CLASS XII
SAMPLE PAPER
MATHEMATICS

Duration: 3 hr. M.M: 100

General Instructions:

(i) All questions are compulsory.

(ii) This question paper contains **29** questions.

(iii) Question **1–4** in **Section A** are very short-answer type questions carrying **1** mark each.

(iv) Question **5–12** in **Section B** are short-answer type questions carrying **2** marks each.

(v) Question **13–23** in **Section C** are long-answer-**I** type questions carrying **4** marks each.

(vi) Question **24–29** in **Section D** are long-answer-**II** type questions carrying 6 marks each.

(vii) Use of calculators is not permitted.

SECTION – A

1. Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i} - 3\hat{j} + 6\hat{k}$.

2. If A is a square matrix such that |A| = 5. Write the value of $|AA^t|$.

3. If f(x) = x + 7 and g(x) = x – 7, x ∈ R, then find fog(7).

4. Find the integrating factor of the differential equation: $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1.$

SECTION – B

5. Let $A=\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B=\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and $C=\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$. Find a matrix D such that CD – AB = 0.

6. If $y = x^{x^{x^{\cdots\infty}}}$, then find that $x\frac{dy}{dx}=\frac{y^2}{1-y\log x}$

7. Sand is pouring from a pipe at the rate of 12 cm^2/sec. The falling sand form a cone on the ground in such a way that the height of the curve is always one-sixth of the radius of the base. How fast in the height of the sand cone increasing when the height is 4cm?

8. Find the approximate change in the volume V of a cube of side 'x' metres caused by increasing the side by 2%.

9. Evaluate: $\int \frac{\sqrt{9-(\log x)^2}}{x}dx$.

10. Form the differential equation of the family of ellipses having foci on x-axis and centre at the origin.

11. If the sum of two unit vectors is a unit vector, show that magnitude of their difference is $\sqrt{3}$.

12. A die is thrown twice and the sum of the numbers appearing is observed to be 7. What is the conditional probability that the number 2 has appeared atleast once?

SECTION – C

13. Evaluate: $\int_0^{\pi} \frac{x\sin x}{1+3\cos^2 x}dx$

OR

Evaluate: $\int_0^1 \cot^{-1}\left(1 - x + x^2\right)dx$

14. Find the shortest distance between the following lines:

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} + (2s+1)\hat{k}$

OR

Find the equation of the plane passing through the line of intersection of the planes 2x + y – z = 3 and 5x – 3y + 4z + 9 = 0 and is parallel to the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{5-z}{-5}$.

15. Evaluate: $\int \frac{\sin x - x\cos x}{x(x+\sin x)}dx$

16. Prove that $2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$.

OR

Solve the equation for x: cos $(\tan^{-1}x)$ = sin $(\cot^{-1}3/4)$.

17. For what value of k is the following function continuous at $x = \frac{-\pi}{6}$?

$$f(x) = \begin{cases} \dfrac{\sqrt{3}\sin x + \cos x}{x + \frac{\pi}{6}} &, \quad x \neq \dfrac{-\pi}{6} \\ k &, \quad x = \dfrac{-\pi}{6} \end{cases}$$

18. The monthly income of Aryan and Babban are in the ratio 3:4 and their monthly expenditure are in the ratio 5:7. If each saves Rs 15000 per month, find their monthly incomes using matrix method. Which value is reflected in this problem?

19. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right) w.r.t. \cos^{-1} x^2.$

20. Find the general solution of the differential equation:
(1 + tany) (dx – dy) + 2x dy = 0.

OR

Solve the following differential equation: $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0.$

21. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.

22. In a certain college 4% of boys and 1% of girls are taller than 1.75 metres. Furthermore, 60% of that students in the college are girls. A student is selected at random from the college and is found to be taller than 1.75 metres. Find the probability that the selected student is a girl.

23. Prove that if E and F are independent events, then so are the events E and F'.

SECTION – D

24. Find the equation of the plane which contains the line of intersection of the planes x + 2y + 3z – 4 = 0 and 2x + y – z + 5 = 0 and whose x – intercept is twice its z – intercept.

Hence write the vector equation of a plane passing through the point (2, 3, -1) and parallel to the plane obtained above.

25. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\sin^{-1}\sqrt{\frac{2}{3}}$.

OR

If the function f(x) = $2x^3 - 9mx^2 + 12m^2x + 1$, where m > 0 attains its maximum and minimum at p and q respectively such that $p^2 = q$, then find the value of m.

26. On the set {0, 1, 2, 3, 4, 5, 6} a binary operation * is defined as:

$$a*b=\begin{cases} a+b, & \text{if } a+b<7 \\ a+b-7, & \text{if } a+b\geq 7 \end{cases}$$

Write the operation table of the operation * and prove that zero is the identity for this operation and each element a ≠ 0 of the set is invertible with 7 – a being the inverse of a.

27. Using property of determinants, prove that

$$\begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

OR

If $A=\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ and $A^3 - 6A^2 + 7A + kI_3 = 0$ find k.

28. A company manufactures two types of cardigans: type A and type B. It costs Rs 360 to make a type A cardigan and Rs 120 to make type B cardigan. The company can make at most 300 cardigans and spend at most Rs 72000 a day. The number of cardigans of type B cannot exceed the number of cardigans of type A by more than 200. The company makes a profit of Rs 100 for each cardigan of type A and Rs 50 for every cardigan of type B. Formulate this problem as a linear programming problem to maximize the profit of a company. Solve it graphically and find maximum profit.

29. Using integration, find the area of the region bounded by the curves $y=\sqrt{4-x^2}$, $x^2 + y^2 - 4x = 0$ and the x-axis.

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