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# CLASS XII
# SAMPLE PAPER-041
# MATHS

Time allowed : 3 hours                                                                                     Maximum
Marks : 100

**General Instructions:**

(i) **All** questions are compulsory.

(ii) This question paper contains **29** questions.

(iii) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.

(iv) Question **5-12** in **Section B** are short-answer type questions carrying **2** marks each.

(v) Question **13-23** in **Section C** are long-answer-**I** type questions carrying **4** marks each.

(vi) Question **24-29** in **Section D** are long-answer-**II** type questions carrying **6** marks each.

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**-A**

**Questions 1 to 4 carry 1 mark each.**

1. What is the principal value of $cos^{-1}\left(cos\frac{2\pi}{3}\right)+\ sin^{-1}\left(sin\frac{2\pi}{3}\right)$ **?**

2. Let $f: R \to R$ be defined as $f(x) =\ 10x + 7$. Find the function $g: R \to R$ such that $gof = fog = I_R$ .

3. Using elementary transformations, find the inverse matrix , A=$\begin{bmatrix}1 & 3\\2 & 7\end{bmatrix}$.

4. What are the direction cosines of a line which makes equal angles with the coordinate axes?

## Section-B

**Questions 5 to 12 carry 2 marks each.**

5. Evaluate: $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \, logsinx \, dx$.
6. If f(x) = $|cosx|$ , find $f'\left(\frac{3\pi}{4}\right)$ .
7. The radius of a circle is increasing at the rate of 0.7cm/s. What is the rate of increase of its circumference?
8. Find the area of the parallelogram determined by the vectors $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ $and$ $3\ \hat{\imath} - 2\hat{\jmath} + \hat{k}$.
9. Find the probability of a diamond card in each of the two consecutive draws from a well-shuffled pack of cards, if the card drawn is not replaced after the first draw.
10. Prove that the function f:R→ $R\ defined\ as\ f(x) = 2x - 3$ is invertible. Also find $f^{-1}$.

11.Find the value of x,y,z , if the matrix A $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation $A^TA = I_3$.

12. Differentiate $tan^{-1}\left(\frac{1+2x}{1-2x}\right)$ with respect to $\sqrt{1+4x^2}$.

## SECTION C

**Questions 13 to 23 carry 4 marks each.**

13.If y(x) is a solution of $\left(\frac{2+sinx}{1+y}\right)\frac{dy}{dx} = -cosx$ and y(0)=1, find the value of $y\left(\frac{\pi}{2}\right)$.

14.Using properties of determinants, show that $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = (x+y+z)(z-x)^2$.

OR

Find the value of $\theta\ satisfying$ $\begin{vmatrix} 1 & 1 & sin3\theta \\ -4 & 3 & cos2\theta \\ 7 & -7 & -2 \end{vmatrix} = 0$.

15.If f(x) $=\begin{cases} \frac{x^3+x^2-16x+20}{(x-2)^2}, & x \neq 2 \\ k, & x = 2 \end{cases}$ is continuous at x=2, find the value of k.

OR

A function f(x) is defined as follows: f(x)= $\begin{cases} x^2 sin\frac{1}{x} & ,if x \neq 0 \\ 0, if\ x = 0 \end{cases}$ show that f(x) is differentiable at x=0.

16.Prove that $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}sin^{-1}\left(\frac{x}{a}\right)\right] = \sqrt{a^2 - x^2}$ .

**OR**

If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, ($x \neq y$) prove that: $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$ .

17.Find the equation of curve passing through the point (1,1) , if the tangent drawn at any point P(x,y) on the curve meets the coordinates axes at A and B such that P is the mid-point of AB.

18.Evaluate : $\int \frac{dx}{\sin(x-a).\cos(x-b)}$ .

**OR**

Evaluate $\int \frac{xe^{2x}}{(1+2x)^2}dx.$

19.Two bikers are running at the speed more than speed allowed on the road along lines $\vec{r}$= (3i+5j+7k)+ $\lambda(\mathrm{i} - 2\mathrm{j} + \mathrm{k})$ and $\vec{\mathrm{r}} = (-\mathrm{i} - \mathrm{j} - \mathrm{k}) + \mu(7\mathrm{i} - 6\mathrm{j} + \mathrm{k})$.Using shortest distance , check whether they meet to an accident or not.

20. An urn contains m white and n black balls . A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn . A ball is again drawn at random . Show that the probability of drawing a white ball does not depends on k.

21.A clever student used a biased coin so that the head is 3 times as likely to occur as tail . If the coin is tossed twice , find the probability distribution and mean of numbers of tails.

22.Let $\vec{a}$=2i+k , $\vec{b}$=i+j+k and $\vec{c}$=4i-3j+7k be three vectors . Find a vector $\vec{r}$ which satisfies $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a}$=0.

23.Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

**SECTION D**

**Questions 24 to 29 carry 6 marks each.**

24.Solve the equation sin[$2cos^{-1}${cot($2tan^{-1}x$)}]=0.

25.A diet for a sick person must contain atleast 4000 units of vitamins , 50 units of minerals and 1400 calories. Two foods A and B are available at a cost of Rs.4 and Rs.3 per unit, respectively. 1 unit of food A contains 200 units of vitamins, 1 unit of minerals and 40 calories, Food B contains 100 units of vitamins , 2 units of minerals and 40 calories. Find what combination of food should be used to have the least cost. Why a proper diet required for us.

**OR**

A small firm manufactures shirts and trouser. Total number of shirts and trousers that it can handle per day is atmost 24. It takes 1hr to make a trouser and half an hour to make a shirt. The maximum number of hours available per day is 16. If the profit on a shirt is Rs.100 and that on a trouser is Rs.300, then how many of each should be produced daily to maximise the profit?

26.For $x > 0, let\ f(x) = \int_1^x \frac{\log_e t}{1+t} dt. Find\ the\ function\ f(x) + f\left(\frac{1}{x}\right)$ and show that $(e) + f\left(\frac{1}{e}\right) = \frac{1}{2}$.

27. Find A$^{-1}$, if A=$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$and show that $A^{-1}$= $\frac{A^2-3I}{2}$.

28. Find the distance of the point (-2,3,-4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane 4x+12y-3z+1=0.

29.Using integration , find the area of the region enclosed between the circles $x^2+y^2= 4$ and $(x-2)^2+y^2 = 4$.

By: Ankit Kumar Agrawal