Series RMM

Code No. **RSPL/1**

Roll No. | | | | | | | |

Candidates must write the Code on the title page of the answer-book.

- Please check that this question paper contains **5** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **26** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper.

MATHEMATICS

Time allowed : 3 hours *Maximum Marks : 100*

General Instructions :

(i) *All questions are compulsory.*

(ii) *The question paper consists of **26** questions divided into three sections A, B and C. Section A comprises of **6** questions of **one mark** each, Section B comprises of **13** questions of **four marks** each and Section C comprises of **7** questions of **six marks** each.*

(iii) *All questions in **Section A** are to be answered in one word, one sentence or as per the exact requirement of the question.*

(iv) *Use of calculators is **not** permitted. You may ask for logarithmic tables, if required.*

SECTION – A

1. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$ then find $(A + 2B)'$.

2. Find the differential equation for $y = A\cos\alpha x + B\sin\alpha x$, where A and B are arbitrary constant.

3. Find the position vector of a point R which divides the line joining points $P(\hat{i} + 2\hat{j} - \hat{k})$ and $Q(-\hat{i} + \hat{j} + \hat{k})$ in the ratio 2 : 1.

4. Using differentials, find the approximate value of $\sqrt{25.2}$.

5. Evaluate : $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

6. If the lines $\frac{x-1}{-2} = \frac{y-4}{3p} = \frac{z-3}{4}$ and $\frac{x-2}{4p} = \frac{y-5}{2} = \frac{z-1}{-7}$ are perpendicular to each other, then find the value of p.

SECTION – B

7. Let T be the set of all triangles in a plane. Let us define a relation $R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2;\ T_1, T_2 \in T\}$ Show that R is an equivalence relation.

OR

Consider $f : R_+ \longrightarrow [4, \infty)$ given by $f(x) = x^2 + 4$. show that f is invertible with the inverse (f^{-1}) of f given by $f^{-1}(y) = \sqrt{y-4}$, where R_+ is a set of all non-negative real numbers.

8. Solve the equation $\sin\left[2\cos^{-1}\left\{\cot\left(2\tan^{-1}x\right)\right\}\right]=0$

OR

Show that $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=2\left(\tan^{-1}1+\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}\right)$

9. Let $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}$ *and* $B=\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}$. Find a matrix D such that $CD-AB=O$ where $C=\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}$.

10. If $\phi(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$ show that $\phi(x)\cdot\phi(y)=\phi(x+y)$.

OR

Find the matrix product : $\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$ Hence or otherwise , solve the system of equations: $x-y+z=4$, $x-2y-2z=9$ and $2x+y+3z=1$.

11. If none of a, b, c is zero, then show that $\begin{vmatrix}-bc & b^2+bc & c^2+bc\\ a^2+ac & -ac & c^2+ac\\ a^2+ab & b^2+ab & -ab\end{vmatrix}=(ab+bc+ca)^3$

12. Find the value of a, b , c so that the function $f(x)$ defined below is continues at x = 0

$$f(x)=\begin{cases}\dfrac{\sin(a+1)x+\sin x}{x} & if\ x<0\\ c & if\ x=0\\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}} & if\ x>0\end{cases}$$

13. If $f=Ax+\dfrac{B}{x}$ then show that $x^2\dfrac{d^2f}{dx^2}=f-x\dfrac{df}{dx}$

OR

Verify Rolle's theorem for the function $f(x)=\log\left(\dfrac{x^2+ab}{(a+b)x}\right)$ in $[a,b]$ where $0<a<b$.

14. Evaluate : $\int\dfrac{1}{\sin x-\sin 2x}dx$

15. Evaluate : $\int_0^2\left(e^{2x}+x\right)dx$ limit as a sum.

16. Solve the following differential equation : $ye^{\frac{x}{y}}dx=\left(x\cdot e^{\frac{x}{y}}+y\right)dy$

17. For any three vectors $\vec{a},\vec{b},\vec{c}$ show that $\left[\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}\right]=2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

OR

If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$, find vector $\vec{c}$ such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}\cdot\vec{c}=3$.

18. Find the equation of the plane which passes through the points (3, 4, 1) and (0, 1, 0) and is parallel to the line $\frac{x+3}{2}=\frac{y-3}{7}=\frac{z-2}{5}$.

19. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of the number of successes.

SECTION C

20. Find the distance of the point $(-2,3,-4)$ from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane $4x+12y-3z+1=0$

21. A drunkard man takes a step forward with probability 0.4 and takes a step backward with probability 0.6. He takes 11 steps all. Find the probability that he is just one step away from the initial point. Do ypu think drinking habit can ruin one's family life ?

OR

In a school, 30% of the student has 100% attendance. Previous year result report tell that 70% of all student having 100% attendance attain A grade and 10% of remaining students attain A grade in their annual examination. At the end of the year, one student is chosen at random and he has an A grade. What is the probability that the student has 100% attendance?

22. Draw the rough sketch of the curves $y=\sin x$ and $y=\cos x$ as x varies from 0 to $\frac{\pi}{2}$ and find the area of the region enclosed by them and X –axis.

OR

Using the method of integration find the area bounded by the curves $|x|+|y|=1$.

23. A manufacture produces two types of steel trunks. He has two machines A and B. The first type of the trunk requires 3 hours on machine A and 3 hours on machine B. The second type of the trunk requires 3 hours on machine A and 2 hours on machine B. Machines

A and B are run daily for 18 hours and 15 hours respectively. There is a profit of ₹30 on the first type of the trunk and ₹25 on the second type of the trunk. How many trunks of each type should be produced and sold to make maximum profit.

24. Manufacturer can sell 'x' items at a price of rupees $\left(5-\frac{x}{100}\right)$ each. The cost price of 'x' item in Rs $\left(\frac{x}{5}+500\right)$. Find the number of items he should sell to earn maximum profit.

OR

Show that the volume of the greatest cylinder, which can be inscribed in a cone of height 'h' and semi-vertical angle θ is $\frac{4}{27}\pi h^3\tan^2\theta$.

25. Solve $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$, x > 0.

26. Prove that the curves $y=6+x-x^2$ and $y(x-1)=x+2$ touch each other at $(2,4)$ also find the equation of common tangent.

OR

(i) Show that $f(x)=2x+\cot^{-1}x+\log\left(\sqrt{1+x^2}-x\right)$ is increasing in R.

(ii) If total revenue received from the sale of x units of a product is given by $R(x)=13x^2+26x+17$. Find the marginal revenue when x = 9.