MATHEMATICS ***CLASS: XII***

MAX MARKS: 100 ***MAX TIME: 3 HRS***

Section: A, one mark each. Section: B, four marks each. Section: C, six marks each

1. Show that the binary operation $*$ defined by a*b = ab + 1 on Q is Commutative.

2. Solve: $\tan^{-1}2x + \tan^{-1}3x = \pi/4$.

3. Find a matrix X such that B–2A +X=O, where $A = \begin{bmatrix} 5 & 3 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -2 \\ 3 & 1 \end{bmatrix}$.

4. Evaluate $\int \frac{dx}{x\cos^2(1+\log x)}$.

5. If $|\vec{a}| = 5$; $|\vec{b}| = 13$ and $|\vec{a} \times \vec{b}| = 25$, find $\vec{a}.\vec{b}$

6. The Cartesian equation of a line AB is $\frac{2x-1}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$. Find the direction cosines of a line parallel to AB.

SECTION-B

7. If $f : R - \left\{\frac{7}{5}\right\} \rightarrow R - \left\{\frac{3}{5}\right\}$ be defined as $f(x) = \frac{3x+4}{5x-7}$ and $g : R - \left\{\frac{3}{5}\right\} \rightarrow R - \left\{\frac{7}{5}\right\}$ be defined as $g(x) = \frac{7x+4}{5x-3}$.Show that $g \circ f = I_A$ and $f \circ g = I_B$ where B=$R - \left\{\frac{3}{5}\right\}$ and A=R-$\left\{\frac{7}{5}\right\}$.

8. Using properties of determinants, prove that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

9. Prove that : $2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{5\sqrt{2}}{7} + 2\tan^{-1}\frac{1}{8} = \frac{\pi}{4}$.

OR

Solve for x : $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$.

10. For what value of k is the following function continuous at x = 2?

$$f(x) = \begin{cases} 2x+1; x < 2 \\ k; x = 2 \\ 3x-1; x > 2 \end{cases}$$

11. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$ find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$.

OR

If y = $[\log(x + \sqrt{1+x^2})]^2$, show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dc} - 2 = 0$.

12. Find the intervals in which the function f(x) = sin x – cos x; $0 \le x \le 2\pi$
(i) is increasing (ii) is decreasing.

13. Evaluate $\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8} dx$.

OR

Prove that $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\cos^4 x + \sin^4 x} dx = \frac{\pi^2}{16}$

14. Solve the following differential equation: $(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x$.

15. Find the particular solution of the differential equation $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$, given that y(1) = 0.

16.**if** $\vec{\alpha} = 3\vec{i} + 4\vec{j} + 5\vec{k}$ **and** $\vec{\beta} = 2\vec{i} + \vec{j} - 4\vec{k}$ **,** then express $\vec{\beta}$ in the form $\vec{\beta} = \overline{\beta_1} + \overline{\beta_2}$, where $\overline{\beta_1}$ is parallel to $\vec{\alpha}$ and $\overline{\beta_2}$ is perpendicular to $\vec{\alpha}$

Or

Find a unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ *and* $\vec{a} - \vec{b}$, where $\vec{a} = 3\vec{i} + 2\vec{j} + 2\vec{k}$ **and** $\vec{b} = 3\vec{i} + 2j\text{-}2\vec{k}$

17. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+1}{2}$.

18. A box contains 12 bulbs of which 3 are defective. If 3 bulbs are drawn from the box at random, find the probability distribution of X, the number of defective bulbs drawn. Hence compute the mean of X.

19. solve the differential equation $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) dy = 0$

SECTION-C

20.Given that A = $\begin{vmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{vmatrix}$ and B = $\begin{vmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{vmatrix}$ find AB. Hence using this product solve the system of equations: x – y + z = 4, x – 2y – 2z = 9, 2x + y + 3z = 1

OR

Using elementary row transformation, find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$.

21. Show that the rectangle of maximum area that can be inscribed in a circle of radius r is a square of side $\sqrt{2}\, r$ *units* .

OR

Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere.

22. Evaluate: $\int \sqrt{\tan x}\, dx$.

23. Using the method of integration, find the area of the region bounded by the lines 2x + y = 4, 3x – 2y = 6 and x – 3y + 5 = 0.

24. Find the image of the point (1, 2, 3) in the plane x + 2y + 4z = 38. Also find the perpendicular distance from the point to the plane.

25. a company sells two different products A and B . the two products are produced in a common production process which has a total capacity of 500 man hours . it takes 5 hours to produce a unit of A and 3 hours to produce a unit of B , the demand in the market shows that the maximum number of units of A that can sold is 70 and that of B is 125. Profit on each unit of A

is 20 and that on B is 15. How many units of A and B should be produced to maximum profit ? Solve graphically?

26.A card from a pack of 52 cards is lost. From the remaining cards of the pack, 2 cards are drawn at random and are found to both clubs. Find the probability of the lost card being of club.

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