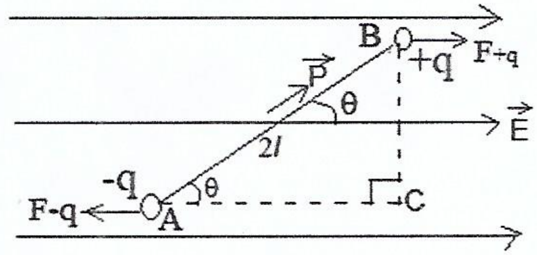
Physics 12th Important Questions for cbse (2nd Term)

Electrostatics

Section - A (of 2 marks)

1. **An electric dipole is placed in uniform electric field at an angle θ with the direction of field. Show that dipole will not done translator motion. Then, obtained expression for torque acting on the dipole.**

Answer:



According to figure, electric force on charge +q of electric dipole due to uniform electric field will,

F_B = qE …………… (i) [along $\vec{E}$]

And electric force on charge -q of electric dipole due to uniform electric field will,

$F_A = -qE$ ………….. (ii) [opposite to $\vec{E}$]

∴ Net force on dipole F = $F_B + F_A$ = qE – qE = 0

Hence, electric dipole will not done translator motion.

But, action lines of forces F_B and F_A are different. So, dipole will want to rotate.

∴ Torque acting on the dipole will,

τ = magnitude of either force × ⊥ distance between forces.

= qE × BC = qE × AB sin θ

[From triangle ABC, sin θ = $\frac{BC}{AB}$ ⇒ BC = AB sin θ]

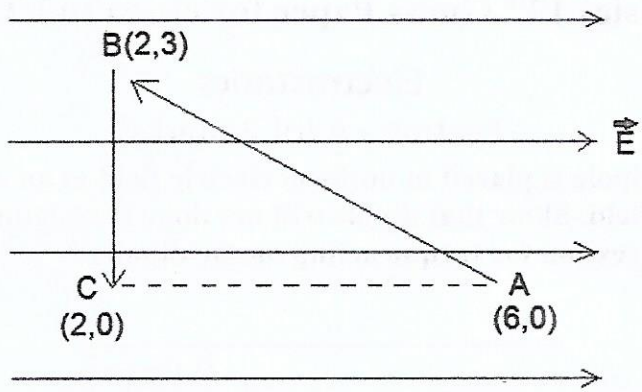
= qE × 2l sin θ = 2ql (E sin θ)

= PE sin θ

∴ τ = PE sin θ ……….. (i)

In vector form, $\vec{\tau} = \vec{P} \times \vec{E}$ ……………. (ii)

2. **A test charge q is moved without acceleration from A to C along the path from A to B and then from B to C in electric field E as shown in the figure,**
   (i) Calculate the potential difference between A and C will V_{AC} = E
   (ii) At what point (of the two) is the electric potential more and why?



Answer:
(i) Potential difference between A and C will,
$V_{AC} = -\,Edr = -\,E(6-2) = -4E$
(ii) At point C, electric potential is more. Because along direction of electric field, electric potential decreases.

3. **(a) Define the term 'electric flux'. Write its SI units.**
(b) What is the flux due to electric field $\vec{E} = 3 \times 10^3\ \hat{\imath}$ N/C through a square of side 10 cm, when it is held normal to $\vec{E}$.

Answer:
(a) The electric lines of force (electric field) passing through a surface normally is called electric flux.
Its SI unit is newton metre^2 per coulomb (Nm^2C^{-1}).
(b) Given, $E = 3 \times 10^3$ along X-axis, $A = (0.1)^2 = 0.01\ m^2$,
θ = angle between $\vec{E}$ and normal drawn on square $= 0^0$, $\phi_B = ?$
$\therefore\ \phi_B = EA\cos\theta = 3 \times 10^3 \times 0.01 \times \cos 0^0 = 30\ Nm^2C^{-1}$

4. **Calculate the amount of work done in arranging a system of three charges 6 μC , 6 μC and −6 μC placed on the vertices of an equilateral triangle of side 10 cm.**

Answer:
Given,
$q_1 = q_2 = 6\ \mu C = 6 \times 10^{-6}\ C$, $q_3 = -6\ \mu C = -6 \times 10^{-6}$, $U = ?$

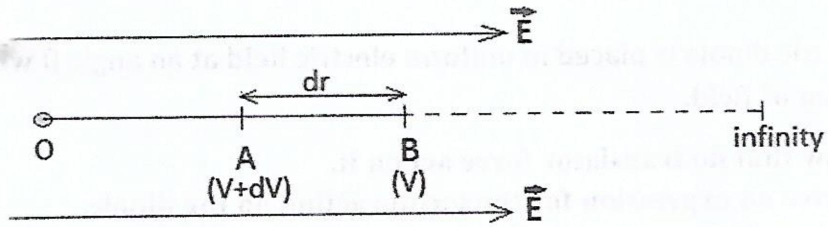
$$\therefore\ U = \frac{K\,q_1q_2}{r_{12}} + \frac{K\,q_1q_3}{r_{13}} + \frac{K\,q_2q_3}{r_{23}}$$

$$= \frac{9 \times 10^9 \times 6 \times 10^{-6} \times 6 \times 10^{-6}}{0.1} + \frac{9 \times 10^9 \times 6 \times 10^{-6} \times (-6 \times 10^{-6})}{0.1} + \frac{9 \times 10^9 \times 6 \times 10^{-6} \times (-6 \times 10^{-6})}{0.1}$$

$= 3.24 - 3.24 - 3.24 = -3.24$ J

5. **Establish the relationship between electric field intensity and electric potential.**

Answer:



Suppose two points A and B are placed at small distance dr in electric field E. The potentials of these points are (V + dV) and V respectively.

∴ Potential difference between points A and B will,

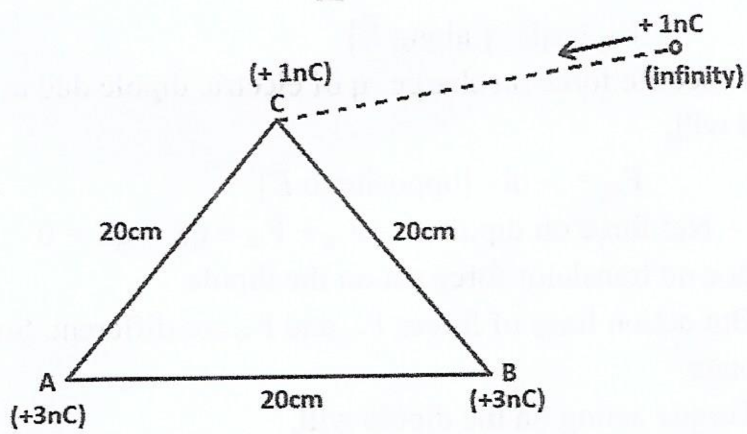
$$dV = \frac{dw}{q} = \frac{-F\,dr}{q} = -\frac{F}{q} \times dr = -E\,dr$$

Negative sign indicates opposite direction.

Or $\quad E = -\frac{dV}{dr}$

6. **The side of an equilateral triangle is 20 cm. Two equal point charges +3 nC are placed at its two corners. What will be the amount of work done in bringing a test charge of + 1 nC from infinity to third corner of the triangle.**

Answer:



Potential at point C due to charges at points A and B will,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{r_{AC}} + \frac{q_B}{r_{BC}}\right] = 9 \times 10^9 \left[\frac{3 \times 10^{-9}}{0.2} + \frac{3 \times 10^{-9}}{0.2}\right]$$

$$= 9 \times 10^9 \left[2 \times \frac{3 \times 10^{-9}}{0.2}\right] = 9 \times 10^9 \times 3 \times 10^{-8}$$

= 270 volt.

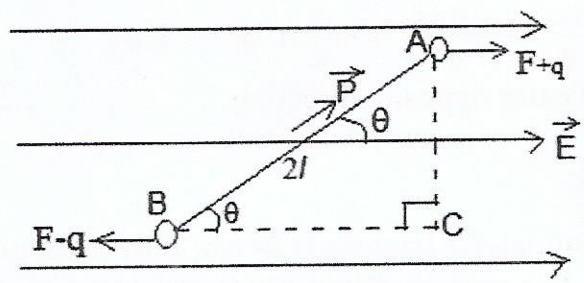
∴ The amount of work done in bringing a test charge of + 1nC from infinity to third corner C will,

$W = q_3 V = 1 \times 10^{-9} \times 270 = 2.7 \times 10^{-7}$ J

Section -B (of 3 marks)

1. **An electric dipole is placed in uniform electric field at an angle θ with the direction of field.**

(a) Show that no translator force act on it.
(b) Derive an expression for the torque acting on the dipole.
(c) Find work done in rotating the dipole through 180^0.
Answer:



(a) According to figure, electric force on charge +q of electric dipole due to uniform electric field will,

$F_{+q} = qE$ [along $\vec{E}$]

And Electric force on charge -q of electric dipole due to uniform electric field will,

$F_{-q} = -qE$ [opposite to $\vec{E}$]

∴ Net force on dipole $F = F_{+q} + F_{-q} = qE - qE = 0$

Hence no translator force act on the dipole.

(b) But action lines of forces F_{+q} and F_{-q} are different. So, dipole will want to rotate.

∴ Torque acting on the dipole will,

τ = magnitude of either force × perpendicular distance between forces.

$= qE \times AC = qE \times AB \sin\theta$ [From triangle ABC, $\sin\theta = \frac{AC}{AB} \Rightarrow AC = AB \sin\theta$]

$= qE \times 2l \sin\theta = 2ql\,(E \sin\theta)$

$= PE \sin\theta$

$\therefore \tau = PE \sin\theta$ .... (i)

In vector form $\vec{\tau} = \vec{P} \times \vec{E}$ .... (ii)

Now, work done on electric dipole by external torque in turning through small angle dθ will,

$$dw = \tau_{ext}\, d\theta = \tau_r\, d\theta = PE \sin\theta\, d\theta$$

∴ Total work done on electric dipole by external torque in turning anticlockwise through angle 0^0 to 180^0 will,

$$W = \int_{0^0}^{180^0} dw = \int_{0^0}^{180^0} pE \sin\theta = PE \int_{0^0}^{180^0} \sin\theta$$

$$= pE\{-\cos\theta\}_{0^0}^{180^0} = -pE\,(\cos 180^0 - \cos 0^0)$$

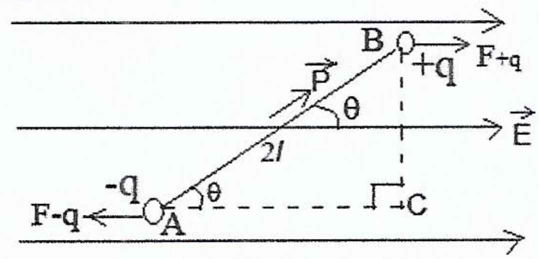
$$= -pE\,(-1-1) = 2pE$$

2. **(a) Obtain an expression for the torque $\vec{\tau}$ experienced by an electric dipole of dipole moment $\vec{p}$ in a uniform electric field $\vec{E}$.**

(b) What will be happen if the field were not uniform ?

Answer:

(a)



According to figure, electric force on charge +q of electric dipole due to uniform electric field will,

$$F_B = qE \;\ldots\ldots\ldots\; \text{(i)} \qquad [\text{along } \vec{E}]$$

And Electric force on charge -q of electric dipole due to uniform electric field will,

$$F_A = -qE \;\ldots\ldots\ldots\text{(ii)} \qquad [\text{opposite to } \vec{E}]$$

∴ Net force on dipole,

$$F_{net} = F_B + F_A = qE - qE = 0$$

Hence, electric dipole will not done translator motion.
But, action lines of forces F_B and F_A are different. So, dipole will want to rotate.

∴ Torque acting on the dipole will,

τ = magnitude of either force × ⊥ distance between forces.

$= qE \times BC = qE \times AB \sin\theta$

[From triangle ABC, $\sin\theta = \frac{BC}{AB} \Rightarrow BC = AB \sin\theta$]

$= qE \times 2l \sin\theta = 2ql\,(E \sin\theta)$

$= PE \sin\theta$

∴ $\tau = PE \sin\theta$ ………. (i)

In vector form, $\vec{\tau} = \vec{P} \times \vec{E}$ .... (ii)

(b) When electric field is not uniform, then forces acting on the both Charges are not equal. Therefore, net force and net torque on the dipole will non-zero.

Hence, dipole will do translator as well as rotatory motion.

3. **State Gauss' theorem for electrostatics. Applying this theorem, derive an expression for the electric field intensity due to a uniformly charged spherical conducting shell at a point**

(i) outside the shell and (ii) inside the shell.

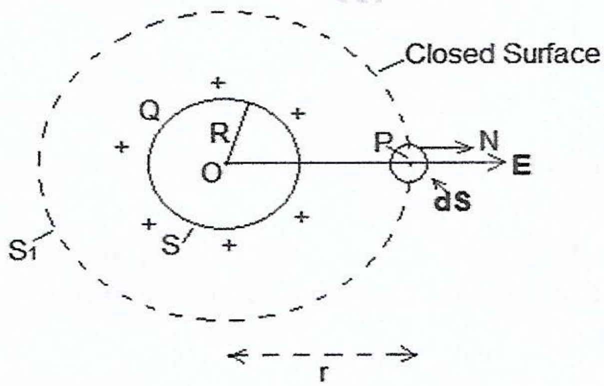
Plot a graph showing variation of electric field as a function of r > R and r < R

Where r is the distance from the centre of the shell.

Answer:

Gauss' theorem: The total electric flux linked with a closed surface is $\frac{1}{\varepsilon_o}$ times of net charge inside the closed surface (q_{net}).

i.e. $\phi_E = \oint_S \vec{E}.\vec{dS} = \frac{q_{net}}{\varepsilon_0}$



Consider a charged spherical shell of radius R placed in air or vacuum. Let we have to determine electric field intensity at point P outside the shell at distance r from centre O. For this we imagine a spherical closed surface S_1 of radius r such that point P is on the surface of S_1. The electric field intensity at every point on the surface of S_1 is **E** directed outside along radius. Let surface S_1 is divide into many equal parts of area ds. Again, consider one area element **ds**.

Now, total electric flux linked with closed surface S_1 will,

$$\phi_E = \oint \vec{E}.\vec{dS} = \oint EdS\cos\theta = \oint EdS\cos 0^0 = \oint E\, dS = \mathrm{E}\oint dS$$

$= \mathrm{E} \times$ total area of closed surface S_1

Or $\phi_E = \mathrm{E} \times 4\pi r^2$ .... (i)

According to Gauss' theorem, $\phi_E = \frac{q_{net}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$ ....(ii)

Comparing equations (i) and (ii), we get

$$E \times 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\therefore \quad E = \frac{Q}{4\pi\varepsilon_0 r^2} \quad .... \quad (iii)$$

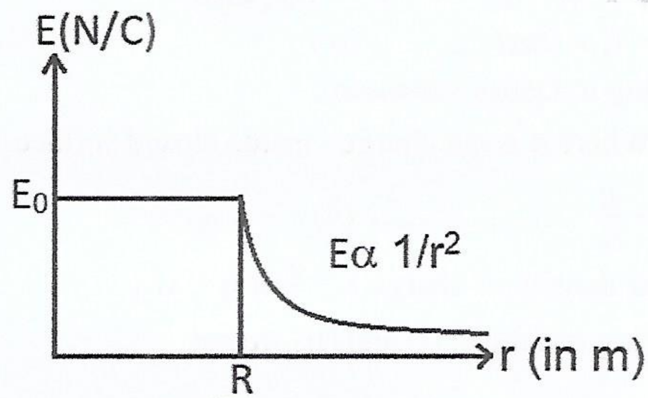
This equation is same as electric field intensity due to a point charge. Hence charged spherical shell can be assumed as a point charge.

If point P is at inside the spherical shell, then $q_{net} = 0$

$$\therefore \quad E \times 4\pi r^2 = \frac{q_{net}}{4\pi\varepsilon_0 r^2} = \frac{0}{4\pi\varepsilon_0 r^2} = 0$$

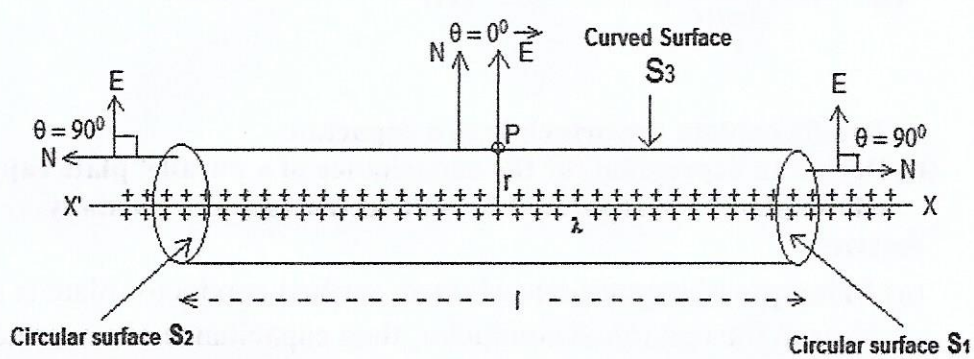
Or $\quad E = 0$

A graph showing variation of electric field as a function of r is shown in the figure:



4. **State Gauss' theorem in electrostatics. Apply this theorem to obtain the expression for the electric field intensity at a point due to an infinitely long, thin, uniformly charged straight wire.**

Ans.



Suppose an infinite long wire is placed in air of surface density of charge λ. We have to calculate electric field intensity at point P at distance r from wire. For this we imagine a cylindrical closed surface of length *l*. The point is situated on this surface as shown in the figure.

Now, electric field flux linked with surface S_1 will,

$\phi_1 = ES_1 \cos\theta \quad [\because \phi_E = ES\cos\theta]$

Or $\phi_1 = ES_1 \cos 90^0 = 0$ ....(i)

Electric field flux linked with surface S_3 will,

$\phi_3 = ES_3 \cos\theta = ES_3 \cos 90^0 = 0$ ....(ii)

And, electric field flux linked with surface S_2 will,

$\phi_2 = ES_2 \cos\theta = ES_2 \cos 0^0 = ES_2$

Or $\phi_2 = E \times 2\pi r l$ ....(iii)

[$\because$ Area of curved surface of cylinder = $2\pi rh$]

$\therefore$ Total electric flux liked with closed surface will,

$$\phi_E = \phi_1 + \phi_3 + \phi_2 = 0 + 0 + E \times 2\pi r l$$

Or $\phi_E = E \times 2\pi r l$ .... (iv)

According to Gauss's theorem,

$\phi_E = \frac{q}{\varepsilon_0}$ Where q is net charge inside closed surface.

Or $\phi_E = \frac{\lambda l}{\varepsilon_0}$ .... (v)

[$\because$ linear density of charge $\lambda = \frac{q}{l}$ or $q = \lambda l$]

Comparing equations (i) and (ii),we get,

$$E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0} \text{ or } E = \frac{\lambda l}{2\pi\varepsilon_0 r l}$$

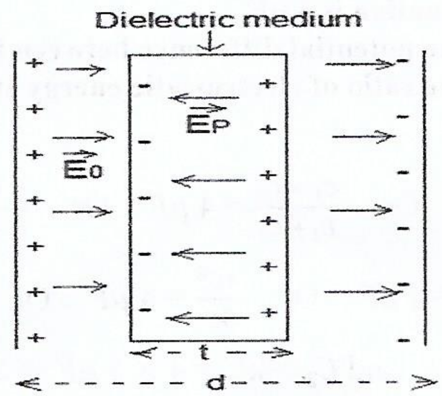
Or $E = \frac{\lambda}{2\pi\varepsilon_0 r}$ .... (vi)

5. **(a) Briefly explain the principle of a capacitor.**
(b) Derive an expression for the capacitance of a parallel plate capacitor, whose plates are separated by a dielectric medium partially.

Answer:

(a) **Principle of capacitor:-** when an earthed conductor plate is brought Near to the unearthed conductor, then capacitance of unearthed conductor is increased extremely.

(b)



Suppose a dielectric medium of permittivity k and thickness t is placed in between plates of capacitor, where t < d.

$\therefore \quad k = \frac{E_0}{E} \Rightarrow E = \frac{E_0}{K}$ .... (i)

Where E = electric field in dielectric medium and

E_0 = electric field in air.

$\therefore$ Potential difference between plates will,

$$V = V_{air} + V_{dielectric}$$

$$= E_0 (d - t) + E t = E_0 (d - t) + \frac{E_0}{K} t \quad [\text{ since } V = Ed]$$

$$= E_0 [d - t + \frac{t}{K}] = \frac{\sigma}{\varepsilon_0} [d - t + \frac{t}{K}]$$

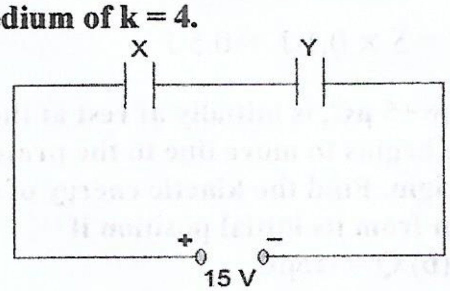
$$\Rightarrow V = \frac{q}{\varepsilon_0 A} [d - t + \frac{t}{K}] \text{ .... (ii)} \quad [\text{since } \sigma = \frac{q}{A}]$$

$\therefore$ Capacitance of capacitor will,

$$C = \frac{q}{V} = \frac{q}{\frac{q}{\varepsilon_0 A} [d - t + \frac{t}{K}]} = \frac{\varepsilon_0 A q}{q [d - t + \frac{t}{K}]} = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

$$\Rightarrow C = \frac{\varepsilon_0 A}{d - t (1 - \frac{1}{K})} \text{ .... (iii)}$$

6. **Two parallel plate capacitors X and Y have the same area of plates and same separation between them. X has air between the plates while Y contains a dielectric medium of k = 4.**



(i) Calculate capacitance of each capacitor if equivalent capacitance

of the combination is 4 μF.
(ii) Calculate the potential difference between the plates of X and Y.
(iii) Estimate the ratio of electrostatic energy stored in X and Y.

Answer:

(i) $C_s = 4\ \mu F$ Or $\frac{C_1 \times C_2}{C_1 + C_2} = 4\ \mu F$ Or $\frac{C \times kC}{C + kC} = 4\ \mu F$

Or $\frac{C \times 4C}{C + 4C} = 4\ \mu F$ Or $\frac{4C^2}{5C} = 4\ \mu F$ Or $C = 5\ \mu F$

$\therefore C_1 = 5\ \mu F$ and $C_2 = kC = 4 \times 5\ \mu F = 20\ \mu F$

(ii) Charge (q) on each capacitor in series combination is same

$\therefore q = C_S V = 4\ \mu F \times 15 = 4 \times 10^{-6} \times 15$

$= 60 \times 10^{-6}$ coulomb $= 0.6\ \mu C$

$\therefore \quad V_1 = \frac{q}{C_1} = \frac{60 \times 10^{-6}}{5 \times 10^{-6}} = 12$ volt

And $V_2 = \frac{q}{C_2} = \frac{60 \times 10^{-6}}{20 \times 10^{-6}} = 3$ volt

(iii) $\frac{U_1}{U_2} = \frac{\frac{1}{2} C_1 V_1^2}{\frac{1}{2} C_2 V_2^2} = \frac{5 \times 10^{-6} \times 12^2}{20 \times 10^{-6} \times 3^2} = 4:1$

7. **A parallel plate capacitor of capacitance 20 μF is connected to a 100 V supply. After sometime, the battery is disconnected, and the space between the plates of the capacitor is filled with a dielectric of dielectric constant 5. Calculate the energy stored in the capacitor.**
   (i) before (ii) after the dielectric has been put in between its plates.

Answer:

(i) $C = 20\ \mu F = 20 \times 10^{-6}$ F, V = 100 volt, E = ?

$\therefore \quad E = \frac{1}{2} CV^2 = 0.5 \times 20 \times 10^{-6} \times 100^2 = 0.1$ J

(ii) $C = 20\ \mu F = 20 \times 10^{-6}$ F, k = 6, V = 100 volt, E = ?

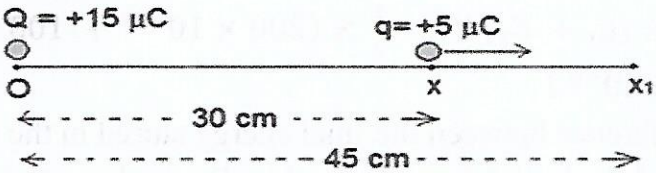
$\therefore \quad E = \frac{1}{2} C_m V^2 = \frac{1}{2} kCV^2 \quad [C_m = kC]$

$= k \times \frac{1}{2} CV^2 = 5 \times 0.1$ J $= 0.5$ J

8. **A particle, having a charge +5 μC, is initially at rest at the point x = 30 cm on the x axis. The particle begins to move due to the presence of a charge Q that is kept fixed at the origin. Find the kinetic energy of the particle at the instant it has moved 15 cm from its initial position if**
   (a) Q =+15μC and (b) Q = -15μC

Answer:

(a)



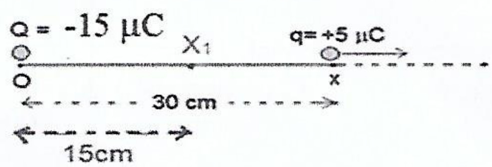
(a) Kinetic energy of the particle at final position (x_1) if Q = + 15 μC,

$$E_1 = Q\,(V_i - V_f) = Q\left(\frac{kq}{r_i} - \frac{kq}{r_f}\right) = kqQ\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$= 9 \times 10^9 \times 5 \times 10^{-6} \times 15 \times 10^{-6}\left(\frac{1}{30 \times 10^{-2}} - \frac{1}{45 \times 10^{-2}}\right)$$

$$= 45 \times 15 \times 10^{-3}\left(\frac{100}{30} - \frac{100}{45}\right) = 45 \times 15 \times 10^{-3} \times \frac{100}{90} = 0.75\ \text{J}$$

(b) Kinetic energy of the particle at final position (x_1) if Q = −15 μC,



$$E_2 = 9 \times 10^9 \times 5 \times 10^{-6} \times 15 \times 10^{-6}\left(\frac{1}{30 \times 10^{-2}} - \frac{1}{15 \times 10^{-2}}\right)$$

$$= 45 \times 15 \times 10^{-3}\left(\frac{100}{30} - \frac{100}{15}\right) = 45 \times 15 \times 10^{-3} \times \frac{-100}{30} = -2.25\ \text{J}$$

9. **A capacitor of 200 pF is charged by a 300 V battery. The battery is then disconnected and the charged capacitor is connected to another uncharged capacitor of 100 pF. Calculate the difference between the final energy stored in the combination system and the initial energy stored in the single capacitor.**

Answer:

The initial energy stored in the single capacitor,

$$E_1 = \frac{1}{2} C_1 {V_1}^2$$

$$= \frac{1}{2} \times (200 \times 10^{-12}) \times 300^2 = 9 \times 10^{-6} J$$

Again, Common potential of combination after connecting with uncharged capacitor,

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{200\times10^{-12}\times300 + 100\times10^{-12}\times0}{200\times10^{-12} + 100\times10^{-12}}$$

$= \frac{6\times10^{-8}}{3\times10^{-10}} = 200$ volt

The final energy stored in the combination system,

$E_2 = \frac{1}{2} \times (C_1 + C_2)\, V^2 = \frac{1}{2} \times (200 \times 10^{-12} + 100 \times 10^{-12})\, 200^2$

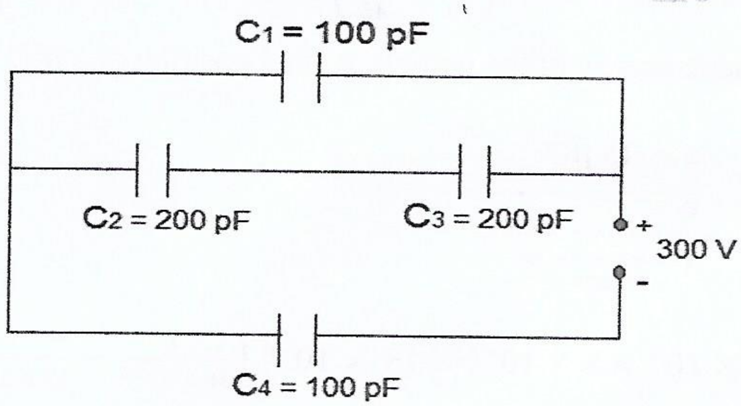
$= 3\times 10^{-6}$ J

∴ The difference between the final energy stored in the combination system and the initial energy stored in the single capacitor.

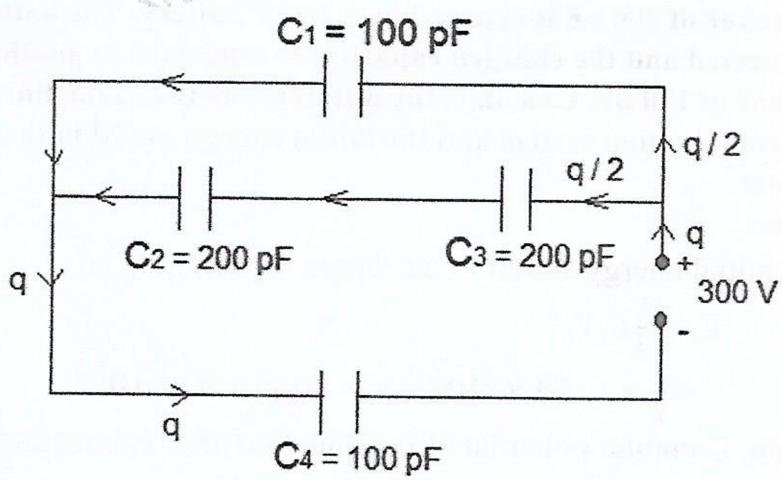
$E = E_1 - E_2 = -6\times 10^{-6}$ J

10. **(a) Obtain equivalent capacitance of the following network given in the figure below.**

(b) For a 300 V supply, determine the charge and voltage cross each capacitor.



Answer:



(a) Equivalent capacitance of C_2 and C_3 is

$$C_{23} = \frac{C_2 . C_3}{C_2 + C_3} = \frac{200 \times 200}{200+200} = \frac{200 \times 200}{2 \times 200} = 100 \text{ pF}$$

C_{23} is connected with capacitor C_1 in parallel.

Therefore, equivalent capacitance of C_{23} and C_1 will,

$$C_{123} = C_{23} + C_1 = 100\text{ pF} + 100\text{ pF} = 200\text{ pF}$$

Again, C_{123} is connected with C_4 in series.

∴ Equivalent resistance of Circuit will,

$$C = \frac{C_{123} \times C_4}{C_{123} + C_4} = \frac{200 \times 100}{200 + 100} = \frac{200 \times 100}{300} = \frac{200}{3}\text{ pF}$$

(b) Charge flowing in the circuit will,

$$q = CV = \frac{200}{3}\text{ pF} \times 300\text{ V}$$

$$= \frac{200}{3} \times 10^{-12} \times 300\text{ V} = 2 \times 10^{-8}\text{ coulomb}$$

Since charge does not distribute in series,

∴ Charge on each plate of C_{123} and $C_4 = 2 \times 10^{-8}$ coulomb

i.e. $q_{123} = q_4 = 2 \times 10^{-8}$ coulomb

Again, charge distribute equally in parallel , when they are of equal capacitances

∴ Charge on each plate of C_1 and $C_{23} = \frac{q}{2} = \frac{2 \times 10^{-8}}{2} = 10^{-8}$ coulomb

i.e. $q_1 = q_2 = q_3 = 10^{-8}$ coulomb

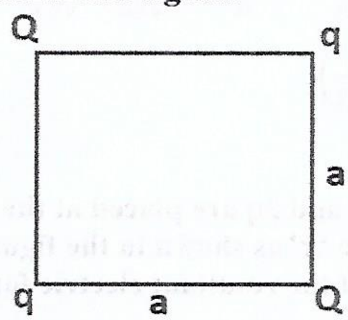
Now, voltage across C_1 will, $V_1 = \frac{q_1}{C_1} = \frac{10^{-8}}{100 \times 10^{-12}} = 100$ volt

Voltage across C_2 will, $V_2 = \frac{q_2}{C_2} = \frac{10^{-8}}{200 \times 10^{-12}} = 50$ volt

Voltage across C_3 will, $V_3 = \frac{q_3}{C_3} = \frac{10^{-8}}{200 \times 10^{-12}} = 50$ volt

And voltage across C_4 will, $V_4 = \frac{q_4}{C_4} = \frac{2 \times 10^{-8}}{100 \times 10^{-12}} = 200$ volt

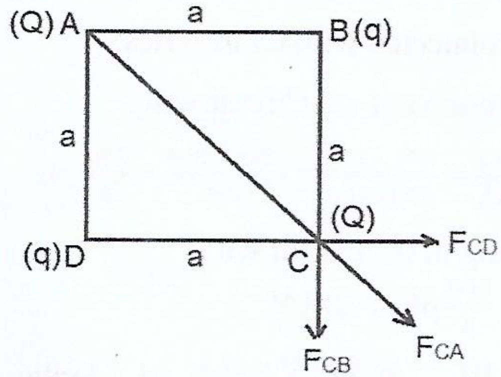
11. **Four point charges Q, q, Q and q are placed at the corners of a square of side 'a' as shown in the figure.**



Find the (a) resultant electric force on charge Q and (b) potential energy of this system.

Answer:

(a)



Electric force on charge Q at corner C due charge q at corner B,

$$F_{CB} = \frac{k\, q_1 q_2}{r^2} = \frac{K\, Qq}{a^2} = F \text{ (let)} \quad \text{along BC produced}$$

Electric force on charge Q at corner C due charge q at corner D,

$$F_{CD} = \frac{K\, Qq}{a^2} = F \quad \text{along DC produced}$$

Resultant force of F_{CB} and F_{CD} will,

$$F_{CBD} = \sqrt{F^2 + F^2} = \sqrt{2F^2} = \sqrt{2}\,F \text{ along AC produced}$$

Electric force on charge Q at corner C due charge Q at corner A,

$$F_{CA} = \frac{K\, q_1 q_2}{r^2} = \frac{K\, Q.\, Q}{(\sqrt{2}\, a)^2} = \frac{KQ^2}{2a^2} \quad \text{along AC produced}$$

∴ Net force on charge Q at corner at C = $\sqrt{2}\,F + \frac{KQ^2}{2a^2}$

$$= \sqrt{2}\frac{K\, Qq}{a^2} + \frac{KQ^2}{2a^2} = \frac{KQ}{a^2}\left(\sqrt{2}\, q + Q/2\right) \text{ along AC produced}$$

(b) Potential energy of system,

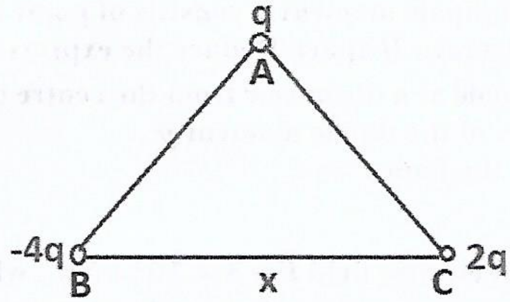
$$U = \frac{KQq}{r_{AB}} + \frac{KQ.Q}{r_{AC}} + \frac{KQq}{r_{AD}} + \frac{KqQ}{r_{BC}} + \frac{Kq.q}{r_{BD}} + \frac{KQq}{r_{CD}}$$

$$= \frac{KQq}{a} + \frac{KQ.Q}{\sqrt{2}\, a} + \frac{KQq}{a} + \frac{KQq}{a} + \frac{Kq^2}{\sqrt{2}\, a} + \frac{KQq}{a}$$

$$= 4\frac{KQq}{a} + \frac{KQ.Q}{\sqrt{2}\, a} + \frac{Kq^2}{\sqrt{2}\, a} = 4\frac{KQq}{a} + \frac{KQ^2}{\sqrt{2}a} + \frac{Kq^2}{\sqrt{2}\, a}$$

$$= \frac{K}{a}\left(4Qq + \frac{Q^2}{\sqrt{2}} + \frac{q^2}{\sqrt{2}}\right)$$

12. **(a) Three point charges q , - 4q and 2q are placed at the vertices of an equilateral triangle ABC of side '*x*' as shown in the figure. Obtained the expression for the magnitude of the resultant electric force acting on the charge q.**



(b) Find out the amount of the work done to separate the charges at infinite distance.

Answer:

Electric force on charge q at corner A due to charge (2q) at corner C,

$$F_{AC} = \frac{k\,|q_1|.|q_2|}{r^2} = \frac{K\,2q.\,q}{x^2} = \frac{2Kq^2}{x^2} \qquad \text{along CA produced}$$

Electric force on charge q at corner A due to charge (-4q) at corner B,

$$F_{AB} = \frac{k\,|-4q|.|q|}{r^2} = \frac{K\,4q.\,q}{x^2} = \frac{4Kq^2}{x^2} \qquad \text{along AB}$$

Angle between $\overrightarrow{F_{AC}}$ and $\overrightarrow{F_{AB}}$ is 120^0

∴ Resultant force on charge q will,

$$F = \sqrt{\left(\frac{2Kq^2}{x^2}\right)^2 + \left(\frac{4Kq^2}{x^2}\right)^2 + 2.\frac{2Kq^2}{x^2}.\frac{4Kq^2}{x^2}\cos 120^0}$$

$$= \sqrt{\left(\frac{2Kq^2}{x^2}\right)^2 + \left(\frac{4Kq^2}{x^2}\right)^2 + 2.\frac{2Kq^2}{x^2} \times \frac{4Kq^2}{x^2} \times \left(-\frac{1}{2}\right)}$$

$$= \sqrt{4\left(\frac{Kq^2}{x^2}\right)^2 + 16\left(\frac{Kq^2}{x^2}\right)^2 - 8\left(\frac{Kq^2}{x^2}\right)^2}$$

$$= \frac{Kq^2}{x^2}\sqrt{4+16-8} = \frac{Kq^2}{x^2}\sqrt{12} = 2\sqrt{3}\,\frac{Kq^2}{x^2}$$

$$\therefore \quad \vec{F} = 2\sqrt{3}\,\frac{Kq^2}{x^2}$$

(b) $U = \frac{K|q_A|.\,|q_B|}{r_{AB}} + \frac{K|q_A|.\,|q_C|}{r_{AC}} + \frac{K|q_B|.\,|q_C|}{r_{BC}}$

$$\left[\because \textit{for three point charges } U = \frac{k\,q_1\,q_2}{r_{12}} + \frac{k\,q_1\,q_3}{r_{13}} + \frac{k\,q_2\,q_3}{r_{23}}\right]$$

$$= \frac{K|q|.\,|-4q|}{x} + \frac{K|q|.\,|2q|}{x} + \frac{K|-4q|.\,|2q|}{x}$$

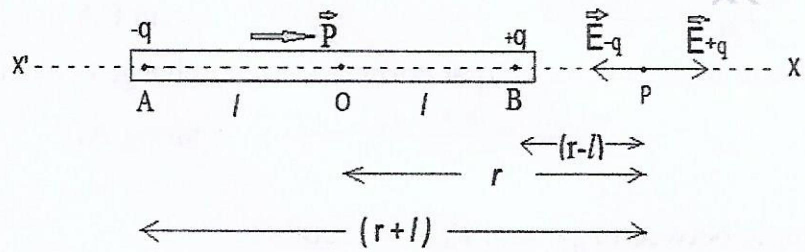
$$= \frac{4Kq^2}{x} + \frac{2kq^2}{x} + \frac{8kq^2}{x} = \frac{14Kq^2}{x}$$

Section - C (5 marks)

1. **(a) An electric dipole of dipole moment $\vec{p}$ consists of point charges +q and –q separated by a distance $2l$ apart. Deduce the expression for the electric field $\vec{E}$ due to the dipole at a distance r from the centre of the dipole on its axial line in terms of the dipole moment $\vec{p}$.**
   Hence, show that in the limit r >> l,
   $$\vec{E} = \frac{p}{4\pi\varepsilon_0 r^3}$$
   (b) Consider a uniform electric field $\vec{E} = 3 \times 10^3\ \hat{i}$ N/C. what is the flux of this field through a square of 10 cm on a side whose plane is parallel to the y-z plane ?

Answer:

(a) **Electric field at any point P on the axial line**



Suppose an electric dipole AB of charges ±q , dipole length $2l$ and dipole moment $\vec{p}$ is placed in air. We have to calculate electric field at point P on the axial line at distance r from O. For this, we imagine a unit positive charge at point P.

Now, Magnitude of electric field at point P due to charge +q will,

$$E_{+q} = \frac{q}{4\pi\varepsilon_0 (BP)2} = \frac{q}{4\pi\varepsilon_0 (r-l)^2} \quad \text{...... (i)} \quad [\text{along P to X}]$$

And magnitude of electric field at point P due to charge -q will,

$$E_{-q} = \frac{q}{4\pi\varepsilon_0 (AP)2} = \frac{q}{4\pi\varepsilon_0 (r+l)^2} \quad \text{... (ii)} \quad [\text{along P to X'}]$$

∴ Magnitude of net electric field at point P due to whole dipole will,

$$E = E_{+q} - E_{-q} = \frac{q}{4\pi\varepsilon_0 (r-l)^2} - \frac{q}{4\pi\varepsilon_0 (r+l)^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+l)^2-(r-l)^2}{(r-l)^2\,(r+l)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{4rl}{(r^2-l^2)^2}\right] \qquad [\because (a+b)^2 - (a-b)^2 = 4ab]$$

$$= \frac{2r(2ql)}{(r^2-l^2)^2} = \frac{2r \times p}{(r^2-l^2)^2} \qquad [\because p = 2ql]$$

$$\therefore \quad E = \frac{2pr}{(r^2-l^2)^2} \quad \text{.................................... (iii)}$$

If r ≫ l then neglecting l^2, we get from equation (iii),

$$E = \frac{2pr}{r^4} \quad \text{or} \quad E_A = \frac{2p}{r^3} \quad \text{........................ (iv)}$$

In vector form, $\vec{E_A} = \frac{2\vec{p}}{r^3}$

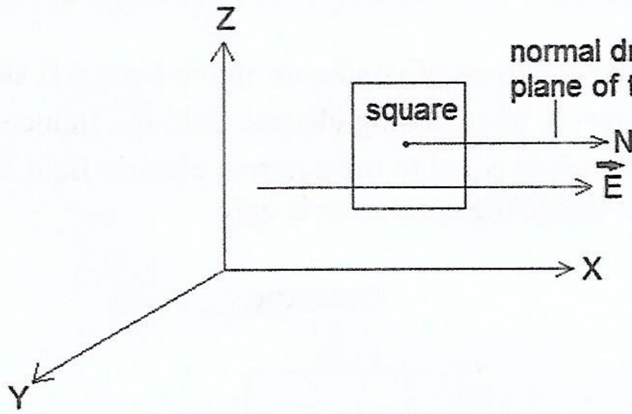
Its direction is along P to X i.e. along $\vec{p}$.

(b) Given,

Electric field intensity $\vec{E} = 3 \times 10^3\ \hat{\imath}$ N/C or E = 3× 10^3 N/C,

Area of square A = side² = $10^2 = 100$ cm² = 100 × 10^{-4} m² = 10^{-2} m²,

Electric flux passing through the plane of square ϕ = ?

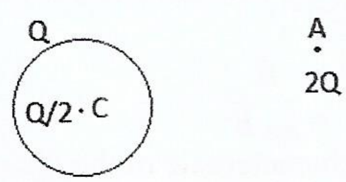


Since, the plane of the square is parallel to the y-z plane and direction of electric field is along X- axis. Therefore, angle between the normal drawn on plane of square and direction of $\vec{E}$ is $\theta = 0^0$.

$\therefore \phi = EA\cos\theta = 3\times 10^3 \times 10^{-2} \times \cos 0^0 = 30\ Nm^2C^{-1}$.

2. **(a) Explain, using suitable diagrams, the difference in the behavior of a (i) conductor and (ii) dielectric ; in the presence of external electric field. Define the terms polarization of a dielectric and write its relation with susceptibility.**

(b) A thin metallic spherical shell of radius R carries a charge Q on its surface. A point charge $\frac{Q}{2}$ is placed at its Centre C and another charge +2Q is placed outside the shell at a distance x from the Centre as shown in the figure.

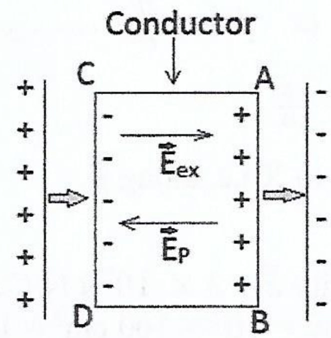


Find (i) The force on the charge at the Centre of shell and at the point A, (ii) the electric flux through the shell.

Answer:

(a)

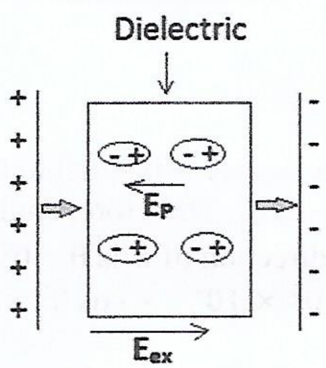
Conductor:



When a conductor ABCD is placed in an external electric field of intensity
$\overrightarrow{E_{ex}}$ then free electrons of conductor move from AB side to CD side.

As a result, an opposing electric field $\overrightarrow{E_P}$ induces inside the conductor, which is equal to the external electric field $\overrightarrow{E_{ex}}$. Therefore, net electric field inside the conductor is zero.

Dielectric:



When a dielectric is placed in an external electric field of intensity $\overrightarrow{E_{ex}}$ then some molecules of dielectric are polarized.

As a result, an opposing electric field $\overrightarrow{E_P}$ induces inside the dielectric, which is less than external electric field $\overrightarrow{E_{ex}}$. Therefore, net electric field inside the conductor is not zero.

Polarization:

The dipole moment per unit volume is called polarization ($\vec{P}$). For a linear isotropic dielectrics,

$$\vec{P} \propto \vec{E}$$

$$\text{Or} \quad \vec{P} = \chi_e \vec{E}$$

where χ_e is a constant characteristic of the dielectric and is known as the electric susceptibility of the dielectric medium.

(b) Charge $\frac{Q}{2}$ at Centre induces a charge $-\frac{Q}{2}$ at the inside surface of the shell and $+\frac{Q}{2}$ at the outside surface. Therefore, net charge on the outer surface will be

$(Q + \frac{Q}{2})$.

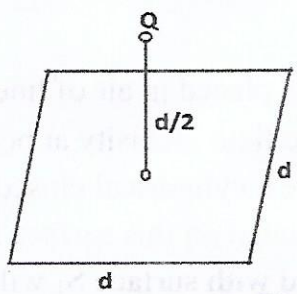
Now, electric field at point A will be, $E = \frac{K(Q+\frac{Q}{2})}{x^2}$

(i) Since, the electric field inside a hollow conductor is zero. Therefore, force experienced by the charge $\frac{Q}{2}$ at the Centre will be zero.
And, the force on charge 2Q at point A will be,

$$F = qE = 2Q \times \frac{K(Q+\frac{Q}{2})}{x^2} = \frac{3Q^2K}{x^2}$$

(ii) The electric flux through the shell $= \frac{q_{net}}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$

3. **(a) Define electric flux. Is it a scalar or vector quantity ?**
A point charge Q is at a distance of d/2 directly above the centre of a square of side "d' as shown in the figure. Use Gauss's law to obtain the expression for the electric flux through the square.



(b) If the point charge is now moved to a distance 'd' from the centre of the square and the side of the square is doubled , explain how the electric flux will be affected.

Answer:

(a) The electric field lines passing normally through a surface is called electric flux.
It is a scalar physical quantity.

According to Gauss' law, total electric flux passing through a closed surface is

$$\phi_E = \frac{q_{net}}{\varepsilon_0}$$

∴ Electric flux passing through given square,

$$\phi_E = \frac{1}{6} \times \frac{q_{net}}{\varepsilon_0} = \frac{Q}{6\,\varepsilon_0}$$

(b) Since, net charged within closed surface made by cube of side 2d is same as net charge within closed surface by cube of side d.
Therefore, electric flux does not change, when side of square would be doubled and distance of charge would be increased by distance d.

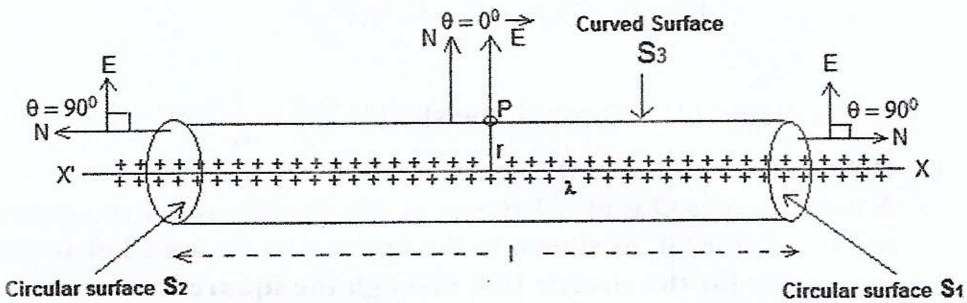
4. **(a) Use Gauss's law to derive the expression for the electric field due to a straight uniformly charged infinite line of charge of charged density λ C/m.**

(b) Draw a graph to show the variation of E with perpendicular distance r from the line of charge.

(c) Find the work done in bringing a charge q from perpendicular distance r_1 to r_2 ($r_2 > r_1$).

Answer:

(a)



Suppose an infinite long wire is placed in air of linear density of charge λ. We have to calculate electric field intensity at point P at distance r from wire. For this, we imagine a cylindrical closed surface of length l and radius r. The point P is situated on this surface as shown in the figure.

Now, electric flux linked with surface S_1 will,

$\phi_1 = ES_1 \cos\theta$ $[\because \phi_E = ES\cos\theta]$

Or $\phi_1 = ES_1 \cos 90^0 = 0$ ....(i)

[Since, angle between normal drawn on S_1 and direction of E is 90^0.]

Electric field flux linked with surface S_2 will,

$\phi_2 = ES_2 \cos\theta = ES_2 \cos 90^0 = 0$ ....(ii)

[Since, angle between normal drawn on S_2 and direction of E is 90^0.]

And, electric field flux linked with surface S_3 will,

$\phi_3 = ES_3 \cos\theta = ES_3 \cos 0^0 = ES_3$

[Since, angle between normal drawn on S_3 and direction of E is 0^0.]

Or $\phi_3 = E \times 2\pi r l$ ....(iii)

[$\because$ Area of curved surface of cylinder $= 2\pi rh = 2\pi r l$]

$\therefore$ Total electric flux liked with closed surface will,

$\phi_E = \phi_1 + \phi_2 + \phi_3 = 0 + 0 + E \times 2\pi r l$

Or $\phi_E = E \times 2\pi r l$ .... (iv)

According to Gauss's theorem,

$$\phi_E = \frac{q_{net}}{\varepsilon_0}$$

Where q_{net} is net charge inside the closed surface.

Or $\phi_E = \frac{\lambda l}{\varepsilon_0}$ .... (v)

[∵ linear density of charge $\lambda = \frac{q}{l}$ or $q = \lambda l$]

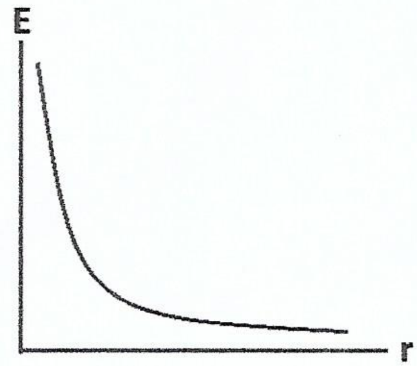
Comparing equations (iv) and (v),we get

$$E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

Or $E = \frac{\lambda l}{2\pi\varepsilon_0 r l}$

Or $E = \frac{\lambda}{2\pi\varepsilon_0 r}$ .... (vi)

(b)



(c)

Required work done ,

$$W = \int_{r_1}^{r_2} F\, dr = \int_{r_1}^{r_2} qE\, dr = \int_{r_1}^{r_2} q \frac{\lambda}{2\pi\varepsilon_0 r} dr = \frac{\lambda q}{2\pi\varepsilon_0} \int_{r_1}^{r_2} \frac{1}{r} dr$$

$$= \frac{\lambda q}{2\pi\varepsilon_0} \int_{r_1}^{r_2} r^{-1} dr = \frac{\lambda q}{2\pi\varepsilon_0} \left[\frac{r^{-2}}{-2}\right]_{r_1}^{r_2} = \frac{-\lambda q}{4\pi\varepsilon_0} \left[r^{-2}\right]_{r_1}^{r_2} = \frac{-\lambda q}{4\pi\varepsilon_0} \left[\frac{1}{r^2}\right]_{r_1}^{r_2}$$

$$= \frac{-\lambda q}{4\pi\varepsilon_0} \left[\frac{1}{{r_2}^2} - \frac{1}{{r_1}^2}\right]$$

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