Series : PTS/11

Code No. 14/11/11

Roll No. | 2 | 1 | 4 | 1 | 2 | | |

Candidates must write the Code on the title page of the answer-book.

PLEASURE TEST SERIES XII – 11

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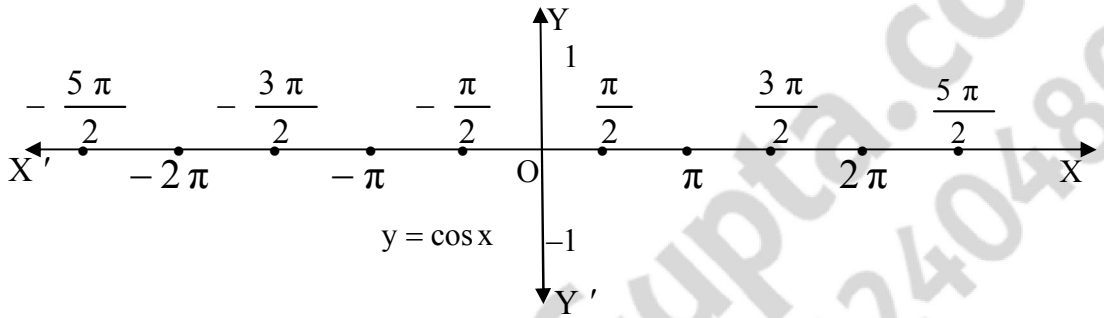
Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION – A

Q01. Draw the graph of $y = \cos x$ and, identify the intervals of x in which the function can be inverted?



Q02. If * is binary operation defined as a*b = GCF of a and b then, write the value of 13*19.

Q03. If $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ and $A^2 = \begin{bmatrix} -5 & -18 \\ 18 & 7 \end{bmatrix}$, find $A^2 - 6A + 17\,I$.

Q04. Write the value of λ, if the vectors $\vec{a} = \hat{i} + \lambda\hat{j} + 3\hat{k}$ and $\vec{b} = 4\hat{i} - 5\hat{j} + 2\hat{k}$ are orthogonal vectors.

Q05. Check if (3,–5, 1), (–1, 0, 8) and (7,–10,–6) are collinear points or not.

Q06. Write the value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x\cos^3 x}{\log|x|}dx$. Mention the property used to evaluate this integral.

SECTION – B

Q07. Evaluate : $\int_0^{\pi} \frac{dx}{3 + 2\sin x + \cos x}$. **OR** Evaluate : $\int \frac{\cos^2 x\,dx}{1 + \tan x}$.

Q08. A kite is moving horizontally at a height of 151.5 meters. If the speed of kite is 10 m/s, how fast is the string being let out; when the kite is 250 m away from the boy who is flying the kite? The height of boy is 1.5 m.

Q09. Form the differential equation satisfied by the equation $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x - y)$, where 'a' is any arbitrary constant. **OR** Solve : $(1 + y + x^2y)dx + (x + x^3)dy = 0$, where $y = 0$ when $x = 1$.

Q10. Find $\frac{dy}{dx}$, if $y = \sqrt{a + \sqrt{a + \sqrt{a + x^2}}}$, where 'a' is a constant.

Q11. Let R be a relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v) if and only if xv = yu. Show that R is an equivalence relation.

OR Let A = Q × Q, Q being the set of rationals. Let * be a binary operation on A, defined by (a, b) *(c, d) = (ac, ad + b). Show that

(i) * is not commutative (ii) * is associative (iii) The identity element w.r.t. * is (1, 0).

Q12. A drunkard man takes a step forward with probability 0.6 and takes a step backward with probability 0.4. He takes 9 steps in all. Find the probability that he is just one step away from the initial point.

Q13. Solve : $(1 + \tan y)(dx - dy) + 2xdy = 0$.

Q14. Discuss the continuity and differentiability of $|3x + 4|$ at $x = -4/3$.

Q15. Find the intervals in which the value of the determinant of matrix $\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$ lies.

Q16. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, $\vec{a} \neq \vec{0}$ then show that $\vec{b} = \vec{c}$.

OR If $\vec{p}$ and $\vec{q}$ are the diagonals of a parallelogram with sides $\vec{a}$ and $\vec{b}$, find the area of parallelogram in terms of its diagonals. Hence find the area in terms of its sides.

Q17. Determine the distance of the point (2, 4,–1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{6-z}{9}$.

Q18. Solve the equation : $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = -\pi/2$.

Q19. Discuss the applicability of Lagrange's Mean Value Theorem for $f(x) = |\sin x|$ on $x \in [-\pi/2, \pi/2]$.

SECTION – C

Q20. Prove that the curves $y = x^2$ and $x = y^2$ divide the square bounded by $x = 0$, $y = 0$, $x = 1$ and $y = 1$ into three parts that are equal in area. Also find the area of each equal part.

Q21. Consider an isosceles triangle ABC inscribed in a circle of radius r. If the vertical angle of ΔABC is 2θ, show that the maximum area of triangle is obtained when $\pi = 6\theta$.

Q22. A village has 500 hectares of land to grow two types of plants X and Y. The contribution of total amount of oxygen produced by plant X and Y are 60% and 40% per hectare respectively. To control weeds, a liquid herbicide has to be used for the plants X and Y at the rate of 20 litres and 10 litres per hectare, respectively. Further no more than 8000 litres of herbicides should be used in order to protect aquatic animals in a pond which collects drainage from this land. How much land should allocated to each crop so as to maximize the total production of oxygen?

Q23. A factory manufactures screws. Machines X, Y and Z manufacture respectively 1000, 2000 and 3000 of the screws. 1%, 1.5% and 2% of their outputs are respectively defective. A screw is selected at random from the product and is found to be defective. What is the probability that it is manufactured by the machine X?

Q24. Evaluate : $\int_{-1}^{1/2} \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}\,dx$. **OR** Evaluate : $\int_0^{2\pi} \log(1+\sin x)\,dx$.

Q25. Two farmers Ramkrishna and Hari Prasad cultivated three varieties of rice namely Basmati, Permal and Naura . The sale (in Rupees) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices 'A' and 'B' :

September Sales (in Rupees)

$$A = \begin{matrix} \text{Basmati} & \text{Permal} & \text{Naura} \\ \end{matrix}$$
$$\begin{pmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \text{Ramkrishna} \\ \text{Hari Prasad} \end{matrix}$$

October Sales (in Rupees)

$$B = \begin{matrix} \text{Basmati} & \text{Permal} & \text{Naura} \\ \end{matrix}$$
$$\begin{pmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{pmatrix} \begin{matrix} \text{Ramkrishna} \\ \text{Hari Prasad} \end{matrix}$$

(i) Find the combined sale in September and October for each farmer in each variety.

(ii) Find the decrease in sales from September to October.

(iii) If both farmers receive 2% profit on gross sales, compute the profit for each farmer and for each variety sold in October.

(iv) Which farmer gets more profit in the overall sales for both the months?

(v) Which farmer in your opinion is more resourceful and why?

Q26. The plane $ax + by = 0$ is rotated about its line of intersection with the plane $z = 0$ through an angle of α. Prove that the equation of the plane in its new position is $ax + by \pm [\sqrt{a^2 + b^2}\tan\alpha]z = 0$.

OR Find the equation(s) of line(s) through the origin intersecting the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at an angle of $\pi/3$.

Q01. $[-\pi,0],[0,\pi],[\pi,2\pi]$ etc. **Q02.** 1 **Q03.** O

Q04. 2 **Q05.** Points are collinear

Q06. 0, Property used : $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, \text{ if } f(x) \text{ is even function i.e., } f(-x) = f(x) \\ 0, \text{if } f(x) \text{ is even function i.e., } f(-x) = -f(x) \end{cases}$.

Q07. Use $\sin A = \frac{2\tan(A/2)}{1+\tan^2(A/2)}, \cos A = \frac{1-\tan^2(A/2)}{1+\tan^2(A/2)} \Rightarrow I = \frac{\pi}{4}$

OR Obtain $I = \int \frac{\cos^3 x\,dx}{\sin x + \cos x} = \frac{1}{2}\int \frac{(\cos^3 x + \sin^3 x) + (\cos^3 x - \sin^3 x)}{\sin x + \cos x}dx$

i.e., $I = \frac{x}{2} + \frac{1}{4}\log|\sin x + \cos x| + \frac{1}{8}(\cos 2x + \sin 2x) + k$

Q08. **See NCERT Exemplar Solutions by OP Gupta Ex6.3 Q03**

Q09. Put $x = \sin\theta, y = \sin\beta \Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ **OR** $4(xy + \tan^{-1}x) = \pi$. **Q10.** $\frac{x}{4y(y^2-a)\sqrt{a+x^2}}$

Q11. **See NECRT Textbook Part I Chapter 01 Example 42**

OR Let A = Q × Q, Q being the set of rationals. Let * be a binary operation on A, defined by (a, b) *(c, d) = (ac, ad + b). Show that

(i) * is not commutative (ii) * is associative (iii) The identity element w.r.t. * is (1, 0).

(i) Commutativity : (a, b) *(c, d) = (ac, ad + b) $\forall a, b, c, d \in A$

And, (c, d)* (a, b) = (ca, cb + d) $\neq$ (a, b) *(c, d). $\therefore$ * is not commutative.

(ii) Associativity : Let a, b, c, d, e, f $\in$ A

(a, b) * [(c, d) * (e, f)] = (a, b) * (ce, cf + d) = (ace, acf + ad + b)

Also [(a, b)* (c, d)] * (e, f) = (ac, ad + b) * (e, f) = (ace, acf + ad + b)

$\because$ (a, b) * [(c, d) * (e, f)] = [(a, b)* (c, d)] * (e, f) $\therefore$ * is associative.

(iii) Let (x, y) be an identity element in *.

So, (a, b)*(x, y) = (a, b) $\Rightarrow$ (ax, ay + b) = (a, b) $\Rightarrow$ ax = a, ay + b = b $\Rightarrow$ x = 1, y = 0

$\therefore$ the identity element is (1, 0).

Q12. **See Value Based Question (Probability) in OP Gupta's MATHEMATICIA Vol. 2**

Q13. **See NECRT Exemplar Solutions by OP Gupta Ex9.3 Q26**

Q14. Continuous but not differentiable at x = – 4/3

Q15. Obtain the value of $\Delta = 2(1+\sin^2\theta)$ then, $\Delta \in [2,4]$

Q16. **See OP Gupta's Challenge 20 on Vectors**

Q17. Foot of $\perp^{er}$: (–4, 1,–3) and distance : 7 units

Q18. **See NECRT Exemplar Chapter 02 Example 19**

Q19. MV Theorem is not applicable as given function is not differentiable at all $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Note that the function is not differentiable at $x = 0 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Q20. **See similar question in NCERT Textbook Part II Chapter 08 Example 13.**

Here 1/3 Sq. units for each part.

Q21. **See NECRT Exemplar Chapter 06 Example 18**

Q22. Let X and Y hectares of land be allocated to plant X and Y respectively.

To maximize : $Z = ₹\left(\frac{60}{100}x + \frac{40}{100}y\right)$

Subject to constraints : $x, y \geq 0$, $x + y \leq 500$, $20x + 10y \leq 8000$

Maximum value of Z = ₹260/- is attained at (300, 200).

Q23. Let E_1, E_2 & E_3 represent the production by machines X, Y & Z respectively
And E represents the defective item . By using Bayes' Theorem :

$$P(E_1|E) = \frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2) + P(E|E_3)P(E_3)} = \frac{\frac{1}{6}\times\frac{1}{100}}{\frac{1}{6}\times\frac{1}{100} + \frac{2}{6}\times\frac{1.5}{100} + \frac{3}{6}\times\frac{2}{100}} = \frac{1}{10} \text{ or } 10\%$$

Q24. See OP Gupta's Challenge 30 on Definite Integrals

Q25. (i) Obtain $A + B = \begin{pmatrix} \text{Basmati} & \text{Permal} & \text{Naura} \\ 15000 & 30000 & 36000 \\ 70000 & 40000 & 20000 \end{pmatrix} \begin{matrix} \\ \text{Ramkrishna} \\ \text{Hari Prasad} \end{matrix}$

(ii) Obtain $A - B = \begin{pmatrix} \text{Basmati} & \text{Permal} & \text{Naura} \\ 5000 & 10000 & 24000 \\ 30000 & 20000 & 0 \end{pmatrix} \begin{matrix} \\ \text{Ramkrishna} \\ \text{Hari Prasad} \end{matrix}$

(iii) 2% of B $= 0.02 \times B = B = \begin{pmatrix} \text{Basmati} & \text{Permal} & \text{Naura} \\ 100 & 200 & 120 \\ 400 & 200 & 200 \end{pmatrix} \begin{matrix} \\ \text{Ramkrishna} \\ \text{Hari Prasad} \end{matrix}$

Hence in October Ramkrishna receives ₹100, ₹200 & ₹120 as profit in the sale of each variety of rice respectively. Also Hari Prasad receives profit of ₹400, ₹200 & ₹200 in the sale of each variety of rice, respectively.
(iv) Hari Prasad gets more profit on sales of both varieties of rice for both the months.
(v) Hari Prasad is more resourceful as he makes more profit in same two months of time.

Q26. See NECRT Exemplar Solutions by OP Gupta Ex11.3 Q23
OR See NECRT Exemplar Solutions by OP Gupta Ex11.3 Q11.

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☞ For any clarification(s), please contact :
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