

CLASS XII MATHS

DIFFERENTIATION-I

1-MARK QUESTIONS

1. If $x^y = y^x$ find $\frac{dy}{dx}$
2. Differentiate $\sin x^x$ w.r.t. $\log x$
3. Differentiate $\cos^{-1}\left(\frac{1+x}{1-x}\right)$
4. Differentiate: $\cot^{-1}\left(\frac{1+x}{1-x}\right)$
5. Differentiate: $\log\left[\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right]$
6. Differentiate: $\log_{\sin x} \cos x + \tan x^0$ **
7. If $y = \log(\log_7 x)$ find $\frac{dy}{dx}$

4-MARKS QUESTIONS

- **1. If $y = \sin^{-1}\left(\frac{5x + 12\sqrt{1-x^2}}{13}\right)$, find $\frac{dy}{dx}$.
2. If $y = e^{a \sin^{-1} x}$ prove that $(1-x^2)y_2 - xy_1 - a^2y = 0$.
3. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for $-1 < x < 1$ show that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$ **
4. If $f(x) = \left(\frac{3+x}{1+x}\right)^{2+3x}$, find $f'(0)$
5. $y = x^{\sin x} + (\sin x)^x$, Find $\frac{dy}{dx}$ **
6. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ then prove that $\frac{dy}{dx} = \sqrt{\frac{1-x^2}{1-y^2}}$.

7. Differentiate $\tan^{-1} \frac{2x}{1-x^2}$ w.r.t $\sin^{-1} \frac{2x}{1+x^2}$ 8. If $y = \frac{\log x}{x}$, Show that $\frac{d^2y}{dx^2} = \frac{2 \log x - 3}{x^3}$ **

9. Differentiate w.r.t.x: $\tan^{-1} \left(\frac{2x}{1+15x^2} \right) + \tan^{-1} \left(\frac{5ax}{a^2 - 6x^2} \right)$

10. Differentiate $\sin^{-1} \left(\frac{1-x^2}{1+x^2} \right)$ w.r.to $\tan^{-1} \frac{\sqrt{1+x^2} - 1}{x}$

11. If $y = \left[\log(x + \sqrt{x^2 + 1}) \right]^2$ prove that $(1+x^2) y_2 + xy_1 = 2$. **

12. If $x = \sin t$, $y = \sin pt$, prove that $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2y = 0$ **

13. If $y = e^{ax} \sin bx$, then prove that $\frac{d^2y}{dx^2} - 2a \frac{dy}{dx} + (a^2+b^2)y = 0$

14. Find $\frac{dy}{dx}$, if $x = \frac{1}{2} e^t (\cos t + \sin t)$, $y = \frac{1}{2} e^t (\cos t - \sin t)$

15. If $x = a \cos^3 \theta$, $y = a \sin^3 \theta$, find $\frac{d^2y}{dx^2}$ 16. If $x = a(\theta - \sin \theta)$, $y = a(1 + \cos \theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$.

17. If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y} \left(\frac{dy}{dx} \right)^2 - \frac{y}{x} = 0$. 18. If $y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$, find $\frac{dy}{dx}$

19. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$

20. If $y = (\sin^{-1} x)^2$, prove that $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2 = 0$

21. If $x^p y^q = (x+y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$ and $\frac{d^2y}{dx^2} = 0$

22. If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$ prove that $\frac{dy}{dx} = -\frac{y}{x}$

23. If $y = (\tan^{-1} x)^2$, prove that $(1+x^2)y_2 + 2x(1+x^2)y_1 = 2$

24. If $x^y = e^{x-y}$ then show that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

25. Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$

26. Differentiate with respect to x: $\tan^{-1} \left[\frac{\sqrt{1+x^2} - 1}{x} \right]$

27. If $\cos y = x \cos(a + y)$, Show that $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a}$

28. Differentiate $\tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right)$ w.r.t. $\sin^{-1} \left(\frac{2x}{1+x^2} \right)$

29. Differentiate $\tan^{-1} \frac{2x}{1-x^2}$ w.r.t $\sin^{-1} \frac{2x}{1+x^2}$ **

30. If $x=3\sin t - \sin 3t$, $y=3\cos t - \cos 3t$ find $\frac{d^2y}{dx^2}$

36. If $y = \sin(m(\sin^{-1}x))$, prove that $(1-x^2)y_2 - xy_1 + m^2y = 0$.

37. If $x = a(\cos\theta + \log \tan \frac{\theta}{2})$ and $y = a \sin\theta$ then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

38. If $y = \sin^{-1}x$, Prove that $\frac{d^2y}{dx^2} = \frac{x}{(1-x^2)^{\frac{3}{2}}}$ 39. Differentiate: $\tan^{-1} \left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right]$

40. If $x=2\sin t - \sin 2t$, $y=2\cos t - \cos 2t$ find $\frac{d^2y}{dx^2}$

41. If $x = \frac{1 + \log t}{t^2}$, $y = \frac{3 + 2 \log t}{t}$, $t > 0$ prove that $yy_1 - 2xy_1^2 = 1$

**42. Find $\frac{dy}{dx}$ if $y = \left(x + \frac{1}{x}\right)^x + x^{\left(1 + \frac{1}{x}\right)}$ ** 43. If $y = b \tan^{-1} \left(\frac{x}{a} + \tan^{-1} \frac{y}{x} \right)$, find $\frac{dy}{dx}$

**44. If $y = x \log \left(\frac{x}{a + bx} \right)$ prove that $\frac{d^2y}{dx^2} = \frac{1}{x} \left(\frac{a}{a + bx} \right)^2$

**45. If $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$, $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$, prove that $\frac{dy}{dx} = -\cot 3t$

**46. If $y = \log\left(\frac{x}{a+bx}\right)^x$, then prove that $x^3 \frac{d^2y}{dx^2} = \left(x \frac{dy}{dx} - y\right)^2$

**47. If $(a+bx) e^{\frac{y}{x}} = x$ then prove that $x^3 \frac{d^2y}{dx^2} = \left(x \frac{dy}{dx} - y\right)^2$

48. If $y = \left[x + \sqrt{x^2 + a^2}\right]^n$ then prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}$.

49. If $y\sqrt{x^2 + 1} = \log\left[\sqrt{x^2 + 1} - x\right]$, show that $(x^2 + 1)\frac{dy}{dx} + xy + 1 = 0$

**50. If $f(1) = 4$, $f'(1) = 2$, find the value of derivative of $\log f(e^x)$ w.r.t x at the point $x = 0$.

**51. If $x^2 + y^2 = t - \frac{1}{t}$, $x^4 + y^4 = t^2 + \frac{1}{t^2}$, prove that $xyy_1 = 1$

**52. If $\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3 - y^3)$ prove that $\frac{dy}{dx} = \frac{x^2\sqrt{1-y^6}}{y^2\sqrt{1-x^6}}$

**53. If $y = \cos^{-1}\left(\frac{3\cos x - 4\sin x}{5}\right)$, find $\frac{dy}{dx}$. **54. If $\sin y = \sqrt{x}\sqrt{1-x^4} - x^2\sqrt{1-x}$, find $\frac{dy}{dx}$.

55. If $y = \left[x + \sqrt{x^2 + 1}\right]^n$ then prove that $(1+x^2)y_2 + xy_1 = n^2y$.

**56. If $x = a\sin 2t(1 + \cos 2t)$ and $y = b\cos 2t(1 - \cos 2t)$, show that $\frac{dy}{dx} = b/a$ at $t = \pi/4$.

57. If $x = \sin\left(\frac{1}{a} \log y\right)$, show that $(1-x^2)y_2 - xy_1 - a^2y = 0$.

**58. If $y = \tan^{-1}\left(\frac{1-x}{1+x}\right) - \tan^{-1}\left(\frac{x+2}{1-2x}\right)$, find $\frac{dy}{dx}$.

59. If $y = e^x \tan^{-1}x$, then prove that $(1+x^2)\frac{d^2y}{dx^2} - 2(1-x+x^2)\frac{dy}{dx} + (1-x)^2y = 0$

**60. If $y = \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1}\left[\sqrt{\frac{a-b}{a+b}} \tan \frac{x}{2}\right]$ prove that $\frac{dy}{dx} = \frac{1}{a + b \cos x}$, $a > b > 0$