MATHEMATICS
CLASS-XII

Time:-2:15 hrs M.M: 100

General instructions:

(i) *All questions are compulsory.*

(ii) *The question paper consists of 27 questions divided into 3 sections — A, B, and C. Section A comprises of ten questions of 1 mark each, Section B comprises of 12 questions of 4 marks each,*
Section C comprises of 7 questions of 6 marks each.

(iii) *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*

(iv) *In question on construction, the drawing should be neat and exactly as per the given measurements.*

(v)Use of calculators is not permitted

Section – A

Questions numbers 1 to 10 carry 1 mark each

1. Let f, g and h be functions from R to R. Show that (f+g) oh=foh+goh.

2. Evaluate: $\left(\begin{bmatrix}1 & 3\\ -1 & -4\end{bmatrix}+\begin{bmatrix}3 & -2\\ -1 & 1\end{bmatrix}\right)\begin{bmatrix}1 & 3 & 5\\ 2 & 4 & 6\end{bmatrix}$.

3. Find the rate of change of the area of a circle with respect to its radius when r=3 cm.

4. Find the value of x if $[1 \;\; x \;\; 1]\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix}=0$.

5. If A= $\begin{pmatrix}2 & 3\\ 1 & 2\end{pmatrix}$ $and\, I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$. Find x and y such that A^2=xA+yI.

6. Evaluate: $\int \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}\,dx$

7. Evaluate: $\int \frac{x^2+1}{x^4+1}dx$.

8. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, prove that x+y+z=xyz.

9. Evaluate: $\int \frac{\cos x}{(1-\sin x)(2+\sin x)}dx.$

10. Find the foot of the perpendicular drawn from the point P (1,6,3) on the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Also find its distance from P.

Section – B

Questions numbers 11 to 22 carry 4 marks each

11. If A = R-{3} and B=R-{2} and f:A→ B is a mapping defined by f(x)= $\frac{x-2}{x-3}$. Show that f is bijective.

12. Two unbiased dice are tossed simultaneously. Find the probability that the sum of the numbers will be a multiple of 3 or 5.

Or

There are two bags. The first bag contains 4 white and 2 black balls, while the second bag contains 3 white and 4 black balls. A bag is picked up at random and a ball is drawn out. Find the probability that it is a white ball.

13. Solve the differential equation: $x(1+y^2)\,dx - y\,(1+x^2)dy=0$, given that y=0 when x=1.

14. Solve the differential equation: $\frac{dY}{dX}-\frac{Y}{x}=2x^2$.

15. Discuss the continuity of the function at x=0. $f(x)=\begin{cases}\frac{|x|}{x} & , if\ x\neq 0\\ 0 & , if\ x=0\end{cases}$

16. Differentiate √sinx w.r.t. x from first principle.

Or

Differentiate the following w.r.t. x: $(x)^{\cos x}+(\cos x)^x$.

17. A point source of light along a straight road is at a height of 'a' metres. A boy 'b' metres in height is walking along the road. How fast is his shadow increasing if he is walking away from the light at the rate of c metres per minute?

18. If $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}\neq=0$, show that $\vec{b}=\vec{c}+t\vec{a}$, for some scaler t.

19. Using properties of determinants, prove the following: $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$.

20. Evaluate: $\int_0^{\pi/2} x^2 \cos 2x\, dx.$

Or

Using properties of definite integrals, evaluate the following:

$\int_0^{\pi/2} \sin 2x \log \tan x\, dx.$

21. Prove that: $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x, x \in \left[0, \frac{\pi}{4}\right]$.

22. A coin is tossed 12 times. Find the probability of getting exactly 10 tails.

Section – C

Questions numbers 23 to 29 carry 6 marks each

23. If a, b and c are the lengths of sides to ∠A, ∠B and ∠C respectively of ΔABC, then show that $\cos C = \frac{a^2+b^2-c^2}{2ab}$.

24. An urn contains 5 white and 3 red balls. Find the probability distribution of the number of red balls, with replacements, in three draws.

25. Using matrices, solve the following system of linear equations: 3x+4y+2z=8, 2y-3z=3, x-2y+6z=-2

26. Find the largest possible area of the right angled triangle whose hypotenuse is 5 cm.

Or

Prove that the radius of the right circular cylinder of the greatest curved surface that can be inscribed in a given cone is half of the radius of the cone.

27. Using integration, find the area of the region enclosed between two circles $x^2+y^2=1$ and $(x-1)^2+y^2=1$.

Or

Using integration, find the area bounded by the curve x2-4y and the straight line x=4y-2.

28. Find the equation of the plane that passes through the points (1,1,0) (1,2,1) and (-2,2,-1).

29. A furniture dealer deals only in two items – tables and chairs. He has Rs. 10,000 to invest and a space to store at most 60 pieces. A table cost him Rs. 500 and a chair Rs. 200. He can sell a table at a profit of Rs. 50 and a chair at a profit he buys. Using linear programming formulates the problem for maximum profit and solve it graphically.