pearl is white and the second is black.

Section - IV

29. Find all points of discontinuity of f, where f is defined as follows :

$$f(x) = \begin{cases} |x| + 3, & x \le -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \ge 3 \end{cases}$$

30. Solve the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1, x\neq 0.$

31. If $y=(\log_{\cos x}\sin x)(\log_{\sin x}\cos x)^{-1}+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, find $\frac{dy}{dx}$ at $x=\frac{\pi}{4}$.

OR

If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{2}$.

32. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$.

33. Let R be a relation on the set N be defined by $\{(x, y) : x, y \in N, 2x + y = 41\}$. Show that R is neither reflexive nor symmetric.

34. Find the interval on which the function $f(x)=\frac{3}{10}x^4-\frac{4}{5}x^3-3x^2+\frac{36}{5}x+11$ is increasing. $= 2,$

OR

Show that $f(x) = \cos(2x + \pi/4)$ is an increasing function on $(3\pi/8, 7\pi/8)$.

35. Evaluate : $\int_0^{\pi/4}\frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

Section - V

36. Find the cartesian equations of the plane through the intersection of the planes $\vec{r}\cdot(2\hat{i}+6\hat{j})+24=0$ and $\vec{r}\cdot(3\hat{i}-\hat{j}+4\hat{k})=0$, which are at a distance of 2 units from the origin.

OR

If the shortest distance between the lines $L_1:\frac{x-1}{1}=\frac{y}{-1}=\frac{z}{2}$ and $L_2:\frac{x+1}{2}=\frac{y}{2}=\frac{z-3}{\lambda}$ is unity, then find the value of λ.

37. Solve the following LPP graphically.

Maximize $Z = 50x + 40y$

Subject to constraints :

$1000x + 1200y \leq 7600$

$12x + 8y \leq 72$

$x, y \geq 0$

OR

Solve the following LPP graphically.

Minimize $Z = 5x + 7y$

Subject to constraints :

$2x + y \leq 8$

$x + 2y \geq 10$

and $x, y \geq 0$

38. If $A=\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$, then find AB. Hence, solve the system of equations :

$x - y = 6, 2x + 3y + 4z = 34, y + 2z = 14$

OR

Solve the system of the following equations : $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$

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