S R Study Material

SAMPLE PAPER 1

Class 12 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and a $\in$ A, then the set A is **[1]**

a) {0} b) $\mathbb{R}$

c) {4} d) $\mathbb{R}$ - {4}

2. If A is a square matrix of order 3 and |A| = 6, then the value of |adj A| is: **[1]**

a) 216 b) 6

c) 27 d) 36

3. $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then the value of |adj A| is **[1]**

a) 16 b) -8

c) 0 d) 64

4. The value of det. $\begin{bmatrix} a & 0 & 0 & 0 \\ 2 & b & 0 & 0 \\ 4 & 6 & c & 0 \\ 6 & 8 & 10 & d \end{bmatrix}$ is **[1]**

a) a + b + c + d b) 1

c) abcd
d) 0

5. If a line makes an angle of 30^o with the positive direction of x-axis, 120^o with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is: **[1]**

a) 0^o
b) 60^o
c) 90^o
d) 120^o

6. The solution of $\frac{dy}{dx}=\sqrt{1-x^2-y^2+x^2y^2}$ is, where, C is an arbitrary constant. **[1]**

a) $2\sin^{-1} y = x\sqrt{1-x^2} + \cos^{-1} x + C$
b) $2\sin^{-1} y = \sqrt{1-x^2} + \sin^{-1} x + C$
c) $2\sin^{-1} y = x\sqrt{1-x^2} + \sin^{-1} x + C$
d) $\sin^{-1} y = \sin^{-1}x + C$

7. A Linear Programming Problem is as follows: **[1]**
Maximize/Minimize objective function Z = 2x - y + 5
Subject to the constraints
3x + 4y $\leq$ 60
x + 3y $\leq$ 30
x $\leq$ 0, y $\geq$ 0
In the corner points of the feasible region are A(0, 10), B(12, 6), C(20, 0) and O(0,0), then which of the following is true?

a) Minimum value of Z is -5
b) At two corner points, value of Z are equal
c) Maximum value of Z is 40
d) Difference of maximum and minimum values of Z is 35

8. The two lines x = ay + b, z = cy + d; and x = $a'y+b', z=c'y+d'$ are perpendicular to each other, if: **[1]**

a) $aa'+cc'=-1$
b) aa' + cc' = 1
c) $\frac{a}{a'}+\frac{c}{c'}=-1$
d) $\frac{a}{a'}+\frac{c}{c'}=1$

9. $\int \frac{1}{x \log x}$ dx is equal to **[1]**

a) $\frac{1}{\log x}+c$
b) log |x log x| + c
c) log |log x| + c
d) $\frac{(\log x)^2}{2}+c$

10. If a matrix A = [1 2 3], then the matrix AA' (where A' is the transpose of A) is: **[1]**

a) [14]
b) 14
c) $\begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix}$
d) $\begin{bmatrix}1&2&3\\2&3&1\\3&1&2\end{bmatrix}$

11. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5).Let F = 4x + 6y be the objective function. The Minimum value of F occurs at **[1]**

a) (0, 2) only
b) (3, 0) only
c) any point on the line segment joining the points (0, 2) and (3, 0).
d) the mid-point of the line segment joining the points (0, 2) and (3, 0) only

12. A unit vector $\hat{a}$ makes equal but acute angles on the co-ordinate axes. The projection of the vector $\hat{a}$ on the **[1]**

vector $\vec{b} = 5\hat{i} + 7\hat{j} - \hat{k}$ is

a) $\frac{3}{5\sqrt{3}}$

b) $\frac{11}{15}$

c) $\frac{4}{5}$

d) $\frac{11}{5\sqrt{3}}$

13. If A = $\begin{vmatrix} 3 & 1 \\ 2 & 5 \end{vmatrix}$ then find A^{-1}. **[1]**

a) $\frac{1}{2}\begin{vmatrix} 5 & -1 \\ -2 & 3 \end{vmatrix}$

b) $\begin{vmatrix} 5 & -1 \\ -2 & 3 \end{vmatrix}$

c) $\frac{1}{13}\begin{vmatrix} 5 & 2 \\ -1 & 3 \end{vmatrix}$

d) $\frac{1}{13}\begin{vmatrix} 5 & -1 \\ -2 & 3 \end{vmatrix}$

14. If for two events A and B, P(A - B) = $\frac{1}{5}$ and P(A) = $\frac{3}{5}$, then P $\left(\frac{B}{A}\right)$ is equal to **[1]**

a) $\frac{1}{2}$

b) $\frac{2}{5}$

c) $\frac{2}{3}$

d) $\frac{3}{5}$

15. For the differential equation $xy\frac{dy}{dx} = (x+2)(y+2)$ find the solution curve passing through the point (1, –1). **[1]**

a) $y + x + 2 = log\left(x^2(y+2)^2\right)$

b) $y - x - 2 = log\left(x^2(y-2)^2\right)$

c) $y - x + 2 = log\left(x^2(y+2)^2\right)$

d) $y - x - 2 = log\left(x^2(y+2)^2\right)$

16. For any two vectors $\vec{a}$ and $\vec{b}$ which of the following statements is always true? **[1]**

a) $\vec{a} \cdot \vec{b} \geq |\vec{a}||\vec{b}|$

b) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$

c) $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$

d) $\vec{a} \cdot \vec{b} < |\vec{a}||\vec{b}|$

17. The vertical asymptotes to curve y = $\frac{e^x}{x}$ is **[1]**

a) x = 2

b) x = 1

c) Curve has no any asymptotes

d) x = 0

18. If a line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\theta$ is equal to **[1]**

a) $\frac{1}{3}$

b) $\frac{2}{3}$

c) $\frac{8}{3}$

d) $\frac{4}{3}$

19. **Assertion (A):** A particle moving in a straight line covers a distance of x cm in t second, where x = $t^3 + 3t^2 - 6t + 18$. The velocity of particle at the end of 3 seconds is 39 cm/s. **[1]**

Reason (R): Velocity of the particle at the end of 3 seconds is $\frac{dx}{dt}$ at t = 3.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. If R is the relation in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a - b| is even}, **[1]**

Assertion (A): R is an equivalence relation.

Reason (R): All elements of {1, 3, 5} are related to all elements of {2, 4}.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$. **[2]**

OR

Using the principal values, write the value of $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$.

22. A particle moves along the curve 6y = x^3 + 2. Find the points on the curve at which y-coordinates is changing 2 times as fast as x - coordinates. **[2]**

23. Find the interval in which the function $f(x)=x^8+6x^2$ is increasing or decreasing. **[2]**

OR

The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm.

24. Find $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$ **[2]**

25. Prove that the function f given by f(x) = x^2 - x + 1 is neither strictly increasing nor strictly decreasing on (-1, 1). **[2]**

Section C

26. Evaluate: $\int_1^2\left(x^2-1\right)dx$ **[3]**

27. In a set of 10 coins, 2 coins are with heads on both the sides. A coin is selected at random from this set and tossed five times. If all the five times, the result was heads, find the probability that the selected coin had heads on both the sides. **[3]**

28. Evaluate: $\int_0^1|5x-3|dx$ **[3]**

OR

Prove that $\int_0^{\pi/2}\log\sin x dx=\int_0^{\pi/2}\log\cos x dx=-\frac{\pi}{2}\log 2$

29. Show that the differential equation (x^3 + y^3)dy - x^2ydx = 0 is homogeneous and solve it. **[3]**

OR

Form the differential equation representing the family of curves given by $(x-a)^2$ + $2y^2$ = a^2, where a is an arbitrary constant.

30. Minimize Z = 3x + 2y subject to the constraints: **[3]**

$x+y\geq 8$

$3x+5y\leq 15$

$x\geq 0,\ y\geq 0$

OR

Solve the following linear programming problem graphically:

Maximise z = 3x + 9y

subject to constraints

$x+3y\leq 60$

$x+y\geq 10$

$x\leq y$

$x,y\geq 0$

31. If x= a(cos t + t sin t) and y = a (sin t - t cos t), then find $\frac{d^2x}{dt^2}$, $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$. **[3]**

Section D

32. Find the area enclosed by the parabola 4y = $3x^2$ and the line 2y = 3x + 12. **[5]**

33. Let L be the set of all lines in xy plane and R be the relation in L define as R = {(L_1, L_2) : $L_1 \parallel L_2$}. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x + 4. **[5]**

OR

Let A = [-1, 1]. Then, discuss whether the following functions defined on A are one-one, onto or bijective:

i. $f(x) = \frac{x}{2}$

ii. g(x) = |x|

iii. h(x) = x|x|

iv. k(x) = x^2

34. Solve the system of equations **[5]**

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$

$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$

$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

35. Find the shortest distance between the lines given by $\overrightarrow{r} = (8 + 3\lambda)\hat{i} - (9 + 16\lambda)\hat{j} + (10 + 7\lambda)\hat{k}$ and $\overrightarrow{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$. **[5]**

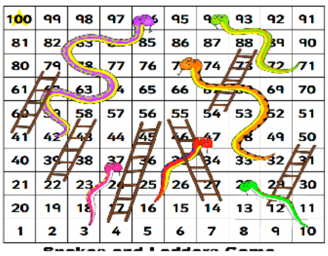
OR

Find the shortest distance between the lines $\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$ and $\vec{r} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Akshat and his friend Aditya were playing the snake and ladder game. They had their own dice to play the game. Akshat was having red dice whereas Aditya had black dice. In the beginning, they were using their own dice to play the game. But then they decided to make it faster and started playing with two dice together.



Aditya rolled down both black and red die together.

First die is black and second is red.

i. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5. (1)

ii. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4. (1)

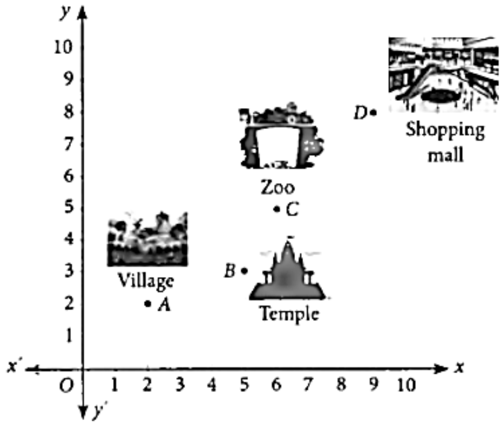
iii. Find the conditional probability of obtaining the sum 10, given that the black die resulted in even number. (2)

OR

Find the conditional probability of obtaining the doublet, given that the red die resulted in a number more than 4. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Girish left from his village on weekend. First, he travelled up to temple. After this, he left for the zoo. After this he left for shopping in a mall. The positions of Girish at different places is given in the following graph.



i. Find position vector of B (1)

ii. Find position vector of D (1)

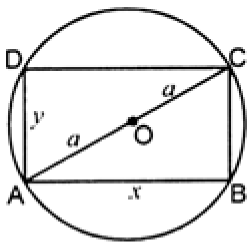
iii. Find the vector $\overrightarrow{BC}$ in terms of $\hat{i}$, $\hat{j}$. (2)

OR

iv. Find the length of vector $\overrightarrow{AD}$. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)



i. Find the perimeter of rectangle in terms of any one side and radius of circle. (1)

ii. Find critical points to maximize the perimeter of rectangle? (1)

iii. Check for maximum or minimum value of perimeter at critical point. (2)

OR

If a rectangle of the maximum perimeter which can be inscribed in a circle of radius 10 cm is square, then the perimeter of region. (2)