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SAMPLE TEST PAPER 03 FOR CLASS X (2020-21)
(SAMPLE ANSWER)

Max. marks: 80 **Time Allowed: 3 hrs**

General Instruction:
1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part – A:
1. It consists three sections- I and II.
2. Section I has 16 questions of 1 mark each.
3. Section II has 4 questions on case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B:
1. Question No 21 to 26 are Very short answer Type questions of 2 mark each,
2. Question No 27 to 33 are Short Answer Type questions of 3 marks each
3. Question No 34 to 36 are Long Answer Type questions of 5 marks each.

PART - A
SECTION-I
Questions 1 to 16 carry 1 mark each.

1. If the pair of equations 2x + 3y = 7 and kx + $\frac{9}{2}$y = 12 have no solution, then find the value of k.
Ans: 3

2. Find the number of solutions of the following pair of linear equations:
$x + 2y – 8 = 0$
$2x + 3y = 16$
Ans: One solution

3. Does the rational number $\frac{441}{2^2.5^7.7^2}$ has a terminating or a non-terminating decimal representation?

$\frac{441^9}{2^2.5^7.7^2} = \frac{9}{2^2.5^7}$

Now numerator and denominator are co-prime and denominator is of the form $2^m \times 5^n$. Hence, given rational number is terminating.

4. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is –1, then find the product of the other two zeroes.
Ans: b – a +1

5. If $\frac{1}{2}$ is a root of the equation $x^2 + kx – \frac{5}{4} = 0$, then find the value of k.

$x^2 + kx - \frac{5}{4} = 0$

$\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0 \Rightarrow \frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$

$\frac{1+2k-5}{4} = 0 \Rightarrow 2k - 4 = 0 \Rightarrow k = 2$

6. Write the nature of roots of the quadratic equation $9x^2 – 6x – 2 = 0$.
D = $(-6)^2 – 4(9)(-2) = 36 + 72 = 108 > 0$
Roots are real and unequal.

7. The nth term of an AP is 7 – 4n. Find its common difference.

Given $a_n = 7 - 4n \Rightarrow a_2 = 7 - 4 \times 2 = -1$ and $a_1 = 7 - 4 \times 1 = 3$

$\therefore$ Common difference $d = a_2 - a_1 = -1 - 3 = -4$.

8. If tan A = 5/12, find the value of (sin A + cos A) . sec A.

Ans: 17/12

9. If the angle between two tangents drawn from an external point 'P' to a circle of radius 'r' and centre O is 60^0, then find the length of OP.

Ans: OP = 2r

10. Find the length of the tangent drawn from a point whose distance from the centre of a circle is 25 cm. Given that radius of the circle is 7 cm.

Ans: 24 cm

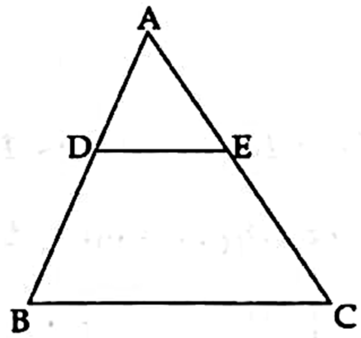
11. To divide a line segment PQ in the ratio 4 : 5, first a ray PX is drawn so that ∠QPX is an acute angle and then at equal distances points are marked on the ray PX and the last point is joined to Q. Write the minimum number of these equal distances points on ray PX.

Ans: 9

12. If sin θ = 1/3, then find the value of $(2 \cot^2 \theta + 2)$

Ans: 18

13. If adjoining figure, DE || BC and AD = 1 cm, BD = 2 cm. What is the ratio of the area of ∆ABC to the area of ∆ADE?



$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADE} = \left(\frac{AB}{AD}\right)^2 = \left(\frac{AD + BD}{AD}\right)^2 = \left(\frac{1+2}{1}\right)^2 = \frac{9}{1} \qquad (\because \Delta ABC \sim \Delta ADE)$$

$\therefore$ Area of ∆ABC : Area of ∆ADE = 9 : 1

14. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the length of the arc.

$$\text{Length of arc} = 2\pi r \cdot \frac{\theta}{360^\circ} \quad = 2 \times \frac{22}{7} \times 21 \times \frac{60^\circ}{360^\circ} \quad = 22 \text{ cm}$$

15. Two cylindrical cans have equal base areas. If one of the can is 15 cm high and other is 20 cm high, find the ratio of their volumes.

Ans: 3: 4

16. If the probability of an event is p, find the probability of its complementary event.

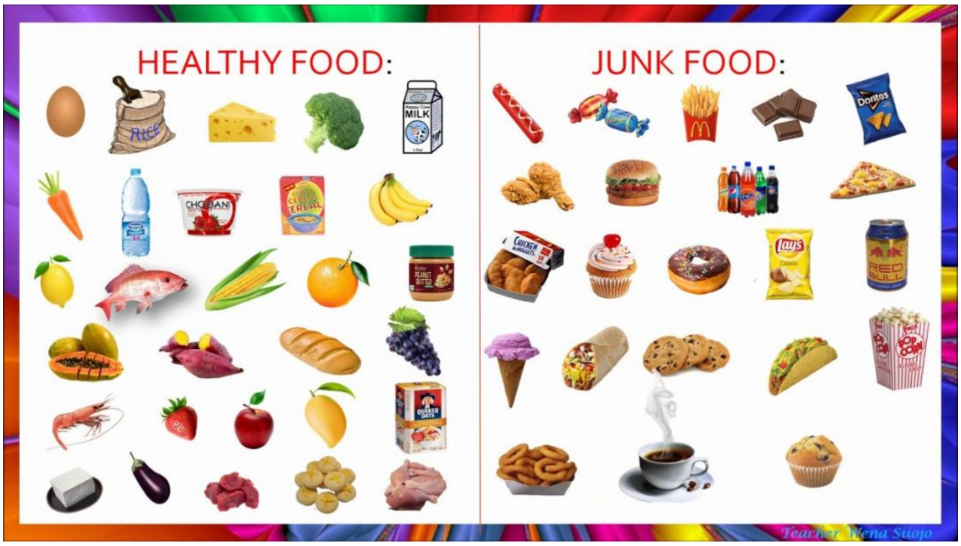
Ans: 1 – p

SECTION-II

Case study-based questions are compulsory. Attempt any four sub parts of each question. Each subpart carries 1 mark

17. Case Study based-1:

Junk food is unhealthful food that is high in calories from sugar or fat, with little dietary fiber, protein, vitamins, minerals, or other important forms of nutritional value. A sample of few students have taken. If α be the number of students who take junk food, β be the number of students who take healthy food such that $\alpha > \beta$ and α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 7x + 10$, then answer the following questions:



(a) Name the type of expression of the polynomial in the above statement?

(i) quadratic (ii) cubic (iii) linear (iv) bi-quadratic

Ans: (i) quadratic

(b) Find the number of students who take junk food.

(i) 5 (ii) 2 (iii) 7 (iv) None of these

Ans: (i) 5

(c) Find the number of students who take healthy food.

(i) 5 (ii) 2 (iii) 7 (iv) None of these

Ans: (ii) 2

(d) Find the quadratic polynomial whose zeros are -3 and -4.

(i) $x^2 + 4x + 2$ (ii) $x^2 - x - 12$

(iii) $x^2 - 7x + 12$ (iv) None of these

Ans: (iii) $x^2 - 7x + 12$

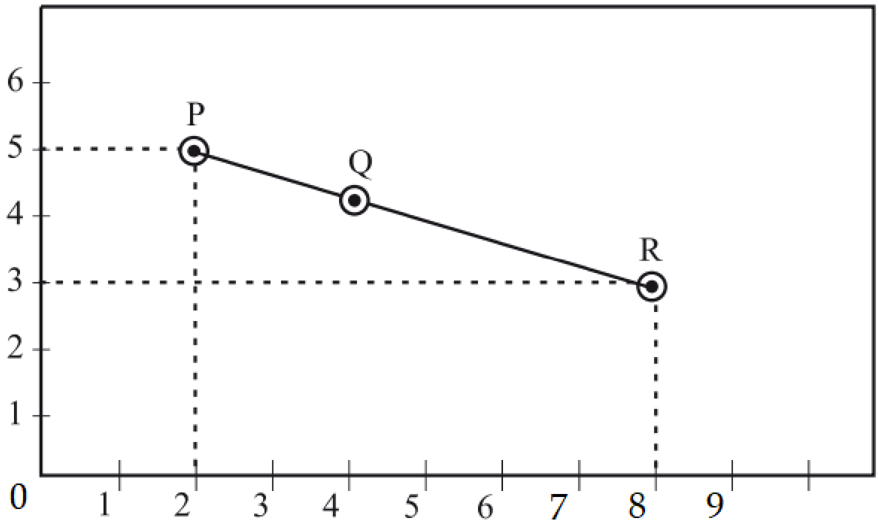
(e) If one zero of the polynomial $x^2 - 5x + 6$ is 2 then find the other zero.

(i) 6 (ii) -6 (iii) 2 (iv) None of these

Ans: (iv) None of these

18. Case Study based-2:

A group of Class X students goes to picnic during vacation. There were three different slides and three friends Ajay, Ram and Shyam are sliding in the three slides. The position of the three friends shown by P, Q and R in three different slides are given below:



Consider O as origin, answer the below questions:

(a) Find the co-ordinates of the point 'Q' which divides the line segment PR in the ratio 1 : 2 internally.

(i) $\left(4, \frac{13}{3}\right)$ (ii) $\left(\frac{13}{3}, \frac{11}{3}\right)$ (iii) $\left(\frac{10}{3}, \frac{13}{3}\right)$ (iv) $\left(\frac{13}{3}, 4\right)$

Ans: (i) $\left(4, \frac{13}{3}\right)$

(b) Find the distance PR.

(i) $2\sqrt{10}$ (ii) $\sqrt{36}$ (iii) $\sqrt{38}$ (iv) $\sqrt{20}$

Ans: (i) $2\sqrt{10}$

(c) Find the co-ordinates of point on x-axis which is at equal distance PQ.

(i) $\left(\frac{11}{9}, 0\right)$ (ii) (3, 0) (iii) $\left(\frac{13}{9}, 0\right)$ (iv) (1, 3)

Ans: (iii) $\left(\frac{13}{9}, 0\right)$

(d) Find the coordinates of the midpoints of PQ.

(i) (–4, 5) (ii) (4, 5) (iii) (5, 4) (iv) (4, 4)

Ans: (iii) (5, 4)

(e) If we shift origin 'O' by 2 units towards right and 1 unit towards North. Then fomd the co-ordinates of point R.

(i) (2, 6) (ii) (6, 2) (iii) (–6, 2) (iv) (–2, 6)

Ans: (ii) (6, 2)

19. Case Study based-3: Collection of Weight Data

Overweight and obesity may increase the risk of many health problems, including diabetes, heart disease, and certain cancers. The basic reason behind is the laziness, eating more junk foods and less physical exercise. The school management give instruction to the school to collect the weight data of each student. During medical check of 35 students from Class X-A, there weight was recorded as follows:

Weight (in kg)	No. of Students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

(a) Find the median class of the given data.

(i) 44-46 (ii) 46-48 (iii) 48-50 (iv) None of these

Ans: (ii) 46-48

(b) Calculate the median weight of the given data.

(i) 46.5 (ii) 47.5 (iii) 46 (iv) 47

Ans: (i) 46.5

(c) Find the mean of the given data.

(i) 45 (ii) 45.8 (iii) 46.2 (iv) 46.8

Ans: (ii) 45.8

(d) Find the modal class of the given data.

(i) 44-46 (ii) 46-48 (iii) 48-50 (iv) 50-52

Ans: (ii) 46-48

(e) While computing the mean of grouped data, we assumed that the frequencies are

(i) evenly distributed over all the classes

(ii) centred at the class marks of the classes

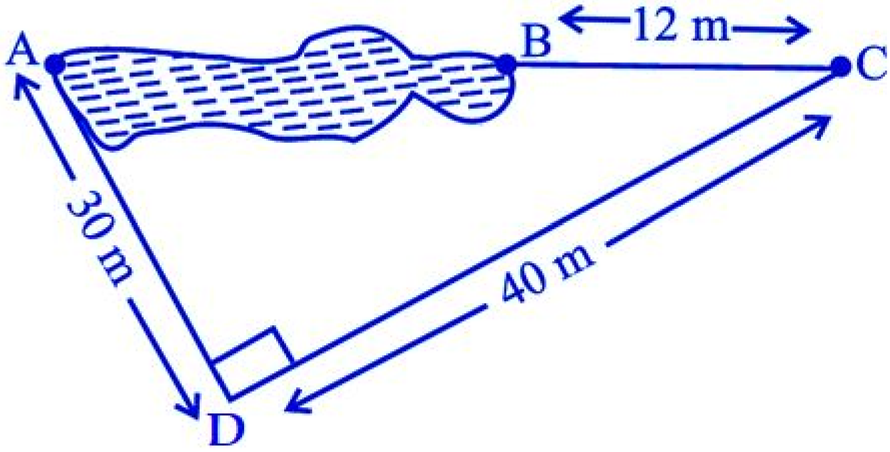
(iii) centred at the upper limits of the classes

(iv) centred at the lower limits of the classes

Ans: (ii) centred at the class marks of the classes

20. Case Study based-4: POND

Points A and B are on the opposite edges of a pond as shown in below figure. To find the distance between the two points, Ram makes a right-angled triangle using rope connecting B with another point C at a distance of 12 m, connecting C to point D at a distance of 40 m from point C and then connecting D to the point A which is at a distance of 30 m from D such that $\angle ADC = 90^0$.



(a) Which property of geometry will be used to find the distance AC?

(i) Similarity of Triangles (ii) Thales Theorem

(iii) Pythagoras Theorem (iv) Quadratic Equation

Ans: (iii) Pythagoras Theorem

(b) What is the distance AC?

(i) 50 m (ii) 12 m (iii) 100 m (iv) 70 m

Ans: (i) 50 m

(c) Which is the following does not form a Pythagoras triplet?

(i) (7, 24, 25) (ii) (15, 8, 17) (iii) (5, 12, 13) (iv) (21, 20, 28)

Ans: (iv) (21, 20, 28)

(d) Find the length AB.

(i) 12 m (ii) 38 m (iii) 50 m (iv) none of these

Ans: (ii) 38 m

(e) Find the length of the rope used.

(i) 120 m (ii) 70 m (iii) 82 m (iv) none of these

Ans: (iii) 82 m

PART – B

(Question No 21 to 26 are Very short answer Type questions of 2 mark each)

21. Two equilateral triangles have the sides of lengths 34 cm and 85 cm respectively. Find the greatest length of tape that can measure the sides of both of them exactly.

Ans: Length of tape = HCF of 34 and 85

$34 = 2 \times 17$ and $85 = 5 \times 17$

$\therefore$ HCF = 17

Hence length is 17.

22. Find the ratio in which the line segment joining (2, – 3) and (5, 6) is divided by x-axis.

Let the required ratio be $k : 1$.

Then the coordinates of the point of division are $\left(\frac{2k+5}{k+1}, \frac{-3k+6}{k+1}\right)$.

This point lies on the x-axis whose equation is $y = 0$.

$\therefore \frac{-3k+6}{k+1} = 0 \Rightarrow 3k = 6$, or $k = 2$.

$\therefore$ Line segment joining the two points is divided in the ratio 2 : 1 internally by x-axis.

23. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$; $0° < A + B \le 90°$; $A > B$, find A and B.

Given $\tan(A+B) = \sqrt{3} \quad \Rightarrow \quad A + B = 60° \cdots (1)$

$\tan(A-B) = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad A - B = 30° \cdots (2)$

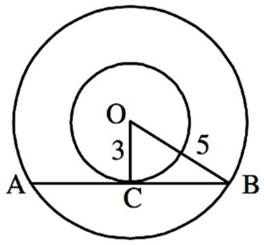
$(1) + (2) \quad \Rightarrow \quad 2A = 90° \quad \Rightarrow \quad A = 45°$

Put A value in (1)

$\Rightarrow \quad B = 60° - 45° = 15°$

24. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

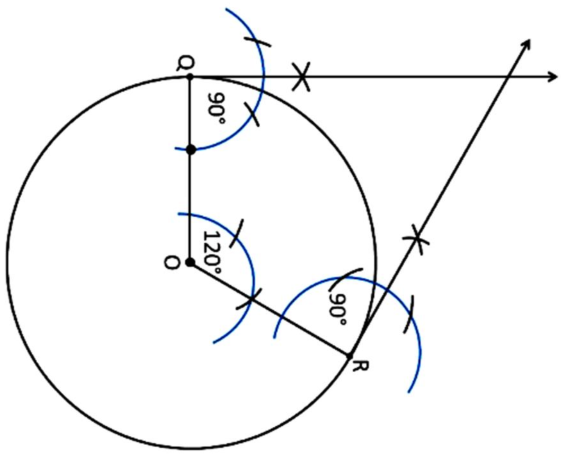
Ans:



Here $BC = \sqrt{5^2 - 3^2} = 4$ cm

$\therefore \quad AB = 2BC = 8$ cm

25. Draw a pair of tangents to a circle of radius 4 cm which are inclined to each other at an angle of 60°.



26. In an AP, the sum of first n terms is $\frac{5n^2}{2} + \frac{3n}{2}$. Find its 20th term.

$S_1 = 4 = a_1$, $S_2 = 13 = a_1 + a_2$
$a_2 = S_2 - S_1 = 13 - 4 = 9$
$d = a_2 - a_1 = 9 - 4 = 5$
Now, $a_{20} = a + 19d = 4 + 95 = 99$

27. Prove that $\sqrt{5}$ is an irrational number.

Let $\sqrt{5}$ be a rational number, then

$\sqrt{5} = \frac{p}{q}$, where p, q are integers, $q \neq 0$ and p, q have no common factors (except 1)

$\Rightarrow \quad 5 = \frac{p^2}{q^2} \quad \Rightarrow \quad p^2 = 5q^2 \quad ...(i)$

As 5 divides $5q^2$, so 5 divides p^2, but 5 is prime.

$\Rightarrow$ 5 divides p

Let $p = 5m$, where m is an integer.

Substituting this value of p in (i), we get

$(5m)^2 = 5q^2 \quad \Rightarrow \quad 25m^2 = 5q^2 \quad \Rightarrow \quad 5m^2 = q^2$

As 5 divides $5m^2$, so 5 divides q^2, but 5 is prime

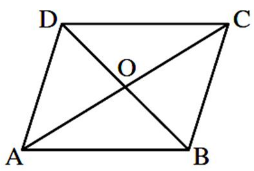
$\Rightarrow$ 5 divides q

Thus p and q have a common factor 5. This contradicts that p and q have no common factors (except 1)

Hence, $\sqrt{5}$ is not a rational number.

28. Prove that the sum of the squares of the sides of a rhombus is equal to sum of the squares of its diagonals.

Ans:



ABCD is a rhombus where diagonals intersect at O.

Let AB = a, AO = x and OB = y

In ΔAOB, $AO^2 + OB^2 = AB^2$

$\Rightarrow \quad x^2 + y^2 = a^2 \quad \Rightarrow \quad 4x^2 + 4y^2 = 4a^2$

$\Rightarrow \quad (2x)^2 + (2y)^2 = 4a^2 \quad \Rightarrow \quad AC^2 + BD^2 = 4a^2$

Thus sum of the squares of the sides of a rhombus is equal to sum of the squares of diagonals.

29. Prove that $\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{2\sin^2 A - 1}$.

$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{2\sin^2 A - 1}$$

$$\text{LHS} = \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A}$$

$$= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{\sin^2 A - \cos^2 A}$$

$$= \frac{\sin^2 A + \cos^2 A + 2\sin A \cos A + \sin^2 A + \cos^2 A - 2\sin A \cos A}{\sin^2 A - \cos^2 A} = \frac{1+1}{\sin^2 A - \cos^2 A}$$

$$= \frac{2}{\sin^2 A - 1 + \sin^2 A} = \frac{2}{2\sin^2 A - 1} = \text{RHS.}$$

30. A motor boat whose speed is 20 km/h in still water takes 1 hour more to go 48 km upstream than to return downstream to the same spot. Find the speed of the stream.

Let the speed of the stream be x km/h.

Given, speed of the motor boat in still water = 20 km/h.

Then, the speed of the boat upstream = $(20 - x)$ km/h

and the speed of the boat downstream = $(20 + x)$ km/h

Time taken by the motor boat to cover 48 km upstream = $\frac{48}{20-x}$ hours,

time taken by the motor boat to cover 48 km downstream = $\frac{48}{20+x}$ hours.

According to given, $\frac{48}{20-x} - \frac{48}{20+x} = 1$

$\Rightarrow \quad 48(20 + x) - 48(20 - x) = (20 - x)(20 + x) \quad \Rightarrow \quad 48(2x) = 20^2 - x^2$

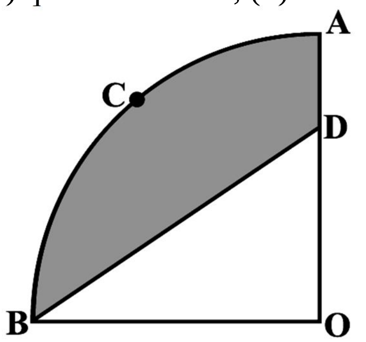
$\Rightarrow \quad x^2 + 96x - 400 = 0 \quad \Rightarrow \quad (x - 4)(x + 100) = 0$

$\Rightarrow \quad x = 4$ or $x = -100$ but x being the speed of the stream cannot be negative

$\Rightarrow \quad x = 4$

Hence, the speed of the stream is 4 km/h.

31. In the below figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the (i) quadrant OACB, (ii) shaded region.



Ans: (i) 9.625 cm^2 or 77/8 cm^2 (ii) 6.125 cm^2 or 49/8 cm^2

(i) Area of quadrant OACB $= \frac{\text{Area of Circle}}{4} = \frac{\pi \times r^2}{4}$

$$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 = 9.625\,\text{cm}^2$$

(ii) Area of the shaded region $=$ Area of Quadrant $-$ Area of $\triangle BDO$

$$= 9.625 - \left(\frac{1}{2} \times 3.5 \times 2\right) = 6.125\,\text{cm}^2$$

32. Find the mode of the following data:

Runs scored	2000-4000	4000-6000	6000-8000	8000-10000	10000-12000
No. of batsmen	9	8	10	2	1

The maximum class frequency is 10 and the class corresponding to this frequency is 6000 – 8000.
So, the modal class is 6000 – 8000.
Here, $l = 6000$, $h = 2000$, $f_1 = 10$, $f_0 = 8$ and $f_2 = 2$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 6000 + \frac{10-8}{2\times10-8-2} \times 2000$$

$$= 6000 + \frac{2}{10} \times 2000 = 6000 + 400 = 6400$$

$\therefore$ Mode = 6400.

33. The king, queen and jack of clubs are removed from a deck of 52 playing cards and the remaining cards are shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of (i) hearts (ii) queen.

After removing king, queen and jack of clubs, the number of cards left = 49

(*i*) The number of cards of hearts left in the remaining cards = 13

$\therefore$ P (a card of hearts) = $\frac{13}{49}$

(*ii*) The number of cards of queens left in the remaining cards = 13

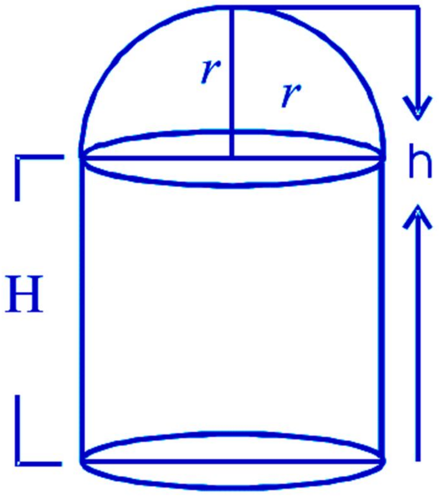
$\therefore$ P (a card of queen) = $\frac{3}{49}$

(Question no 34 to 36 are Long Answer Type questions of 5 marks each.)

34. Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Ans: Let the speed of train and bus be u km/h and v km/h.
According to the Question, 60/u + 240/v = 4 ... (i)
100/u + 200/v = 25/6 ... (ii)
Putting 1/u = p and 1/v = q in the equations, we get
⇒ 60p + 240q = 4 ... (iii)
⇒ 100p + 200q = 25/6 ⇒ 600p + 1200q = 25 ... (iv)
Multiplying equation (iii) by 10, we get 600p + 2400q = 40 .... (v)
Subtracting equation (iv) from (v), we get 1200q = 15
⇒ q = 15/200 = 1/80 ... (vi)
Putting equation (iii), we get 60p + 3 = 4 ⇒ 60p = 1 ⇒ p = 1/60
⇒ p = 1/u = 1/60 and q = 1/v = 1/80
⇒ u = 60 and v = 80
Hence, Speed of train = 60 km/h and Speed of bus = 80 km/h.

35. A building is in the form of cylinder surmounted by a hemispherical dome (see below figure). The base diameter of the dome is equal to $\frac{2}{3}$ of the total height of the building. Find the height of the building if it contains $67\frac{1}{21}$ m^3 of air.



Let the radius of the spherical dome be r metres and the total height of the building be h metres.

Since the base diameter of the dome is $\frac{2}{3}$ of the total height of the building, therefore,

$$2r = \frac{2}{3}h \Rightarrow r = \frac{h}{3}.$$

$\therefore$ Height of the cylinder $= \left(h - \frac{h}{3}\right)$ m $= \frac{2h}{3}$ m.

Volume of air inside the building = volume of hemisphere + volume of cylinder

$= \frac{2}{3}\pi r^3 + \pi r^2 H$, where H is height of cylinder

$$= \left(\frac{2}{3}\pi \times \left(\frac{h}{3}\right)^3 + \pi \times \left(\frac{h}{3}\right)^2 \times \frac{2h}{3}\right) \text{ m}^3 = \frac{8\pi}{27}h^3 \text{ m}^3$$

But the volume of air inside the building $= 67\frac{1}{21}$ m^3 $= \frac{1408}{21}$ m^3,

$$\therefore \quad \frac{8}{81} \times \frac{22}{7} \times h^3 = \frac{1408}{21} \Rightarrow h^3 = 216 \Rightarrow h = 6.$$

Hence, the height of the building = 6 m.

36. If sec θ + tan θ = p, then find the value of cosec θ.

Ans:

Given: sec θ + tan θ = p ----- (i)

We know that sec²θ – tan²θ = 1 ⇒ (secθ + tanθ)(secθ – tanθ) = 1

$$\Rightarrow (p)(\sec\theta - \tan\theta) = 1 \Rightarrow \sec\theta - \tan\theta = \frac{1}{p} \quad \text{----- (ii)}$$

On solving (i) & (ii), we get

$$\sec\theta + \tan\theta + \sec\theta - \tan\theta = p + \frac{1}{p} \Rightarrow 2\sec\theta = \frac{p^2+1}{p} \Rightarrow \sec\theta = \frac{p^2+1}{2p}$$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{2p}{p^2+1}$$

$$\text{Now, } \sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{2p}{p^2+1}\right)^2 = \left(\frac{p^2-1}{p^2+1}\right)^2 \Rightarrow \sin\theta = \frac{p^2-1}{p^2+1}$$

$$\text{Now, we know that } \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{p^2+1}{p^2-1}$$