

NAME : \_\_\_\_\_

ROLL. NO. : \_\_\_\_\_

**Instructions:**

1. All questions are compulsory.
2. The question paper is printed on two pages and consists of 29 questions divided into four sections A, B, C and D.  
Section A contains 4 questions of 1 mark each, Section B is of 8 questions of 2 marks each, Section C is of 11 questions of 4 marks each and Section D is of 6 questions of 6 marks each.
3. There is no overall choice. However, internal choices are provided in section C and section D only.
4. Write the serial number of the question before attempting it.
5. Use of calculators is not permitted. However, you may ask for Mathematical tables if needed.
6. Cancel the previous question, if attempted again, for any reason.

Q. No.	Question	Max Marks
<b>SECTION A</b>		
<b>Q.1</b>	A relation R on set $N \times N$ is given by $(a, b)R(c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$ . Find the ordered pair related to (1,2)	1
<b>Q.2</b>	For what value of k, the matrix $\begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}$ has no inverse.	1
<b>Q.3</b>	Write the value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{i} \times \hat{j})$	1
<b>Q.4</b>	If * is a binary operation defined on $\mathbf{R}$ given by $a * b = \frac{ab}{5}$ . Find the identity element w.r.t *	1
<b>SECTION B</b>		
<b>Q.5</b>	Prove that: $\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \cos^{-1}\left(\frac{36}{85}\right)$	2
<b>Q.6</b>	If $A = \begin{bmatrix} 0 & -\tan \alpha/2 \\ \tan \alpha/2 & 0 \end{bmatrix}$ and I is a unit matrix, then prove that $I + A = (I - A)\cos\alpha - \sin\alpha \sin\alpha \cos\alpha$	2
<b>Q.7</b>	If $x = a(\cos \theta + \log \tan \frac{\theta}{2})$ and $y = a \sin \theta$ , find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$	2
<b>Q.8</b>	Water is leaking from a conical funnel at the rate of $5 \text{ cm}^3/\text{s}$ . If the radius of the base of funnel is 5 cm and height 10 cm, find the rate at which the water level is dropping when it is 2.5 cm from the top.	2
<b>Q.9</b>	Evaluate $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}} dx$ .	2
<b>Q.10</b>	Solve $xdy + ydx = \sqrt{x^2 + y^2} dx$	2
<b>Q.11</b>	If $\hat{a}$ and $\hat{b}$ are unit vectors inclined at an angle $\theta$ , then prove that $\cos \frac{\theta}{2} = \frac{1}{2}  \hat{a} + \hat{b} $	2
<b>Q.12</b>	Four cards are drawn from a well shuffled pack of 52 cards. Find the probability of drawing all the four cards of the same suit if a card is replaced after each draw.	2

Q. No.	Question	Max Marks
Q.13	Find the product of matrices $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ , $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and use it for solving the equations: $x + y + 2z = 1, 3x + 2y + z = 7, 2x + y + 3z = 2$ .	4
Q.14	Let $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & x < 0 \\ a, & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}-4}}, & x > 0 \end{cases}$ . For what value of $f$ is continuous at $x = 0$	4
Q.15	If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ , then prove that: $(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$	4
Q.16	Does the straight line $\frac{x}{a} + \frac{y}{b} = 2$ touch the curve $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 2$ ? If it touches then determine the point of contact. OR Show that $y = \log(1+x) - \frac{2x}{2+x}$ , $x > -1$ is an increasing function of $x$ , throughout its domain.	4
Q.17	Find the area of the greatest isosceles triangle that can be inscribed in a given ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex coinciding with one extremity of the major axis.	4
Q.18	Find $\int \frac{\sqrt{x^2+1} [\log(x^2+1) - 2 \log x]}{x^4} dx$ . OR Evaluate $\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$ .	4
Q.19	Solve the differential equation: $\frac{dy}{dx} + y \sec^2 x = \tan x \sec^2 x$ ; $y(0) = 1$	4
Q.20	If $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ , $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ , find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ such that $\vec{c} \cdot \vec{d} = 18$	4
Q.21	Find the shortest distance between the following lines $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$ .	4
Q.22	Assume that a factory has two machines. Machine I produces 20% of output and machine II produce 80% of output. 6% of items produced by machine I were defective and 1% of items produced by machine II were defective. If an item was drawn and found to be defective, find the probability that it was produced by machine I.	4
Q.23	Suppose a fair dice are tossed and let $X$ represents "The sum of points". Find the mean and standard deviation of $X$ .	4
<b>SECTION D</b>		
Q.24	Prove that the relation $R$ on the set $N \times N$ defined by $(a, b)R(c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$ is an equivalence relation.	6
Q.25	Show that: $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix} = (a^3 + b^3)^2$	6
Q.26	Draw a sketch of the region $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$ and calculate its area	6

Q. No.	Question	Max Marks
Q.27	Evaluate $\int_0^2 (x^2 + 2x + 1)dx$ , as limit of sum. OR Evaluate $\int_0^1 \cot^{-1}(1 - x + x^2)dx$ .	
Q.28	Find the equation of plane passing through the point (-1,-1,2) and perpendicular to the planes $3x + 2y - 3z = 1$ and $5x - 4y + z = 5$ . OR Find the equation of the plane passing through the point (0,0,0) and (3,-1,2) and parallel to the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$	6
Q.29	A merchant plans to sell two types of personal computers : a desktop model and a portable model that will cost Rs. 25000 and Rs. 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs. 70 lakhs and his profit on the desktop model Rs. 4500 and on the portable model Rs. 5000. Make an LPP and solve it graphically.	6

\*\*\*\*\*ALL THE BEST\*\*\*\*\*