Class: 12th # Assignment-1(Maths) **Chapter: 1**

1. Give an example of a relation which is
   (a) Reflexive neither symmetric nor transitive. (b) Symmetric but neither reflexive nor transitive.
   (c) Transitive but neither reflexive nor symmetric. (d) Reflexive and symmetric but not transitive.
   (e) Reflexive and transitive but not symmetric. (f) Symmetric and transitive but not reflexive.
2. If $R = \{(x, y):\ x + 2y = 8\ \}$ is a relation on N. Write the range of R.
3. Let T be the set of all triangles in a plane with R a relation in T given by $R = \{(T_1, T_2):\ T_1 \cong T_2\}$. Show that R is an equivalence relation.
4. Let Z be the set of all integers and relation defined as $R = \{(a, b):\ a - b\ is\ divisible\ by\ 5;\ a, b \in Z\}$ Prove that R is an equivalence relation.
5. Check whether the relation R in R defined by: (a) $R = \{(x, y):\ x \le y^2\ ; x, y \in R\}$ (b) $R = \{(a, b):\ a \le b^3\ ; a, b \in R\}$ is reflexive, Symmetric and transitive.
6. Prove that the relation on the set $A = \{5,6,7,8,9\}$ given by $R = \{(a, b):\ |a - b\ |is\ divisible\ by\ 2\}$ is an equivalence relation. Find all elements related to element 6.
7. Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2):\ L_1 is\ perpendicular\ to\ L_2\}$. Check the relation is reflexive, Symmetric and transitive. Find the set of all lines related to line $y = 3x + 7$.
8. Show that $f: A \to B$ defined by $f(x) = \frac{x-2}{x-3}$ is bijective. where $A = R - \{3\}$ and $B = R - \{1\}$
9. If $f(x) = \frac{4x+3}{6x-4}$; $x \ne \frac{2}{3}$ .Show that $f: R \to R$ is invertible and $fof(x) = x$ .Also find f^{-1}.
10. If $f: R \to R$ defined by $f(x) = \frac{3x+5}{2}$ is an invertible function. Also find $f^{-1}(x)$.
11. Consider $f: R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is an invertible function. Also prove that $f^{-1}(x) = \frac{\sqrt{x+6}-1}{3}$
12. Prove that $f: N \to N$ defined by $f(x) = x^2 + x + 1$ is one-one but not onto.
13. If $f: N \to S$ defined by $f(x) = 4x^2 + 12x + 15$ where S is range of f. show that f is invertible also find f^{-1}.
14. Let R be the set of real numbers and $f: R \to R$ be the function defined by $f(x) = 4x + 5$. Show that f is invertible and find $f^{-1}(z)$.
15. Check the injectivity and surjectivity of $f: R \to R$ given by $f(x) = x^2$. Also check if domain replaced by N and co-domain by N.
16. Let $f: N \to N$ defined by $f(n) = \begin{cases} \frac{n+1}{2}\ ; if\ n\ is\ odd \\ \frac{n}{2}\ ;\ if\ n\ is\ even \end{cases}$ for all $n \in N$.
    State whether the function f is bijective. Justify your answer.
17. Show that $f: N \to N$ given by $f(x) = \begin{cases} x + 1\ ; if\ x\ is\ odd \\ x - 1\ ;\ if\ x\ is\ even \end{cases}$ is both one-one onto. find f^{-1}.
18. Show that the function $f: R \to R$ defined by $f(x) = \frac{x}{x^2+1}$; $\forall\ x \in R$ is neither one-one nor onto.
19. If the function $f: R \to R$ is given by $f(x) = x^2 + 3x + 1$ and $g: R \to R$ is given by $g(x) = 2x - 3$ Find (a) fog (b) gof (c) fof (d) gog
20. If the function $f: R \to R$ be given by $f(x) = x^2 + 2$ and $g: R \to R$ be given by $g(x) = \frac{x}{x-1}$; $x \ne 1$. Find fog and gof and hence find $fog(2)$ and $gof(-3)$
21. Let $f: R \to R$ be defined as $f(x) = 10x + 7$.Find the relation $g: R \to R$ such that $fog = gof = I_R$
22. Find hog and goh if $h(x) = 27x^3$ and $g(x) = x^{\frac{1}{3}}$

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23. If the function $f: R \to R$ is given by $f(x) = (3 - x^3)^{\frac{1}{3}}$ then find $fof(x)$.

24. If $f: A \to B$ defined by $f(x) = \frac{3x+4}{5x-7}$ and $g(x) = \frac{7x+4}{5x-3}$ .show that $fog = \mathrm{I_A}$ and $gof = \mathrm{I_B}$ where $= R - \left\{\frac{3}{5}\right\}$, $B = R - \left\{\frac{7}{5}\right\}$ and I is identity function.

25. Let $f, g: R \to R$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x$; $\forall\, x \in R$
Then, find fog and gof.

26. If f be modulus function and g be greatest integer function then evaluate $fog\left(\frac{-5}{3}\right) - gof\left(\frac{-5}{3}\right)$

27. Let $*$ be binary operation defined by $a * b = 3a + 4b - 2$. Find $4 * 5$

28. Let $*$ be binary operation on Q defined by $a * b = \frac{3ab}{5}$. Show that $*$ is commutative as well as associative . Also find identity element and inverse element.

29. Let $A = N \times N$ and $*$ be binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$.
Show that $*$ is commutative and associative . Also find identity element for $*$ if exists.

30. Define a binary operation $*$ on the set $\{0,1,2,3,4,5\}$ as $a * b = \begin{cases} a + b \;; & if\ a + b < 6 \\ a + b - 6 \;; & if\ a + b \geq 6 \end{cases}$
Write the operation table of $*$ and show that 0 is the identity element for $*$ and each element a of the set is invertible with $6 - a$ being the inverse of a.

31. Let X be non empty set. $P(X)$ be its power set. Let $*$ be operation defined on $P(X)$ by
$A * B = A \cap B \quad \forall\, A, B \in P(X)$ then (a) Prove that $*$ be binary operation in $P(X)$
(b) Prove that $*$ is commutative (c) Prove that $*$ is associative (d) Find the identity element in $P(X)$.
(e) Find all invertible elements identity of $P(X)$. (f) If o is another binary operation defined on $P(X)$ as $A\, oB = A \cup B$. Then verify that o distributes itself over $*$

32. In the set N of natural numbers, define the binary operation $*$ by $m * n = g.c.d(m, n)\ \forall\, m, n \in N$
Is the operation * commutative and associative? Also find identity element for $*$ if exists.

Answer Key

1. Relation R defined on A= $\{1,2,3\}$ (a) $R = \{(1,1), (2,2), (3,3), (1,2), (2,3)\}$ (b) $R = \{(1,2), (2,1)\}$
(c) $R = \{(1,2), (2,3), (1,3)\}$ (d) $R = \{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2)\}$
(e) $R = \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3)\}$ (f) $R = \{(1,2), (2,1), (1,1), (2,2)\}$

2. $R = \{(6,1), (4,2), (2,3)\}$ Range $= \{1,2,3\}$ **5.** (a)(b) Not reflexive not Symmetric not transitive.

6. $\{6,8\}$ **7.** Only Symmetric , set of lines $y = \frac{-1}{3}x + \mathrm{c}$ **9.** $f^{-1} = \frac{4x+3}{6x-4} = f$

10. $f^{-1} = \frac{2x-5}{3}$ **13.** $f^{-1}(y) = \frac{\sqrt{y-6} - 3}{2}$ **14.** $f^{-1}(z) = \frac{z-5}{4}$

15. (a) neither one-one nor onto (b) only one-one **16.** Not bijective

17. $f^{-1}(y) = \begin{cases} y - 1 \;; if\ y\ is\ even \\ y + 1 \;; if\ y\ is\ odd \end{cases}$ **19.** (a) $fog = 4x^2 - 6x + 1$ (b) $gof = 2x^2 + 6x - 1$
(c) $fof = x^4 + 6x^3 + 14x^2 + 15x + 5$ (d) $gog = 4x - 9$

20. $fog = \frac{3x^2-4x+2}{x^2-2x+1}$, $gof = \frac{x^2+2}{x^2+1}$, $fog(2) = 6$, $gof(-3) = \frac{11}{10}$ **21.** $g(x) = \frac{x-7}{10}$

22. $hog(x) = 27x$, $goh(x) = 3x$ **23.** $fof(x) = x$

25. $fog(x) = \begin{cases} 0 \;; & if\ x \geq 0 \\ -4x \;; & if\ x < 0 \end{cases}$, $gof(x) = 0$

26. 1 **27.** 30 **28.** $e = \frac{5}{3}, Inverse = \frac{25}{9a}$ **29.** e does not exist

31. (d) $e = X$ (e) only invertible element = X **32.** e does not exist

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