Maple Online Classes

A-25 DLF Loni Ghaziabad UP 201301

TEST PAPER: VECTOR ALGEBRA AND THREE DIMENSIONAL GEOMETRY

Class 12 - Mathematics

Time Allowed: 2 hours **Maximum Marks: 70**

General Instructions:

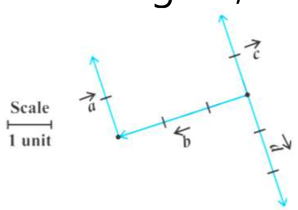
All Questions Are Compulsory.

Section A

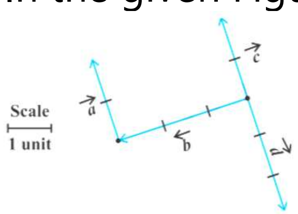
1. Is the measure of 10 Newton a scalar or vector? **[1]**

2. Is the measure of 20 m/s towards north is scalar or vector? **[1]**

3. In the Figure, which are the Collinear vector. **[1]**



4. In the given Figure, which vectors are the Coinitial vectors?. **[1]**



5. Consider two points P and Q with position vectors $\overrightarrow{OP} = 3\vec{a} - 2\vec{b}$ and $\overrightarrow{OQ} = \vec{a} + \vec{b}$. Find the position vector (internally) of a point R which divides the line joining P and Q in the ratio 2 : 1. **[1]**

6. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. **[1]**

7. Find angle θ between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ **[1]**

8. Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of x-axis. **[1]**

a) $-\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}$

b) $\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}$

c) $-\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

d) $\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

9. A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure. **[1]**

a) $\frac{5}{2}\hat{i} - \frac{3\sqrt{3}}{2}\hat{j}$

b) $\frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

c) $\frac{-5}{2}\hat{i} - \frac{3\sqrt{3}}{2}\hat{j}$

d) $\frac{5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

10. Find the value of x for which $x\left(\hat{i} + \hat{j} + \hat{k}\right)$ is a unit vector. **[1]**

a) $\pm\frac{1}{\sqrt{2}}$

b) $\pm\frac{1}{\sqrt{3}}$

c) $\pm\frac{1}{\sqrt{7}}$

d) $\pm\frac{1}{\sqrt{5}}$

11. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\left(2\vec{a}+\vec{b}\right)and\left(\vec{a}-3\vec{b}\right)$ externally in the ratio 1 : 2. Also, show that P is the mid point of the line segment RQ. **[1]**

a) $5\vec{a}+5\vec{b}$

b) $5\vec{a}+3\vec{b}$

c) $3\vec{a}+3\vec{b}$

d) $3\vec{a}+5\vec{b}$

12. The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area. **[1]**

a) $\frac{1}{7}\left(-3\hat{i}-6\hat{j}+2\hat{k}\right);$ $13\sqrt{5}$

b) $\frac{1}{7}\left(3\hat{i}-6\hat{j}-2\hat{k}\right);12\sqrt{5}$

c) $\frac{1}{7}\left(3\hat{i}-6\hat{j}-7\hat{k}\right);15\sqrt{5}$

d) $\frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right);11\sqrt{5}$

13. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if **[1]**

a) $\theta=\frac{\pi}{3}$

b) $\theta=\frac{2\pi}{3}$

c) $\theta=\frac{\pi}{2}$

d) $\theta=\frac{\pi}{4}$

14. The value of $\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{i}\times\hat{k})+\hat{k}\cdot(\hat{i}\times\hat{j})$ is **[1]**

a) -1

b) 1

c) 0

d) 3

15. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $\left|\vec{a}.\vec{b}\right|=\left|\vec{a}\times\vec{b}\right|$ when θ is equal to **[1]**

a) π

b) $\frac{\pi}{2}$

c) $\frac{\pi}{4}$

Section B

16. Show that the points $A\left(2\hat{i}-\hat{j}+\hat{k}\right), B\left(\hat{i}-3\hat{j}-5\hat{k}\right), C\left(3\hat{i}-4\hat{j}-4\hat{k}\right)$ are the vertices of a right angled triangle. **[2]**

17. If $\vec{a}=5\hat{i}-\hat{j}-3\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}-5\hat{k}$, then show that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular. **[2]**

18. If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=8$, then find $|\vec{x}|$. **[2]**

19. Find the area of a triangle having the points A(1, 1, 1), B(1, 2, 3) and C(2, 3, 1) as its vertices. **[2]**

20. If $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}, 2\hat{\mathbf{i}}+5\hat{\mathbf{j}}, 3\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ and $\hat{\mathrm{i}}-6\hat{\mathrm{j}}-\hat{\mathrm{k}}$ are the position vectors of points A, B, C and D respectively, then find the angle between $\vec{AB}$ and $\vec{CD}$. Deduce that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear. **[2]**

21. If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$, $\vec{\alpha}=3\hat{i}-\hat{j}$, $\vec{\beta}=2\hat{i}+\hat{j}-3\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$, where $\vec{\beta}_1$ is || to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$. **[2]**

22. Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Evaluate the quantity $\mu=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$, if $|\vec{a}|=3, |\vec{b}|=4$ and $|\vec{c}|=2$. **[2]**

23. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=3, \left|\vec{b}\right|=4, |\vec{c}|=5$ and each one of them being perpendicular to the sum of the other two, find $\left|\vec{a}+\vec{b}+\vec{c}\right|$. **[2]**

24. Find the direction cosines of a line which makes equal angles with the co-ordinate axes. **[2]**

25. Show that the points (2, 3, 4), (-1, -2, 1), (5, 8, 7) are collinear. **[2]**

Section C

26. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d} = 15$. **[3]**

27. Show that each of the given three vectors is a unit vector: **[3]**
$\frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right), \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right), \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)$
Also, show that they are mutually perpendicular to each other.

28. If either vector $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a} \cdot \vec{b}$ = 0. But the converse need not be true. Justify your answer with an example. **[3]**

29. Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ from the vertices of a right angled triangle. **[3]**

30. Find the distance between the lines l_1 and l_2 given by **[3]**
$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$
and $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

Section D

31. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$ find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$ **[5]**

32. Show that the line through points (4, 7, 8), (2, 3, 4) is parallel to the line through the points (-1, -2, 1), (1, 2, 5). **[5]**

33. Show that the line through the points (1, -1, 2), (3, 4, -2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6). **[5]**

34. Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other. **[5]**