

MOCK BOARD EXAM - 2014

Roll No.
रोल नं.

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AIMS Test Series

Code No.
कोड नं.

AIMS-IV-2014

- Please check that this question paper contains 04 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 29 questions.
- Please write down the serial number of the question before attempting it.
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 04 हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 29 प्रश्न हैं।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें।

MATHEMATICS

गणित

Time allowed : 3 hours
निर्धारित समय : 3 घण्टे

Maximum Marks : 100
अधिकतम अंक : 100

General Instructions: -

- All the questions are compulsory.
- The question paper consists of 29 questions divided into three sections A, B and C. Section A contains 10 questions of 1 mark each, Section B contains 12 questions of 4 marks each and Section C contains 7 questions of 6 marks each.
- There is no overall choice. However an internal choice in any 4 questions of four marks each and any two questions of six marks each has been provided.
- Use of Calculator is not permitted. You may ask for logarithmic tables, if required.

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Section – A

(Questions number 1 to 10 carry 1 marks each)

1. Find the principal value of $\sin^{-1}\left[\left(\sin \frac{3\pi}{5}\right)\right]$.
2. Solve $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$ ($-1 \leq x \leq 1$).
3. Find the order and degree of the differential equation. $\sqrt[5]{\cos\left(1 + \frac{d^2y}{dx^2}\right)} = x$.
4. Write the value of $\begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix}$ which is independent from θ .
5. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B' = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A+2B)'$.
6. Differentiate w.r.t. 'x', $\log_{\log x} \log x$.
7. Find a vector which is perpendicular to both the vectors $2\hat{i} - 3\hat{j} + 5\hat{k}$ and $3\hat{i} + 2\hat{j} - \hat{k}$ and of magnitude one.
8. $A = \begin{bmatrix} 1 & 2 \\ k & 3 \end{bmatrix}$, then write $|2A|$ and find the value of k, if A is singular matrix.
9. Find a unit vector in the direction of vector $\hat{i} + \hat{j} + \hat{k}$ and of magnitude 5 units.
10. Find the angle between the plane $\vec{r} \cdot (3\hat{i} - \hat{j} + 2\hat{k}) = -11$ and line $\vec{r} = (3\hat{i} - 6\hat{j} + \hat{k}) + \lambda(3\hat{i} + 5\hat{j} + 2\hat{k})$.

Section-B

(Questions number 11 to 22 carry 4 marks each.)

11. Show that function $f: \mathbf{R} \rightarrow \{x \in \mathbf{R}: -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in \mathbf{R}$ is one-one and onto function.

OR

Let $A = \mathbf{N} \times \mathbf{N}$ and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$
Show that $*$ is commutative and associative. Find the identity element for $*$ on A, if any.

12. By using the properties of determinants prove that :

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

13. Show that:

$$\frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3} = \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3}$$

OR

Find the value of

$$\tan \frac{1}{2} \left[\sin^{-1} \frac{2x}{1+x^2} + \cos^{-1} \frac{1-y^2}{1+y^2} \right], \quad |x| < 1, y > 0 \text{ and } xy < 1$$

14. Find the differential coefficient of $y = \cot^{-1} \left(\frac{x^x - x^{-x}}{2} \right)$.

15. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$.

16. If $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$ then find the derivative of $f(x)$ at $x=0$.

17. Evaluate $\int \frac{1}{\sqrt{\sin^3 x \cdot \sin(x+\alpha)}} dx$; Write the important trait of integrity in one's character.

18. Evaluate $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx$

19. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of $2\hat{i} + 4\hat{j} - 5\hat{k}$ vectors and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ .

OR

A girl walks 5 km towards west, and then she walks 3 km in a direction 60° east of north and stops. Determine the girl's displacement from her initial point of departure.

20. Find the length of the perpendicular drawn from the point $(5, -3, 2)$ to the line

$$\frac{-x+1}{-2} = \frac{3-y}{-3} = \frac{-2z-4}{2}.$$

21. In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices and only one correct option. The probability that he guess is $\frac{1}{3}$. The probability that he copies the answer is $\frac{1}{6}$. The probability that the answer is correct, given that he copied, is $\frac{1}{8}$. Find the probability that he knows the answer to the question, given that he correctly answered it.

OR

Given that the two numbers appearing on throwing the two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

22. Evaluate $\int \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$ “Differentiate the differences and integrate the integration among the youth and community” justify your comment on it.

Section-C

(Question number 23 to 29 carry 6 marks each)

23. Using elementary transformations, find the inverse of

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}.$$

24. Find the area of the greatest isosceles triangle that can be inscribed in a given ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex coinciding with one extremity of the major axis.

OR

Find the points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes.

25. Using definite integration, find the area of the region: $\{(x, y) : |x-1| \leq y \leq \sqrt{5-x^2}\}$.

26. Find a particular solution of the differential equation

$$(x+1) \frac{dy}{dx} = 2e^{-y} - 1, \text{ given that } y = 0 \text{ when } x = 0$$

OR

Solve the differential equation $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$

27. A bag contains 4 balls. Two balls are drawn at random, and are found to be blue. What is the probability that all the balls are blue?

Why Meditation and Yoga is necessary and sufficient thing for body, mind and soul.

28. Find the vector equation of the line passing through $(1, 2, 3)$ and parallel to the plane $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$.

29. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximize the profit? *How one should respect the hard earn money of parents/guardians in a best economical way.*

___All the Best for Exam On 20 March 2014___