CLASS XII
GUESS PAPER
MATHEMATICS

Time : 3 Hours **Max. Marks : 100**

General Instructions :-

(i) *All questions are compulsory.*

(ii) *The question paper consist of **29** questions divided into three sections A, B and C. Section A comprises of **10** questions of **one mark** each, Section B comprises of **12** questions of **four marks** each and section C comprises of **7** questions of **six marks** each.*

(iii) *There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*

SECTION – A

1. Let $f : \mathrm{R} - \left\{-\frac{3}{5}\right\} \to \mathrm{R}$ be an invertible function defined as $f(x) = \frac{2x}{5x+3}$, find $f^{-1} : \text{Range of } f \to \mathrm{R} - \left\{-\frac{3}{5}\right\}$.

2. Write the value of : $\cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

3. A square matrix A of order 3, has $|A| = 5$. Find $|A.\text{adj}A|$.

4. If a matrix has 13 elements, what are the possible orders it can have?

5. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$, find AB.

6. Evaluate : $\int \frac{3x^2+4x-5}{(x^3+2x^2-5x+1)^2}\,dx.$

7. Evaluate : $\int_{-2}^{1} \frac{|x|}{x}\,dx.$

8. The equation of a line AB is $6x-2=3y+1=2z-2$. What are the direction cosines of a line parallel to the AB ?

9. If $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}|=3$ and $|\vec{b}|=\frac{\sqrt{2}}{3}$ and $\vec{a}\times\vec{b}$ is a unit vector. Then what is the angle between $\vec{a}$ and $\vec{b}$?

10. Find the area of a parallelogram whose arms are represented by the vectors $\hat{i}+2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$.

SECTION – B

11. Find the shortest distance between the lines whose vector equations are : $\vec{r}=(1-\lambda)\hat{i}+(\lambda-2)\hat{j}+(3-2\lambda)\hat{k}$, and $\vec{r}=(\mu+1)\hat{i}+(2\mu-1)\hat{j}-(2\mu+1)\hat{k}$

12. Prove that : $2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{5\sqrt{2}}{7}+2\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$.

OR

Prove that : $\cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}=\frac{x}{2}, x\in\left(0,\frac{\pi}{4}\right)$

13. Using properties of determinants, solve the following for x :

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix}=0$$

14. Solve the differential equation: $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right]\frac{dx}{dy}=1;\ (x\neq 0)$

15. Show that the function $f(x) = |x+2|$ is continuous ate every $x \in \mathrm{R}$ but fails to be differentiable at $x = -2$.

OR

If $x^y = e^{x-y}$, show that $\frac{dy}{dx} = \frac{\log x}{\{\log(ex)\}^2}$.

16. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with the unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to 1. Find the value of λ.

17. Show that the function $f: W \to W$ defined by $f(n) = \begin{cases} n-1, & \text{if n is odd} \\ n+1, & \text{if n is even} \end{cases}$,

is a bijective function.

18. If $x = \tan\left(\frac{1}{a}\log y\right)$, show that $(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$.

19. Find the equation of the normal to the curve $y = x^3 + 2x + 6$ which is parallel to the line $x + 14y + 1 = 0$.

OR

Find the intervals in which the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$

(i) is increasing (ii) is decreasing

20. Solve the differential equation: $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$; $y = 0$ when $x = 1$.

21. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{x+\sin x}{1+\cos x}\,dx$

OR

Evaluate : $\int_0^1 \cot^{-1}(1-x+x^2)\,dx$.

22. A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Determine :

(i) k

(ii) $P(X < 3)$

(iii) $P(X > 6)$

(iv) $P(0 < X < 3)$

SECTION – C

23. Make a rough sketch of the region given below and find the area using Integration :

$$\{(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$$

24. Evaluate : $\int \frac{1}{\sin x(5 - 4\cos x)}\, dx$

OR

Evaluate $\int_1^2 (x^2 + x + 2)dx$ as a limit of sum.

25. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations:

$$x + 2y + z = 4$$
$$-x + y + z = 0$$
$$x - 3y + z = 2$$

26. Find the distance of the point (–1, –5, –10), from the point of intersection of the line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 5$.

27. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be diamonds. What is the probability that the lost card was a card of heart?

OR

Two bags A and B contain 4 white and 3 black balls and 2 white and 2 black balls respectively. From bag A, two balls are drawn at random and then transferred to bag B. A ball is then drawn from bag B and is found to be a black ball. What is the probability that the transferred balls were 1 white and 1 black ?

28. Show that the height of the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

29. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit?