

Mathematics-1

Total Marks -100 Time- 3 h

Section – A, One marks each, Section – B, four marks Each, Section – C, Six marks each

SECTION-A

1. If $\sin^{-1}x + \cos^{-1}\frac{2}{3} = \frac{\pi}{2}$ then the value of x is=?
2. If $a*b = a+b-ab$ then find the value of $3*4$
3. Find the equation of the plane which makes equal intercepts on the coordinate axis and pass through the point $(-2,3,0)$
4. If A is a square matrix of order 3 such that $|A|= 7$ then find $|3A|$
5. Find the degree and order of the differential equation $(\frac{d^2y}{dx^2})^3 + (\frac{dy}{dx})^4 + 2y=0$
6. Evaluate $\int \frac{d(\log x)}{\log x}$

SECTION-B

7. If $(a + bx)e^{y/x} = x$ then prove that $x^3 \frac{d^2y}{dx^2} = (\frac{dy}{dx} - y)^2$
8. Verify the LMVT for the function $f(x)= x^2+2x+3$ in $[4,6]$
9. Solve $\tan^{-1}x + \tan^{-1}2x = \frac{\pi}{4}$
10. Express the matrix as a sum of a symmetric and a skew-symmetric matrix $A = \begin{bmatrix} 3 & -2 & 4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$
11. if $f(x)= \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, show that $f \circ f(x)=x$. also find the inverse of $f(x)$
12. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ then show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$
13. Discuss the continuity of the function

$$F(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases} \quad \text{at } x=0$$

14. Show that
$$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (b+a)^2 \end{vmatrix} = 2abc(a+b+c)$$

15. Solve $(1+x^2) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$ given that $y=0, x=1$

16. Evaluate $\int \frac{x^2}{(x^2+4)(x^2+9)} dx$

17. Evaluate $\int_0^\pi \frac{x \sin x}{1+\sin x} dx$

18. A ladder 13 m is leaning against a vertical wall the bottom of the ladder is dragged away from the wall along the ground at the speed of 2 cm/sec. how fast is its height is decreasing when the foot of the ladder is 5 m away from the wall. **OR**

Find the equation of the plane through the line of intersection of the planes $2x+y-z=3$ & $5x-3y+4z+9$ and parallel to the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$

19. Prove that $|\vec{a} \times \vec{b}|^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} \end{vmatrix}$ **OR**

Cont P2...

Four defective bulbs mixed with 10 good ones three bulbs taken at random at random, find the probability distribution of the defective bulbs

SECTION-C

20. Using limit as sum evaluate $\int_1^3 (2x^2 + x + 9) dx$ **OR**

Find the area of the region bounded by the curves $x^2+y^2=4$ & $y = |x|$.

21. Two companies decide to award there employee for the three values of resourcefulness, competitance and determination in the form of prizes at the rate of Rs x, y, z respectively per person. The first company decide to awarded respectively 4,3,2 employees with a total prize money Rs 37000. The other company decide to awarded respectively 5,3,4 employees with a total prize money of Rs 47000. If all the three prizes per person together amount of Rs 12000 then using the matrix method find the values of x, y, z .

22. Show that the greatest cone that can be inscribe in a given sphere of radius R is $\frac{8}{27}$ part of the volume of the sphere.

23. Evaluate $\int_0^4 \{|x - 1| + |x - 2| + |x - 4|\} dx$ OR $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx$

24. A manufacturer of patent medicines is preparing a production plan on medicines A & B. there are sufficient raw materials available to make 20,000 bottles of A and 40,000 bottles of B, but there are only 45,000 bottles into which either of the medicines can be put. Further , it takes 3 hours to prepare enough material to fill 1000 bottles of A, it takes 1 hour to prepare enough material to fill 1000 bottles of B and there are 66 hours available for this operation. The profit is Rs 8 per bottles of A and Rs 7 per bottles of B. how should the manufacturer schedule his production in order to maximize his profit.

25. There are three probable candidates A, B, C for the post of a Principal. The chances of their selections are in the proportions of 4:2:3 respectively. The probability that A ,if selected, will introduce co-education in collage is 0.3. the probability of B and C doing the same are respectively 0.5 and 0.8. what is the probability that there will be co-education in the college? Find the probability that principal B introduced co-education in college.

26. A point on the hypotenuse of a right angle triangle is a & b distance apart from the two legs of right angle then show that the minimum length of the hypotenuse is $(a^2+b^2)^{3/2}$

27. $f(x)= 2x^3-9x^2+12x+15$; find the interval where f(x) is increasing & decreasing.

Best of Luck

MD. DEBNAJIB