A NOTE

Hey! Hope you will the paper and questions. Most of the questions are available on the internet and some are even higher order thinking skills based ones and some being multidisciplinary evaluative questions. The objective of this paper is not to maximise marks but learnings. The reason it has been published so early is that now is the time for you to practice harder questions and improve critical thinking and learning.

You may not crack all the questions in one go but once you solve all the questions, I am sure you would be one step closer to acing the board exams. Practice this paper in under 3 hours and in case you want me to evaluate your paper, drop me an email in the undersigned and you can also reach out to me for the solutions and further guidance/ question papers if needed.

PLEASE NOTE THAT I AM NOT RUNNING ANY TUITION OR ACADEMY CLASSES AND THE HELP (ZERO CHARGE) WOULD BE ONLY TO AID IN YOUR DEVELOPMENT, NO OTHER MOTIVE

Email: iammyself9310@gmail.com **Telegram: @Methinker**

Class- X
Session- 2022-23
Subject- Mathematics (Standard)
Question Paper

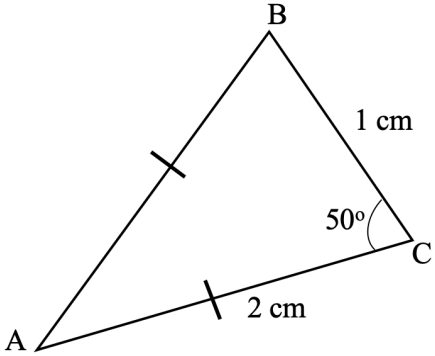
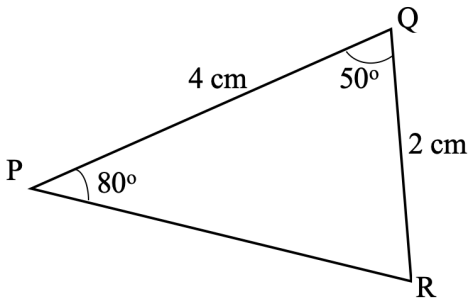
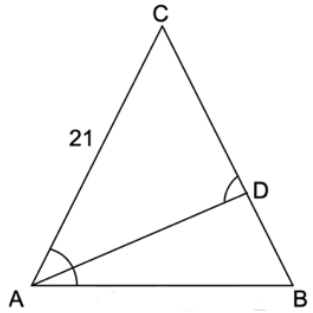
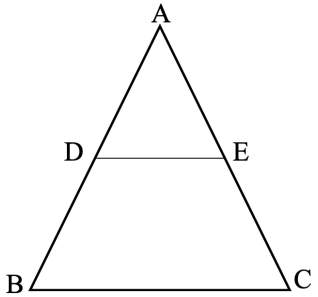
Time Allowed : 3 hrs **Maximum Marks : 80**

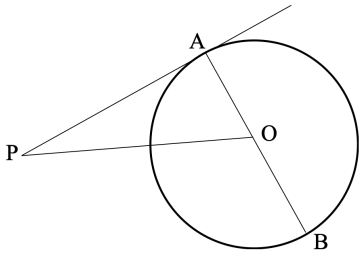
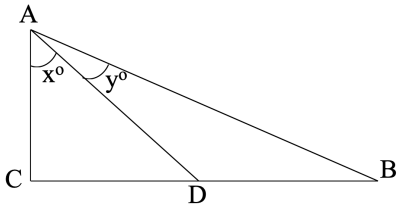
General Instructions

- This Question Paper has 5 Sections A-E.
- Section A has 20 MCQs carrying 01 mark each.
- Section B has 5 questions carrying 02 marks each.
- Section C has 6 questions carrying 03 marks each.
- Section D has 4 questions carrying 05 marks each.
- Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
- All Questions are compulsory.
- Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

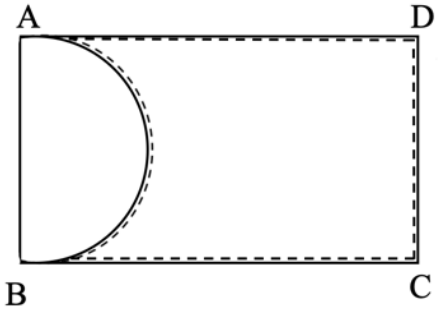
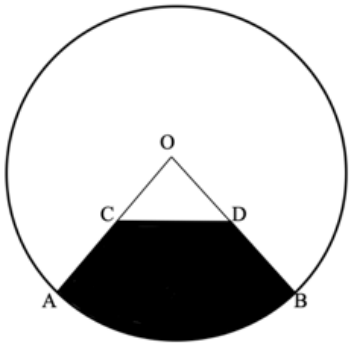
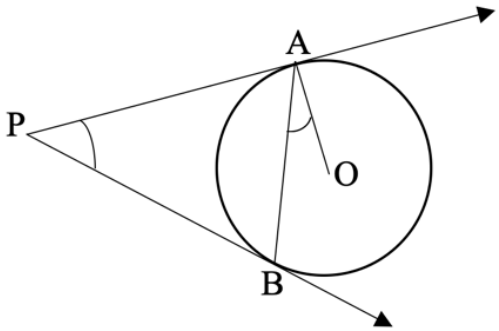
SECTION A (1 Marks Each)

1	Which of the following digits is ruled out in the units place of $12^n + 1$ for every positive integer n? (a) 1 (b) 3 (c) 5 (d) 7	**1**
2	Polynomial $x^2 - 6x + 25$ has zeroes β and α. Equation $2x^2 - px + q = 0$ has roots $\sqrt{\alpha}$ and $\sqrt{\beta}$. What is the value of p? (a) 6 (b) 8 (c) -16 (d) 13	**1**
3	One equation of a pair of dependent linear equations is $-5x + 7y = 2$. The second equation can be : (a) $10x + 14y = -4$ (b) $10x = 4 - 14y$ (c) $14y + 4 = 10x$ (d) $10x - 14y = -4$	**1**
4	Let m and n be positive integers, If $x^2 + mx + 2n$ and $x^2 + 2nx + m$ are perfect squares, then one of the value of m + n is : (a) 8 (b) 6 (c) 5 (d) 7	**1**
5	If centroid of equilateral ΔABC is T (2,3) and one of its vertices is A (-1, 2), find its median's length. (a) $\sqrt{10}$ (b) $\frac{\sqrt{40}}{3}$ (c) $\frac{3\sqrt{5}}{4}$ (d) $\sqrt{\frac{45}{2}}$	**1**

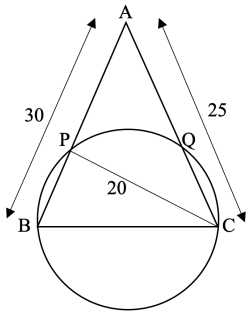
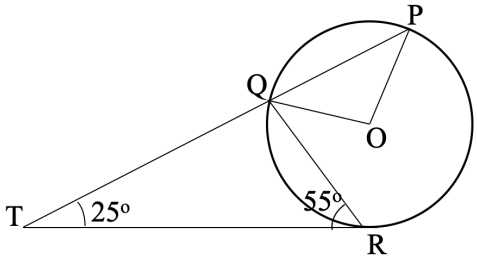
6	With reference to the given figure, which of the following is true?   (a) ΔBAC ~ΔPQR (b) ΔCAB ~ΔRQP (c) ΔACB ~ΔPRQ (d) ΔBCA ~ΔQPR	**1**
7	Which of the following has smallest value? (a) $\frac{\sin 60^0}{\tan 30^0}$ (b) $\frac{\sec 60^0}{\cot 30^0}$ (c) $\frac{\text{cosec } 45^0}{\sec 45^0}$ (d) $\frac{\cos 60^0}{\tan 45^0}$	**1**
8	Let D be a point on the side BC of a triangle ABC such that ∠ADC =∠BAC. If AC=21cm and AD = 15cm, then the area of an equilateral triangle whose area is equal to the area of the rectangle with sides BC and DC is:  (a) 225 cm^2 (b) 441 cm^2 (c) 216 cm^2 (d) 305 cm^2	**1**
9	If sinθ = x and cosecθ = y , then $\{(\frac{2}{12.sin60})(\frac{tan^2 45}{cot^3 60})\}$ is (a) xy (b) $\frac{x}{y}$ (c) $\frac{y}{x}$ (d) $\frac{1}{yx}$	**1**
10	In the figure, if DE‖ BC, AD = 3cm, BD = 4cm and BC= 14 cm, then DE equals:  (a) 7 cm (b) 4 cm (c) 6 cm (d) 3 cm	**1**
11	The LCM of two prime numbers p and q (p > q) is 221. Find the value of 3p – q. (a) 4 (b) 28 (c) 38 (d) 48	**1**

12	In the figure, PA is a tangent from an external point P to a circle with centre O and AB as diameter. If ∠POB = 115° then find ∠APO.  (a) 15^0 (b) 20^0 (c) 25^0 (d) 30^0	**1**
13	The sum of radii of the two circles is 91 cm and the difference between their circumference is 44 cm. What is the radius (in cm) of the larger circle? (a) 56 cm (b) 42 cm (c) 63 cm (d) 49 cm	**1**
14	A solid right circular cone is cut into two parts at the middle of its height by a plane parallel to its base. Find the ratio of the volume of the smaller cone to the whole cone. (a) 1:2 (b) 1:4 (c) 1:8 (d) 2:5	**1**
15	For the following distribution, the upper limit of modal class is: Class | 1-4 | 6-9 | 11-14 | 16-19 | 21-24 Frequency | 10 | 15 | 12 | 20 | 9 (a) 16 (b) 20 (c) 19 (d) 25	**1**
16	The area of the largest circle that can be inscribed in a rectangle of length 8cm and width 6cm is: (a) 36π cm^2 (b) 18π cm^2 (c) 12π cm^2 (d) 9π cm^2	**1**
17	If the difference of Mode and Median of a data is 24, then the difference of median and mean is (a) 8 (b) 12 (c) 24 (d) 36	**1**
18	If two dice are thrown together, the probability that the difference of the numbers appearing on them is a prime number is: (a) $\frac{2}{9}$ (b) $\frac{4}{9}$ (c) $\frac{5}{12}$ (d) $\frac{17}{36}$	**1**
19	In the given figure, D is the mid-point of BC, then the value of $\frac{\tan x}{\tan(x+y)}$ is  (a) 2 (b) 0.5 (c) 0.333.. (d) 0.25	**1**
20	The points (2, 5) and (6, 3) are two end points of a diagonal of a rectangle. If the other diagonal has the equation y = 3x + c, then c is (a) -5 (b) -6 (c) -7 (d) -8	**1**

SECTION B (2 Marks Each)

21	From a rectangle ABCD of area 768 sq. cm, a semi-circular part with diameter AB and area 72π sq. cm is removed. Then find the perimeter of the leftover portion (in terms of π).  	**2**
22	A man travels by a motor boat down a river to his office and back. With the speed of the river unchanged, if he doubles the speed of his motor boat, then his total travel time gets reduced by 75%. Find the ratio of original speed of the motor boat to speed of the river.	**2**
23	Mohanlal, a prosperous farmer, has a circular land with centre O and radius $7\sqrt{2}$ km. There are two lamp posts at points A and point B along the boundary of the land such that the arc AB makes an angle 60 degrees at O. For the current season, he decides to have some fun. He marks two distinct points C and D on OA and OB, respectively, such that OC = OD and OC is $(\frac{\pi}{3\sqrt{3}})^{\frac{1}{2}}$ km. Mohanlal plants potatoes on the area enclosed by points C,D,A and B. Calculate the area on which Mohanlal planted potatoes.  	**2**
24	If $\text{cosec}\theta = \frac{a}{b}$ and $a \neq 0$, then find the value of $\frac{\sqrt{3}\cot\theta+1}{\tan\theta+\sqrt{3}}$ **OR** If $\sec\theta = \frac{a}{b}$ and $b \neq 0$, then find the value of $\frac{1-\tan^2\theta}{2-\sin^2\theta}$	**2**
25	Two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that ∠APB = 2∠OAB  	**2**

SECTION C (3 Marks Each)

26	Prove that $\frac{\sqrt{32}}{\sqrt{50}+7}$ is irrational if it is given that $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ are all irrational.	**3**
27	If α and β are the zeros of the polynomial $x^2 - px + r$ and $\frac{\alpha}{2}$ and 2β are roots of the quadratic equation $x^2 - qx + r = 0$, find the zeroes of the polynomial $(x^3 - r)$.	**3**
28	A circle with a diameter of BC is drawn on the triangle ABC, intersecting AB and AC at points P and Q, respectively. Find the length of BQ in cm using similarity of triangles if the lengths of AB, AC, and CP are 30 cm, 25 cm, and 20 cm, respectively.  	**3**
29	Let $a_1, a_2, \ldots, a_{52}$ be the monthly instalments (in INR) that need to be paid by Anupama to Rakhi Dave in order to repay the loan taken by her from Rakhi and the same are in AP. Instalment a_i signifies the instalment to be paid in month *i*. Suppose, their (instalments) arithmetic mean A is INR 7 short than the arithmetic mean B of $a_3, a_4, \ldots, a_{52}$ (i.e. A < B). If instalment paid in month 26 is INR 180, find value of total loan to be repaid by Anupama.	**3**
30	P, Q, and R are points on a circle with O as its centre in the diagram below. The circle's tangent crosses secant PQ at T. Find the ∠POQ if ∠QRT = 55° and ∠QTR = 25°.  	**3**
31	A bowl contain $(x^2 + 3)$ black and 8 red balls. After $(x + 1)$ more black balls are added to the bowl, a ball is drawn at random. If the probability of not drawing a black ball is $\frac{1}{3}$, find the total number of balls in the bowl. **OR** Babli participated in a lucky draw that had certain number of tickets with tickets numbers in an AP (every ticket has a unique ticket number). The first ticket sold bore the number 22, second one bore 26 and the last ticket sold bore ticket number 1098. Each individual can buy only one ticket. Babli bought ticket number 410. The rule for winning lucky draw is as follows: 4 ticket numbers would be randomly drawn and if the sum of the 4 ticket numbers is between 1000 and 2000, the ticket holders of these 4 ticket numbers win the lottery with Rs.10,000 given to each individual. If the sum of the 4 ticket numbers thus drawn is not between 1000 and 2000, then everyone except the 4 drawn ticket number holders wins the lottery. If it is already given that the first 3 ticket numbers drawn are 30, 322 and 410, what is the probability that Babli wins the lottery?	**3**

SECTION D (5 Marks Each)

32 The distribution below gives the marks of 100 students of a class (out of 40). **5**

Marks	More than 0	More than 5	More than 10	More than 15	More than 20	More than 25	More than 30	More than 35
Cumulative Frequency	100	94	92	$y^2 + 4$	70	10	5	3

The most common and repeated marks appear to be 22.25 marks.
Find the median marks.

33 Prove that $y = \tan^2 x$ by solving the following equation for y: **5**

$$\frac{3sinx-4sin^3x}{4cos^3x-3cosx} = \frac{3tanx-tan^3x}{1-3y}$$

OR

Find the value of $\frac{sec^2\theta(2+tan^2\theta+cot^2\theta)\div(sin^2\theta-tan^2\theta)}{(cosec^2\theta+sec^2\theta)(1+cot^2\theta)^2}$

34 The kingdom of Wakanda has two secluded cities – Rudyarda and Azania. Till now, no business firms exist in the two cities. The total population of Rudyarda is 'a' and total population of Azania is 'b'. The Rule T'Challa (Black Panther) understood that the profitability of the firms situating in either cities would depend on both the population in that city, and the number of firms in that city. **5**

A total of $\{\frac{4ab}{a+b}\}$ have applied to be given license to start business activity in either of the cities (i.e. one firm can establish business in only one city – Rudyarda or Azania).
Profit of a firm in Rudyarda is given by $\{2a^2 – (a - b)x\}$ where x is the number of firms that will be in Rudyarda. Profit of a firm in Azania is given by $\{2b^2 + (a + b)y\}$ where y is the number of firms that will be in Azania.
How many firms should Black Panther give license to start business in Azania so that any firm in Rudyarda would earn same amount of profit as a firm in Azania?

35 A golf ball has radius of *r* cm. It has 175 hemispheres carved from its surface such that the total volume of the ball is 566 π cm^3. If the diameter of a hemisphere is 2cm, calculate the outer surface area of the ball. **5**

SECTION E (CASE STUDY)

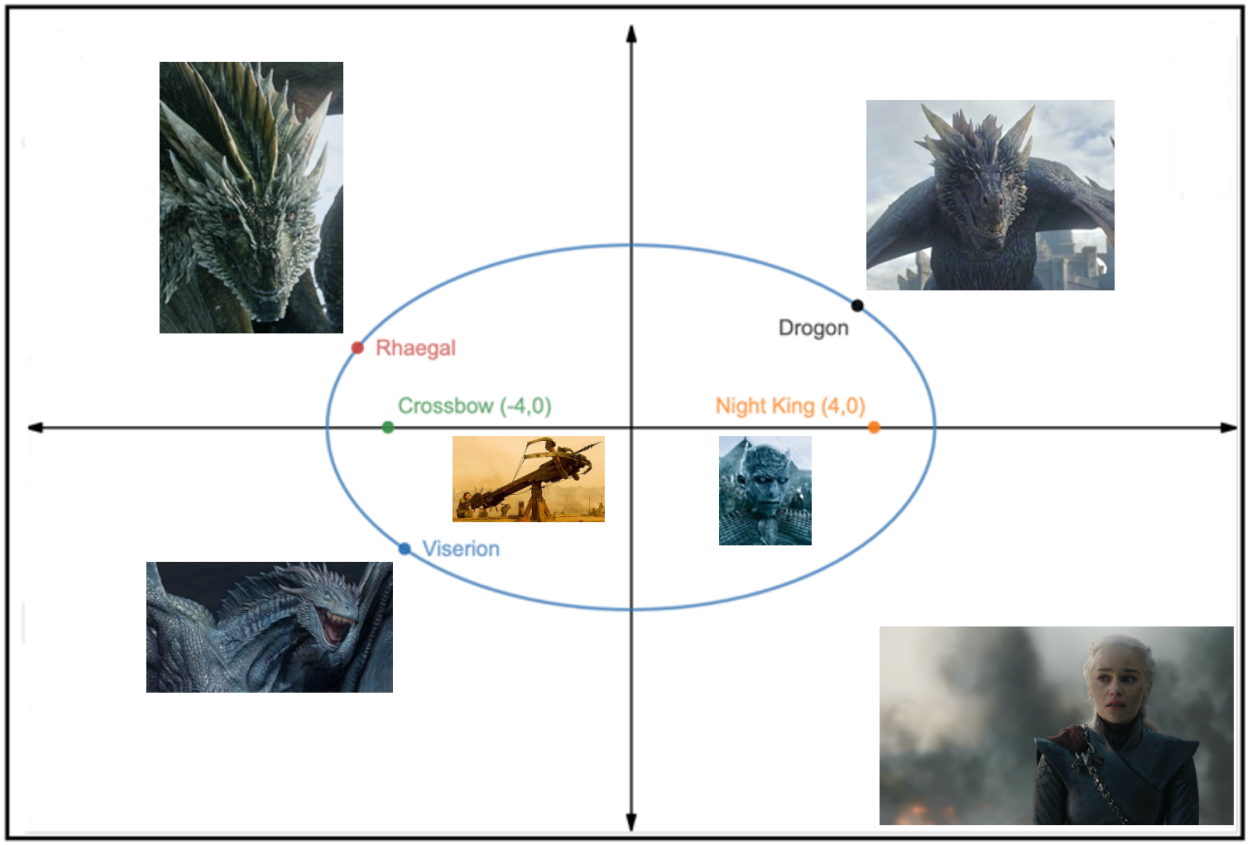
36 There is a battle going on between the city of Westeros, queen Daenerys Targaryen and the Night King of the dead. Queen Daenerys has unleashed her 3 dragons : Drogon, Rhaegal and Viserion to fight the forces of Westeros and the Night King. Daenerys knows from previous experience that the dragons would be safer if they fight from the ground as it would provide localisation and may not be under direct target of Night King or the crossbow of Westeros who and which is capable of injuring the three dragons.

In the battlefield, which can be thought of as a cartesian plane to locate various elements, the crossbow of Westeros is at A(-4,0) and the Night King is fighting from B(4,0).

(i) What is the ratio in which the line {x - 3 = 0} (this line is where Daenerys is fighting in the field) divides the line segment joining the positions of crossbow (-4,0) and the Night King (4,0) **1**

(ii) Based on the abilities of the three dragons, range of the crossbow and vision of the Night King, the advisors of Daenerys Targaryen deduced that if a dragon is always at a point such that the sum of distance of the dragon from the crossbow (-4,0) and the Night King (4,0) is equal to 10 units, the dragon would be completely unharmed in the battle. Assuming that the positions of crossbow (-4,0) and Night King (4,0) are fixed, find the equation of set of points P(x, y), along which Daenerys should position her dragons so as to ensure that they are unharmed from the crossbow and Night King. **2**

(iii) What is the distance between a soldier (5,4) and the crossbow? This is to check whether the soldier will be able to reach the crossbow in time for any attack orders. **1**

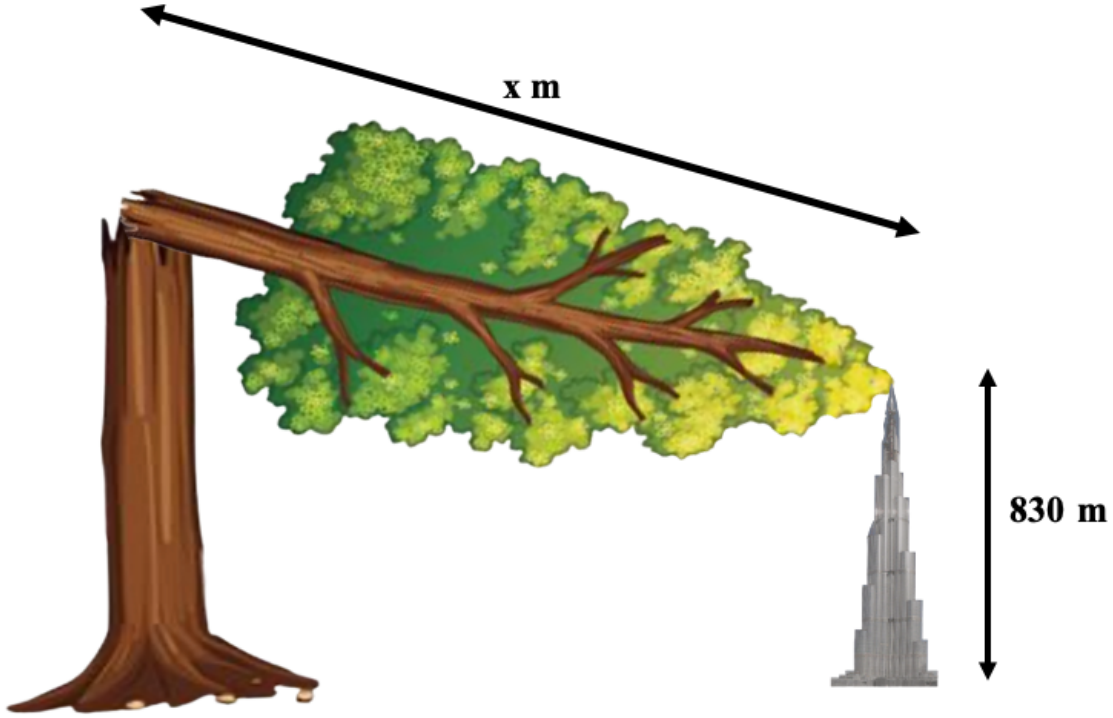


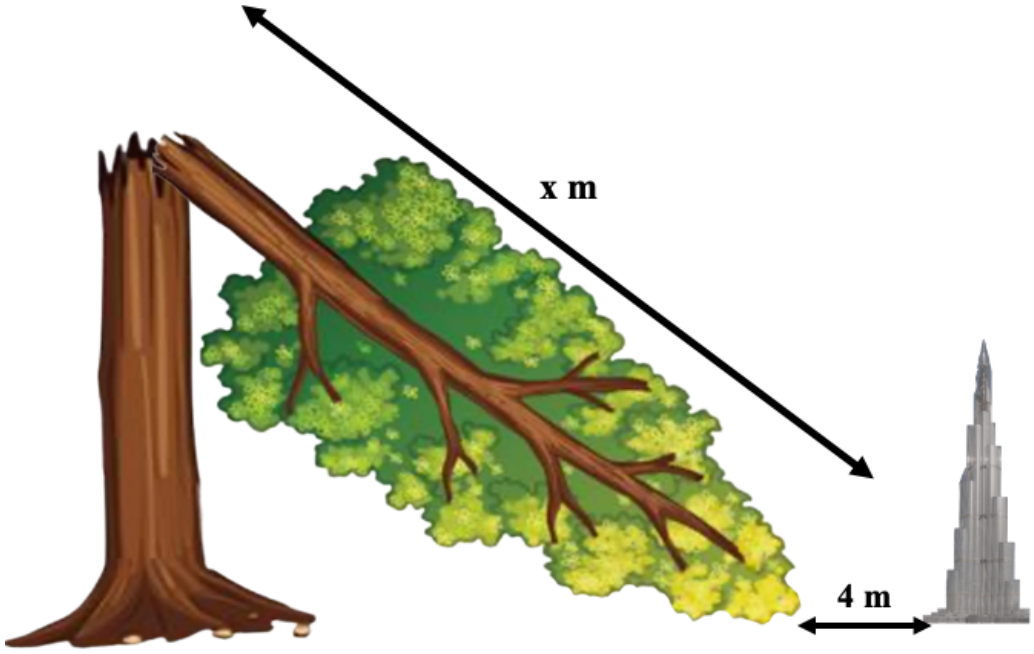
37 A tall tree (the tree of resurrection) and Burj Khalifa are standing opposite to each other in the city of Kasu Kabe. The tree has been considered as sacred and thus remained untouched for years. However, one night, lightning struck the city of Kasu Kabe and a portion of the resurrection tree broke and fell on top of Burj Khalifa (but did not completely detach from the unharmed stem of the tree) making an angle of 30°. Luckily there were no causalities. After a while, the broken part of the tree fell again from the top of the building to the ground in front of the building at a point, 4 m away from foot of the building, making an angle of 45° (but did not completely detach from the unharmed stem of the tree). The height of the Burj Khalifa is 830 m.

(i) Draw a neat labelled figure to show the above situation diagrammatically. **1**

(ii) Find the total height of the tree in meters before it broke. **2**

(iii) What is the distance between foot of the tree and foot of the building? **1**





38 Penny has been working in the cheesecake factory and the wage earned by her is just not enough. She has been good in art and craft and started making flower hair barrettes – branded under the name 'Penny Blossom'. Initially she made one for herself, impressed by which, all waitresses at the cheese cake factory wanted one. Then she went on to partner with a lady who runs cards and home-made jewellery shop and sold some barrettes to her, earning Rs. 15000 in week 1. She made a profit of Rs. 3.75 per penny blossom and set the selling price of Rs.150.

Next week, She further asked her friend Sheldon to advise her whether it could be turned into a viable business to which, he evaluated the operational capabilities and showed concerns. Nevertheless, he supported her and advised to speed up production to increase sales.

Leonard, her brother helped in creating digital space for her through designing website for her business. Few hours since the website launch and they received week long order from east Rutherford LGBTQIA society for their annual event wherein they would use the penny blossoms as accessories. However, Penny charged a price more than Rs.150 i.e. Rs. x. For each 4% rise in the selling price she sold 3 less articles to the east Rutherford LGBTQIA society (since price increase caused the society to buy less).

This week, she only sold to the LGBTQIA society of East Rutherford and made same profit as the week before i.e. Rs. 15000 (Comprising of selling price x and quantity to be calculated after adjusting for each 4% rise in the selling price she sold 3 less articles).

You need to calculate the quantity of penny blossoms and selling price per barrette charged by Penny from the LGBTQIA society.

(i) Write the expression for quantity of penny blossoms sold this week (in terms of x) **1**

(ii) What is the selling price "x" per barrette charged by Penny from the LGBTQIA society of East Rutherford? **2**

(iii) Find the profit percentage earned by Penny this week. **1**