SAMPLE Paper 1

Time Allowed : 3 hrs.] [*Max. Marks* : 90

Questions 1-4 (1 Mark), 5-10 (2 Marks), 11-20 (3 Marks), 21-31 (4 Marks)

Section A

1. Simplify the following equation and identify whether this equation is quadratic or not: $(x - 2)(x + 2) = 12$.
2. Write the AP with given first term a and common difference d for the following: $a = 17$, $d = -6$.
3. Find the distance between the points A(8, – 2) and B(3, – 6).
4. Find the area of a sector of a circle when the radius of the circle is 21 cm and angle of the sector is 60°. (Take $\pi = \frac{22}{7}$)

Section B

5. Find the roots of the following quadratic equation by using the method of factorization: $6x^2 - 13x - 5 = 0$.
6. How many terms of the AP 9, 17, 25, ... must be taken to give a sum of 636?
7. The length of a tangent from a point A, at a distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.
8. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out
   (*i*) an orange flavoured candy? (*ii*) a lemon flavoured candy?
9. A paper is in form of a rectangle ABCD in which AB = 20 cm and BC = 14 cm. A semi-circular portion with BC as diameter is cut off. Find the area of the remaining part. (Take $\pi = \frac{22}{7}$)
10. Two cubes each of side 10 cm are joined end to end. Find the surface area of the resulting rectangular shaped solid.

Section C

11. Solve for x: $\frac{1}{x-2}+\frac{2}{x-1}=\frac{6}{x}$ $(x \neq 0, 1, 2)$.
12. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.
13. Determine the value of p for which the quadratic equation $px^2 - 24x + 16 = 0$ has equal roots.
14. Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.
15. One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting
    (*i*) a king of red colour (*ii*) a face card (*iii*) the queen of diamonds
16. A lot consists of 220 ball pens out of which 22 are defective. Sunita will buy a pen if it is good but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that
    (*i*) she will buy it? (*ii*) she will not buy it?
17. Find the area of the triangle whose vertices are (1, – 1), (– 4, 6) and (3, – 5).
18. Find the value of x for which the distance between the points P(2, – 3) and Q(x, 5) is 10 units.

19. From a circular sheet of 10 cm radius, a disc of radius 3 cm is cut out and the portion of the sheet left behin is painted at the rate of ₹ 0.35 per cm^2. Find the cost of painting the sheet. (Take $\pi = \frac{22}{7}$)

20. The radii of the ends of a frustum of a right circular cone are 5 cm and 8 cm and its lateral heigh (slant height) is 5 cm. Find the volume of the frustum. (Take $\pi = \frac{22}{7}$)

Section D

21. The perimeter of a rectangular piece of land is 130 m and its area is 1000 m^2. Determine its dimensions.

***22.** A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academi performance. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.
What value is reflected by giving cash prizes to students?

23. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is cyclic quadrilateral.

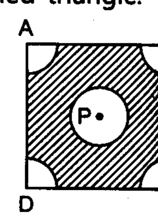
24. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle whic touches the smaller circle.

25. The tangent at any point of a circle is perpendicular to the radius through the point of contact. Prove it.

26. A ladder is placed against a wall of a house such that its upper end is touching the top of the wall. The foo of the ladder is 8 m away from the foot of the wall and the ladder is making an angle of 30° with the leve of the ground. Determine the height of the wall. (Take $\sqrt{3}$ = 1.732)

27. A vertical tower stands on the plane ground and is surmounted by a flagstaff of height 5 m. From a poin on the ground, the angle of elevation of the bottom of the flagstaff is 45° and that of the top of the flagsta is 60°. Find the height of the tower. (Take $\sqrt{3}$ = 1.732)

28. Prove that the points A(– 3, 0), B(1, – 3) and C(4, 1) are the vertices of an isosceles right-angled triangle.



29. From each corner of a square of side 4 cm, a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in fig. Find the area of the remaining portion of the square. (Take $\pi = \frac{22}{7}$)

30. A solid is in the form of a right circular cylinder mounted on a solid hemisphere of radius 14 cm. The radius of the base of the cylindrical part is 14 cm and the vertical height of the complete soli is 28 cm. Find the volume of the solid. (Take $\pi = \frac{22}{7}$)

31. A heap of rice is in the form of a cone of diameter 9 m and height 3.5 m. Find the volume of the rice. Ho much canvas cloth is required to just cover the heap? (Take $\pi = \frac{22}{7}$)

Solutions

1. $(x - 2)(x + 2) = 12 \Rightarrow x^2 - 4 = 12 \Rightarrow x^2 - 16 = 0$

$\therefore$ Given equation is a quadratic equation.

2.
$$a = 17, d = -6$$
$$a_1 = a = 17, a_2 = a + d = 17 + (-6) = 11$$
$$a_3 = a + 2d = 17 + (2 \times -6) = 5$$

$\therefore$ AP is 17, 11, 5, ...

*Value Based Question

3. $$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$\Rightarrow$ $$AB = \sqrt{(3-8)^2 + [-6-(-2)]^2} = \sqrt{(-5)^2 + (-4)^2} = \sqrt{41}$$

4. Here, radius (r) = 21 cm and angle (θ) = 60°

$\therefore$ $$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2 = \frac{60°}{360°} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2$$

$$= \frac{1}{6} \times 22 \times 3 \times 21 = 231 \text{ cm}^2$$

5. Given; $6x^2 - 13x - 5 = 0$

$\Rightarrow$ $6x^2 - 15x + 2x - 5 = 0 \Rightarrow 3x(2x - 5) + 1(2x - 5) = 0$

$\Rightarrow$ $(2x - 5)(3x + 1) = 0$

$\Rightarrow$ Either $2x - 5 = 0$ or $3x + 1 = 0$

$\Rightarrow$ $x = \frac{5}{2}$ or $x = \frac{-1}{3}$

6. Here, $a = 9$, $d = 17 - 9 = 8$, $S_n = 636$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow$ $636 = \frac{n}{2}(18 + 8n - 8)$

$\Rightarrow$ $1272 = n(10 + 8n) \Rightarrow 8n^2 + 10n - 1272 = 0$

$\Rightarrow$ $4n^2 + 5n - 636 = 0$

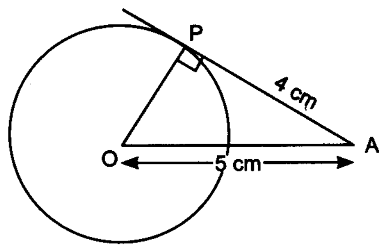
$\Rightarrow$ $4n^2 + 53n - 48n - 636 = 0$

$\Rightarrow$ $n(4n + 53) - 12(4n + 53) = 0$

$\Rightarrow$ $(n - 12)(4n + 53) = 0 \Rightarrow$ Either $n = 12$ or $n = \frac{-53}{4}$ (rejected)

$\therefore$ $n = 12$

7. Here, OA = 5, AP = 4, OP = r (say)



Now in ΔOAP, $OA^2 = AP^2 + OP^2$

$\Rightarrow$ $5^2 = 4^2 + r^2 \Rightarrow r^2 = 9$

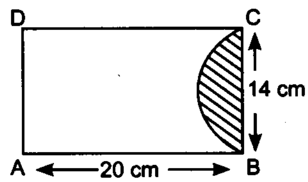
$\Rightarrow$ $r = 3$

$\therefore$ radius = 3 cm

8. (*a*) The probability of orange flavoured candy = 0 since the bag contains only lemon flavoured candies.

(*b*) Required probability = 1 since the bag contains all lemon flavoured candies.

9. Diameter of semicircle = 14 cm



$\therefore$ Radius = $\frac{14}{2}$ = 7 cm

$\therefore$ Area of semicircle = $\frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77 \text{ cm}^2$

Area of paper = $l \cdot b = 20 \times 14 = 280 \text{ cm}^2$

$\therefore$ Area of remaining part = $280 - 77 = 203 \text{ cm}^2$

10. Length of resulting cuboid (l) = 10 + 10 = 20 cm

breadth (b) = 10 cm

height (h) = 10 cm

$\therefore$ Surface area $= 2(l \cdot b + b \cdot h + l \cdot h)$

$= 2(20 \times 10 + 10 \times 10 + 20 \times 10)$

$= 1000 \text{ cm}^2$

11. $$\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$$

$\Rightarrow$ $$\frac{(x-1)+2(x-2)}{(x-2)(x-1)} = \frac{6}{x}$$

$\Rightarrow$ $x(3x - 5) = 6(x^2 - 3x + 2)$

$\Rightarrow$ $3x^2 - 5x = 6x^2 - 18x + 12$

$\Rightarrow$ $3x^2 - 13x + 12 = 0$

$\Rightarrow$ $3x^2 - 9x - 4x + 12 = 0$

$\Rightarrow$ $3x(x - 3) - 4(x - 3) = 0$

$\Rightarrow$ $(3x - 4)(x - 3) = 0$

$\Rightarrow$ Either $x = \frac{4}{3}$ or $x = 3$.

12. Let Ist term of the AP = a and common difference = d

A.T.Q. $a_4 + a_8 = 24$

$\Rightarrow$ $a + 3d + a + 7d = 24$

$\Rightarrow$ $2a + 10d = 24$

$\Rightarrow$ $a + 5d = 12$

Also $a_6 + a_{10} = 44$

$\Rightarrow$ $a + 5d + a + 9d = 44$

$\Rightarrow$ $2a + 14d = 44$

$\Rightarrow$ $a + 7d = 22$

Subtracting (*ii*) from (*i*), we get

$$\begin{array}{r} a + 5d = 12 \\ a + 7d = 22 \\ - \quad - \quad\quad - \\ \hline -2d = -10 \end{array} \Rightarrow d = 5$$

Putting $d = 5$ in eq. (*i*), we get

$a + 5 \times 5 = 12 \Rightarrow a = -13$

$\therefore a_1 = -13$, $a_{12} = a + d = -13 + 5 = -8$

$a_3 = a + 2d = -13 + 2 \times 5 = -3$

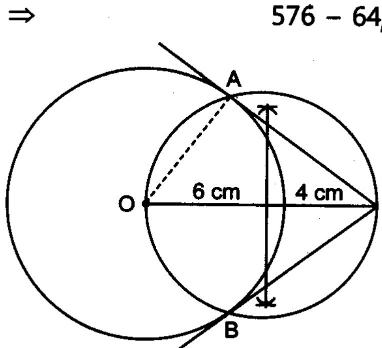
13. For equal roots, $D = 0$

$\Rightarrow$ $b^2 - 4ac = 0$

$\Rightarrow$ $(-24)^2 - 4(p)(16) = 0$

$\Rightarrow$ $576 - 64p = 0 \Rightarrow p = \frac{576}{64} = 9$

14.



15. No. of ways to draw a card = 52.

(*i*) A = card is a king of red colour

$P(A) = \frac{2}{52} = \frac{1}{26}$

(*ii*) B = card is a face card.

$P(B) = \frac{12}{52} = \frac{3}{13}$

(*iii*) C = card is a queen of diamond

$P(C) = \frac{1}{52}$.

16. No. of ball pens = 220

No. of defective ball pens = 22

No. of good ball pens = 220 − 22 = 198

(*i*) Probability that Sunita will buy the ball pen = Probability that ball pen in good

$= \frac{198}{220} = \frac{99}{110} = \frac{9}{10}$

(*ii*) Probability that Sunita will not buy the ball pen = Probability that ball pen is defective

$= \frac{22}{220} = \frac{1}{10}$

17.

$$\text{Area of } \Delta = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2}|1[6-(-5)] + (-4)[-5-(-1)] + 3(-1-6)|$$

$$= \frac{1}{2}|11 + 16 - 21| = \frac{1}{2} \times 6 = 3 \text{ sq units.}$$

18.

$$PQ = 10$$

$\Rightarrow$ $\sqrt{(x-2)^2 + [5-(-3)]^2} = 10$

$\Rightarrow$ $(x-2)^2 + 64 = 100$

$\Rightarrow$ $(x-2)^2 = 36 \Rightarrow x - 2 = \pm 6$

$\Rightarrow$ Either $x - 2 = 6$ or $x - 2 = -6$

$\Rightarrow$ $x = 8$ or $x = -4$

19. Area of disc of radius 3 cm = $\pi r^2 = \pi \times 3^2 = 9\pi$ cm^2

Area of the circular sheet = $\pi \times 10^2 = 100\pi$ cm^2

$\therefore$ Area of the sheet left behind = $100\pi - 9\pi = 91\pi$ cm^2

Rate to paint the sheet = ₹ 0.35 per cm^2

$\therefore$ Total cost = $91\pi \times 0.35$

= ₹ $91 \times \frac{22}{7} \times 0.35$ = ₹ 100.10

20. $r_1 = 8$ cm, $r_2 = 5$ cm, $l = 5$ cm

Now, $l^2 = h^2 + (r_2 - r_1)^2$

$\Rightarrow$ $25 = h^2 + (8-5)^2 \Rightarrow h^2 = 16$

$\Rightarrow$ $h = 4$ cm

$$\text{Volume of frustum} = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 4(8^2 + 5^2 + 8 \times 5) \text{ cm}^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times 4(64 + 25 + 40)$$

$$\frac{88}{21} \times 129 = \frac{88 \times 43}{7} \text{ cm}^3 = \frac{3784}{7} \text{ cm}^3$$

21. Let length = x m and breadth = y m

Now, perimeter = 130 m

$\Rightarrow$ $2(x + y) = 130$ $\Rightarrow$ $x + y = 65$

$\Rightarrow$ $y = (65 - x)$ m ...(

Also Area = 1000 cm^2 $\Rightarrow$ $xy = 1000$

$\Rightarrow$ $x(65 - x) = 1000$ [using eq (

$\Rightarrow$ $65x - x^2 = 1000$

$\Rightarrow$ $x^2 - 65x + 1000 = 0$

$\Rightarrow$ $(x - 25)(x - 40) = 0$

$\Rightarrow$ Either $x = 25$ or $x = 40$

When $x = 25$, $y = 65 - 25 = 40$ m

When $x = 40$, then $y = 65 - 40 = 25$ m

$\therefore$ Dimensions of the field are 40 m and 25 m.

22. Let Ist prize = ₹ a

IInd prize = ₹ $a - 20$

IIIrd prize = ₹ $a - (2 \times 20)$ = ₹ $a - 40$

The prizes are in AP with common difference = –20

No. of prizes = 7

Sum = 700

$\Rightarrow$ $700 = \frac{7}{2}[2 \times a + (7 - 1) \times (-20)]$

$\Rightarrow$ $700 = \frac{7}{2}(2a - 120)$

$\Rightarrow$ $\frac{700 \times 2}{7} = 2a - 120$

$\Rightarrow$ $2a = 200 + 120 = 320$

$a = 160$

$\therefore$ Value of Ist prize = ₹ 160

Value of IInd prize = ₹ 140

Value of IIIrd prize = ₹ 120

Value of IVth prize = ₹ 100

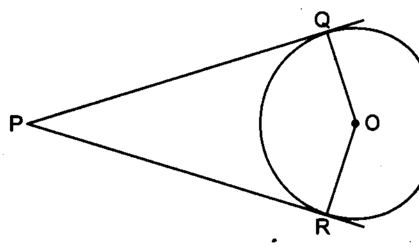
Value of Vth prize = ₹ 80

Value of VIth prize = ₹ 60

Value of VIIth prize = ₹ 40

Value reflected : Encouragement, motivation

23. PQ is a tangent to the circle and OQ is radius.



$\therefore$ OQ $\perp$ PQ

(radius is perpendicular to the tangent at the point of contact)

$\therefore$ $\angle OQP = 90°$

Similarly, $\angle ORP = 90°$

In quadrilateral QORP,

$$\angle RPQ + \angle OQP + \angle ORP + \angle QOR = 360°$$

$\Rightarrow$ $\angle RPQ + 90° + 90° + \angle QOR = 360°$

$\Rightarrow$ $\angle RPQ + \angle QOR = 180°$

$\Rightarrow$ In quadrilateral QORP, opposite angles are supplementary.

$\therefore$ QORP is a cyclic quadrilateral.

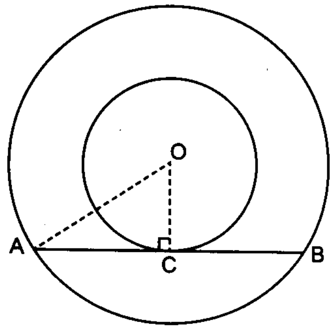
24. AB is tangent to smaller circle.

$\therefore$ OC $\perp$ AB.

Now, AB is a chord of bigger circle and OC $\perp$ AB.

$\therefore$ C is the midpoint of AB.

(perpendicular from the centre of the circle to the chord bisects the chord)



In right ΔOCA, $OA^2 = AC^2 + OC^2$

$\Rightarrow$ $5^2 = AC^2 + 3^2 \Rightarrow AC^2 = 16 \Rightarrow AC = 4$ cm

$\therefore$ AB = 2AC $\Rightarrow$ AB = 8 cm

25. Given: A circle with centre O, line l is tangent to the circle at A.

To Prove: Radius OA is perpendicular to the tangent at A.

Construct: Take a point P, other than A on tangent l. Join OP, meeting the circle at R.

Proof: We know that tangent to the circle touches, the circle at one point and all other points on the tangent lie in the exterior of a circle.

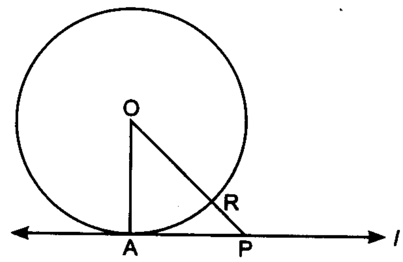
$\therefore$ OP > OR (radius of circle)

$\Rightarrow$ OP > OA ($\because$ OR = OA, radius of circle)

$\Rightarrow$ OA < OP

$\Rightarrow$ OA is the smallest segment, from O to a point on the tangent.

We know that smallest line segment from the centre of the circle to the line is perpendicular segment.



Hence, OA $\perp$ tangent l.

$\Rightarrow$ tangent at any point of a circle is perpendicular to the radius through the point of contact.

26. Let AB is a wall and AC is the ladder.

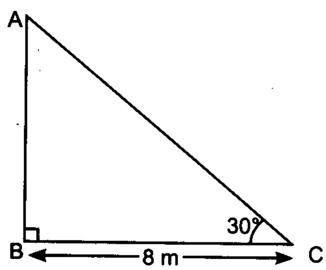
A.T.Q. BC = 8 m

and $\angle ACB = 30°$

In right ΔACB, $\frac{AB}{BC} = \tan 30°$

$\Rightarrow$ $\frac{AB}{8} = \frac{1}{\sqrt{3}} \Rightarrow AB = \frac{8}{\sqrt{3}}$ m

$\Rightarrow$ $AB = \frac{8\sqrt{3}}{3}$ m $= \frac{8 \times 1.732}{3}$ m $= 4.618$ m



27. Let AB is a tower and BC is the flagstaff

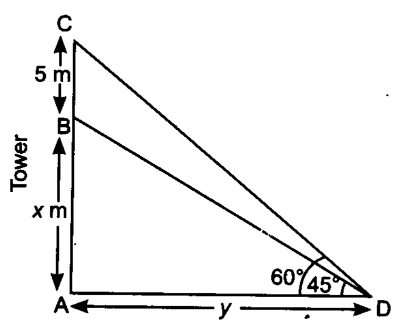
A.T.Q. BC = 5 m, $\angle ADB = 45°$, $\angle ADC = 60°$

Let AB = x m and AD = y m

In right ΔBAD, $\frac{AB}{AD} = \tan 45°$

$\Rightarrow$ $\frac{x}{y} = 1 \Rightarrow x = y$ ...(i)

In right ΔCAD, $\frac{AC}{AD} = \tan 60°$



$\Rightarrow \quad \frac{x+5}{y} = \sqrt{3} \quad \Rightarrow \quad \frac{x+5}{x} = \sqrt{3}$ [using (*i*)]

$\Rightarrow \quad x + 5 = \sqrt{3}x \quad \Rightarrow \quad 5 = x(\sqrt{3}-1)$

$\Rightarrow \quad x = \frac{5}{\sqrt{3}-1} = \frac{5(\sqrt{3}+1)}{2}$

$\Rightarrow \quad x = \frac{5(1.732+1)}{2} = \frac{5 \times 2.732}{2} = 6.83$ m

$\therefore$ Height of the tower = 6.83 m.

28.

$$AB = \sqrt{(1+3)^2 + (-3-0)^2} = \sqrt{25} = 5$$
$$BC = \sqrt{(4-1)^2 + (1+3)^2} = \sqrt{25} = 5$$
$$AC = \sqrt{(4+3)^2 + (1-0)^2} = \sqrt{50} = 5\sqrt{2}$$

Now $AC^2 = (5\sqrt{2})^2 = 50$

Also, $AB^2 + BC^2 = 5^2 + 5^2 = 50$

$\Rightarrow \quad AC^2 = AB^2 + BC^2$

$\therefore$ AB, AC and BC are sides of right Δ.

Also $AB = BC$

$\therefore$ ΔABC is an isosceles right-angled triangle.

29. Area of one quadrant $= \frac{1}{4}\pi r^2 = \frac{1}{4}\pi \times 1^2 = \frac{1}{4}\pi \text{ cm}^2$

$\therefore$ Area of 4 quadrants $= 4 \times \frac{1}{4}\pi = \pi \text{ cm}^2$

Area of circle of diameter 2 cm $= \pi r^2 = \pi \times 1^2 = \pi \text{ cm}^2$

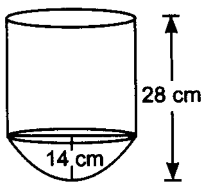
Also, the area of the square of side 4 cm $= 4 \times 4 = 16 \text{ cm}^2$

$\therefore$ Area of remaining portion of the square $= 16 - (\pi + \pi)$

$= 16 - 2\pi = 16 - 2 \times \frac{22}{7}$

$= \frac{112-44}{7} = \frac{68}{7} \text{ cm}^2$

30. Radius of the hemispherical portion = 14 cm



$\therefore$ Volume of hemispherical part $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi \times (14)^3 \text{ cm}^2$

Radius of cylindrical portion = 14 cm

Height of cylindrical portion = 28 − 14 = 14 cm

$\therefore$ Volume of cylindrical portion $= \pi r^2 h$

$= \pi \times (14)^2 \times 14 = \pi \times (14)^3 \text{ cm}^3$

Volume of the solid $= \left[\frac{2}{3}\pi \times (14)^3 + \pi \times (14)^3\right] \text{ cm}^3$

$= \pi \times (14)^3\left(\frac{2}{3}+1\right) = \frac{22}{7} \times (14)^3 \times \frac{5}{3}$

$= \frac{43120}{3} \text{ cm}^3$

31.

$$\text{Diameter} = 9 \text{ m}$$

$$\Rightarrow \quad \text{radius} = \frac{9}{2} \text{ m.}$$

$$\text{Height} = 3.5 \text{ m}$$

$$\therefore \quad \text{Volume} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{9}{2} \times \frac{9}{2} \times 3.5$$

$$= 74.25 \text{ m}^3$$

$$l = \sqrt{r^2 + h^2} = \sqrt{\frac{81}{4} + \frac{40}{4}} = \sqrt{\frac{130}{2}}$$

$$\text{Curved area of canvas (cone)} = \pi r l = \frac{22}{7} \times \frac{9}{2} \times \frac{\sqrt{130}}{2}$$

$$= \frac{22}{7} \times \frac{9}{4} \times 11.40 = 80.61 \text{ m}^2$$