S R Study Material

SAMPLE PAPER 3

Class 12 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. If the matrix A is both symmetric and skew symmetric, then **[1]**

   a) A is a null matrix b) A is a zero matrix

   c) A is a square matrix d) A is a diagonal matrix

2. The value of the determinant of a skew symmetric matrix of even order is **[1]**

   a) A non zero perfect square b) Positive

   c) 0 d) Negative

3. The equations, x + 4y – 2z = 3, 3x + y + 5z = 7, 2x + 3y +z = 5 have **[1]**

   a) no solution b) two solution

   c) a unique solution d) infinitely many solutions

4. If x = 2 cos θ - cos 2θ and y = 2 sin θ - sin 2θ, then $\frac{dy}{dx}$ is: **[1]**

   a) $\frac{\cos\theta-\cos 2\theta}{\sin 2\theta-\sin\theta}$ b) $\frac{\cos 2\theta-\cos\theta}{\sin 2\theta+\sin\theta}$

   c) $\frac{\cos\theta-\cos 2\theta}{\sin\theta-\sin 2\theta}$ d) $\frac{\cos\theta+\cos 2\theta}{\sin\theta-\sin 2\theta}$

5. The angle between the lines $\frac{x-2}{2}=\frac{y-1}{7}=\frac{z+3}{-3}$ and $\frac{x+2}{-1}=\frac{y-4}{2}=\frac{z-5}{4}$ is **[1]**

   a) $\cos^{-1}\left(\frac{3}{8}\right)$ b) $\frac{\pi}{3}$

   c) $\frac{\pi}{6}$ d) $\frac{\pi}{2}$

6. The solution of the differential equation $\frac{dy}{dx} + 1 = e^{x+y}$ is **[1]**

a) $(x + C)e^{x + y} = 0$

b) $(x+y)e^{x+y} = 0$

c) $(x - C) e^{x+y} + 1 = 0$

d) $(x-C)e^{x+y} = 1$

7. The value of objective function is maximum under linear constraints **[1]**

a) at (0, 0)

b) at any vertex of feasible region

c) the vertex which is maximum distance from (0, 0)

d) at the centre of feasible region

8. The value of $(\vec{a} \times \vec{b})^2$ is **[1]**

a) $|\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$

b) $|\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a} \cdot \vec{b})$

c) $|\vec{a}|^2 + |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$

d) $|\vec{a}|^2 + |\vec{b}|^2 - \vec{a} \cdot \vec{b}$

9. $\int$ x sin 2x dx = ? **[1]**

a) - $\frac{1}{3}$ x cos 3x + $\frac{1}{2}$ sin 2x + C

b) $\frac{1}{2}$ x cos 2x - $\frac{1}{4}$ sin 2x + C

c) $\frac{1}{2}$ x cos 2x + $\frac{1}{4}$ sin 2x + C

d) - $\frac{1}{2}$ x cos 2x + $\frac{1}{4}$ sin 2x + C

10. If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, then find (x - y). **[1]**

a) 12

b) 15

c) 10

d) 7

11. The optimal valuie of the objective function is attained at the points **[1]**

a) given by corner points of the feasible region

b) given by intersection of inequations with the axes only

c) given by intersection of inequations with y-axis only

d) given by intersection of inequations with x-axis only

12. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\overrightarrow{\mathrm{C}}$ is a unit vector perpendicular to both $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$. If the angle between $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to **[1]**

a) $\frac{1}{4}|\vec{a}|^2|\vec{b}|^2$

b) $\frac{3}{4}|\vec{a}|^2|\vec{b}|^2$

c) 0

d) 1

13. The roots of the equation det. $\begin{vmatrix} 1-x & 2 & 3 \\ 0 & 2-x & 0 \\ 0 & 2 & 3-x \end{vmatrix} = 0$ are **[1]**

a) 3

b) 2 and 3

c) 1 , 2 and 3

d) 1 and 3

14. If A and B are two events such that $P(A) = \frac{4}{5}$, and $P(A \cap B) = \frac{7}{10}$, then P (B / A) = **[1]**

a) $\frac{1}{10}$

b) $\frac{17}{20}$

c) $\frac{7}{8}$

d) $\frac{1}{8}$

15. What is the solution of the differential equation $\frac{ydx-xdy}{y^2}$ = 0? **[1]**

a) x + y = C
b) x - y = C
c) xy = C
d) y = Cx

16. Vectors $\vec{a}$ and $\vec{b}$ are inclined at angle. $\theta = 120^\circ$. If $|\vec{a}| = 1, |\vec{b}| = 2$, then $[(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})]^2$ is equal to **[1]**

a) 325
b) 225
c) 300
d) 275

17. The function f(x) = $\frac{1}{x-1}$ at x = 1 **[1]**

a) is continuous
b) has asymptotic discontinuity
c) has removable discontinuity
d) has jump discontinuity

18. Direction cosines of a line perpendicular to both x-axis and z-axis are: **[1]**

a) 0, 1, 0
b) 0, 0, 1
c) 1, 1, 1
d) 1, 0, 1

19. **Assertion (A):** Minimum value of (x - 5)(x -7) is -1. **[1]**

Reason (R): Minimum value of $ax^2 + bx + c$ is $\frac{4ac-b^2}{4a}$.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** If A = {0, 1} and N be the set of natural numbers. Then, the mapping f : N $\rightarrow$ A defined by f (2n - 1) = 0, f (2n) = 1, $\forall$ n$\in$ N, is onto. **[1]**

Reason (R): Range = Codomain

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Evaluate $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$ **[2]**

OR

Find the principal value of $\text{cosec}^{-1}(-\sqrt{2})$.

22. Show that $f(x) = \cos(2x + \frac{\pi}{4})$ is an increasing function on $(\frac{3\pi}{8}, \frac{7\pi}{8})$ **[2]**

23. Find the intervals of function $f(x) = 2x^3 - 24x + 5$ is **[2]**

a. increasing
b. decreasing.

OR

Show that $f(x) = (x – 1) e^x + 1$ is an increasing function for all x > 0.

24. Evaluate $\int_1^2 |x-3|dx$ **[2]**

25. Find all the points of local maxima and local minima of the function f given by $f(x) = 2x^3 – 6x^2 + 6x +5$. **[2]**

Section C

26. Evaluate the integral: $\int(x+1)\sqrt{2x^2+3}dx$ **[3]**

27. The contents of three bags I, II and III are as follows: **[3]**

Bag 1 : 1 white, 2 black and 3 red balls,

Bag II : 2 white, 1 black and 1 red ball;

Bag III : 4 white, 5 black and 3 red balls.

A bag is chosen at random and two balls are drawn. What is the probability that the balls are white and red?

28. Evaluate: $\int \frac{(2x+1)}{(x+2)(x-3)} dx.$ **[3]**

OR

Evaluate: $\int \frac{x^2}{(x^2+4)(x^2+25)} dx.$

29. Solve differential equation: $y^2 + \left(x + \frac{1}{y}\right) \frac{dy}{dx} = 0$ **[3]**

OR

Solve the initial value problem: $\tan x \frac{dy}{dx} = 2x \tan x + x^2 - y$; $\tan x \neq 0$ given that y = 0 when $x = \frac{\pi}{2}$

30. Show that the solution set of the linear constraints is empty: $x - 2y \geq 0, 2x - y \leq -2, x \geq 0$ and $y \geq 0$ **[3]**

OR

Maximise Z = 3x + 4y, subject to the constraints: $x + y \leqslant 1, x \geqslant 0, y \geqslant 0$.

31. Find $\frac{dy}{dx}$ of the function $(\cos x)^y = (\cos y)^x$. **[3]**

Section D

32. Find Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the lines x + y = 2. **[5]**

33. Let n be a positive integer. Prove that the relation R on the set Z of all integers numbers defined by $(x, y) \in R \Leftrightarrow$ x - y is divisible by n, is an equivalence relation on Z. **[5]**

OR

Let A be the set of all human beings in a town at a particular time. Determine whether each of the following relations are reflexive, symmetric and transitive:

i. R = {(x, y): x and y work at the same place}
ii. R = {(x, y): x and y live in the same locality}

34. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find A^{-1} **[5]**

35. Find the distance of the point (-1,-5,-10) from the point of intersection of the line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot \left(\hat{i} - \hat{j} + \hat{k}\right) = 5$. **[5]**

OR

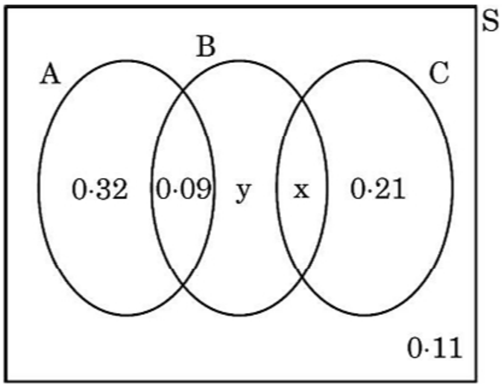
Find the length shortest distance between the lines: $\frac{x-3}{3} = \frac{y-8}{-1} = z - 3$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below:

The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0.44.



i. Find the value of x. (1)

ii. Find the value of y. (1)

iii. Find $P\left(\frac{C}{B}\right)$. (2)

OR

Find the probability that a randomly selected person of the society does Yoga of type A or B but not C. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Three slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (Hub of Learning), B (Creating a better world for tomorrow) and C (Education comes first). The coordinates of these points are (1, 4, 2), (3, -3, -2) and (-2, 2, 6) respectively.

i. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the position vectors of points A, B and C respectively, then find $\vec{a} + \vec{b} + \vec{c}$. (1)

ii. What is the Area of $\triangle$ABC. (1)

iii. Suppose, if the given slogans are to be placed on a straight line, then find the value of $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$. (2)

OR

If $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, then find the unit vector in the direction of vector $\vec{a}$. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Ankit wants to construct a rectangular tank for his house that can hold 80 ft^3 of water. He wants to construct on one corner of terrace so that sufficient space is left after construction of tank. For that he has to keep width of tank constant 5ft, but the length and heights are variables. The top of the tank is open. Building the tank cost ₹20 per sq. foot for the base and ₹10 per sq. foot for the side.

i. Express cost of tank as a function of height(h). (1)

ii. Verify by second derivative test that cost is minimum at critical point. (1)

iii. Find the value of h at which c(h) is minimum. (2)

OR

Find the minimum cost of tank? (2)