

2015
MATHEMATICS (SA-II)
CLASS-X

Full Marks : 90

Time: 3 hours

General Instructions.

- i) All questions are compulsory.
 - ii) The question paper consists of 31 questions which divided into four parts ; A, B ,C and D.
 - iii) Use of CALCULATORS is not permitted.
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Section – A

[Each question carries 1 mark]

1. A number 'x' is chosen at random from -3, 1, 2, 3. Find the probability that $|x| \leq 2$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) 1

2. Suppose α and β be the roots of $x^2 - 5x + 7 = 0$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is

(a) $\frac{5}{7}$ (b) $\frac{7}{5}$ (c) $-\frac{5}{7}$ (d) $\frac{1}{7}$

3. The area of a sector of a circle with radius 7cm , sectorial angle 60° is

(a) $\frac{77}{3} \text{ cm}^2$ (b) $\frac{132}{7} \text{ cm}^2$ (c) $\frac{115}{7} \text{ cm}^2$ (d) 54 cm^2

4. There are 'n' arithmetic means between a and b . The common difference 'd' is

(a) $\frac{n+1}{b+a}$ (b) $\frac{n+1}{b-a}$ (c) $\frac{b-a}{n+1}$ (d) $\frac{b+a}{n+1}$

Section – B

[Each question carries 2 marks]

- 5. The 17th term of an AP exceeds its 10th term by 7. Find its common difference.
- 6. Find the roots of $3x^2 - 2\sqrt{6}x + 2 = 0$
- 7. If Q(0, 1) is equidistant from P(5, -3) and R(x, 6). Find the distance between P and R.
- 8. A wheel makes 5000 revolutions in moving 11 km. Find the diameter of the wheel.

9. Prove that the tangents drawn at the end of chord of a circle make equal angles with the chord.
10. Sabita and Pallab are friends. What is the probability that both will have
- different birth days?
 - the same birthdays? [*ignoring a leap year*]

Section – C

[Each question carries 3 marks]

11. The p^{th} term of an AP is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$. Prove that sum of its pq^{th} term is $\frac{1}{2}(pq + 1)$
12. If the roots of the equation $(c^2 - ab)x^2 + 2(bc - a^2)x + (b^2 - ac) = 0$ are equal then prove that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$.
13. A horse is placed for grazing inside a rectangular field 70m by 52m and is tethered to one corner by a rope 21m long. How much area can it graze?
14. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of entire capsule is 14mm and the diameter of the capsule is 5mm. Find its surface area.
15. Red kings and black aces are removed from a pack of 52 cards. The remaining cards are well shuffled and then a card is drawn from it. Find the probability that the drawn card is - i) a black face card ii) a red card.
16. Prove that the coordinates of centroid of a triangle ABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are given by $\left[\left(\frac{x_1+x_2+x_3}{3}\right), \left(\frac{y_1+y_2+y_3}{3}\right)\right]$
17. Two concentric circles of radii 5cm and 3cm are drawn. Find the length of the chord of the larger circle which touches the smaller circle.
18. The height of a tower is half the height of the flag-staff at its top. The angle of elevation of the top of the tower as seen from a distance of 10m from its foot is 30° . Find the angle of elevation of the top of the flag-staff from the same point.

19. If A be the area of a right triangle and b one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^2+4A^2}}$
20. An 'ice cream cone' is the union of a right circular cone and a hemi sphere that has the same base as the cone. Find the volume of the ice cream if the height of the cone is 9cm and the radius of its base is 2.5cm.

Section – D

[Each question carries 4 marks]

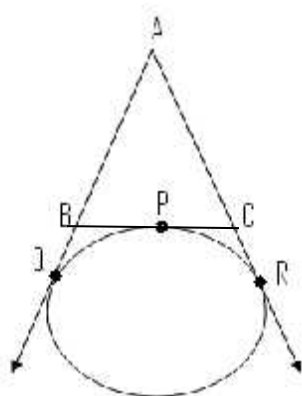
21. The denominator of a fraction is more than twice the denominator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$. Find the fraction.
22. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the next row and so on. In how many rows are 200 logs placed and how many logs are in the top row?
23. Draw a ΔABC in which $BC = 6\text{cm}$, $AB = 4\text{cm}$ and $AC = 5\text{cm}$, draw a triangle similar to ΔABC with its sides equal to $\frac{3}{4}$ of the corresponding sides of ΔABC .
24. If A and B are $(-2, -2)$ and $(2, -4)$ respectively. Find the coordinates of P such that $AP = \frac{3}{7}AB$ and P lies in the line segment AB .
25. The radius of the base of a right circular cone is r . It is cut by a plane parallel to the base at a height h from the base. The distance of the boundary of the upper surface from the centre of the base of the frustum is $\sqrt{h^2 + \frac{r^2}{9}}$. Show that the volume of the frustum is $\frac{13}{27}\pi r^2 h$.
26. In an equilateral triangle of side 24cm, a circle is inscribed touching its sides. Find the area of the remaining portion of the triangle.
27. A round balloon of radius a subtends an angle θ at the eye of the observer while the angle of elevation of its centre is ϕ . Prove that the height of the centre of the balloon is $a \sin \phi \operatorname{cosec} \frac{\theta}{2}$.

28. A bucket has top and bottom diameter of 40cm and 20cm respectively. Find the volume of the bucket if its depth is 12cm. Also find the cost of tin sheet used for making the bucket at the rate of Rs. 0.12 per dm^2 .

29. On tossing three coins simultaneously, find the probability of getting

i) 3 tails ii) 2 tails iii) no tail iv) 2 heads and 1 tail v) at least one head

30. a circle touches the side BC of ΔABC at 'p' and touches AB and AC when produced at 'Q' and 'R' respectively, as shown in figure, show that $AQ = \frac{1}{2} (\text{perimeter of } \Delta ABC)$



31. Prove that the straight line $y - x + 2 = 0$ cuts the straight line joining (3, -1) and (8, 9) in the ratio 2:3.