Series : PTS/1

Code No. 14/9/1

Roll No. | 2 | 1 | 4 | 1 | 2 | | |

Candidates must write the Code on the title page of the answer-book.

- Please check that this question paper contains **3** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of answer-book by the candidate.
- Please check that this question paper contains **26** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.00 a.m. From 10.00 a.m. to 10.15 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

PLEASURE TEST SERIES XII – 01

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Time Allowed : 180 Minutes **Max. Marks : 100**

General Instructions :

(a) *All questions are compulsory.*

(b) *The question paper consist of* ***26 questions*** *divided into* ***three sections A, B and C****. Section A comprises of* ***6 questions of one mark*** *each, section B comprises of* ***13 questions of four marks*** *each and section C comprises of* ***07 questions of six marks*** *each.*

(c) *All questions in Section A are to be answered in one word, one sentence or as per* ***the exact requirement*** *of the question.*

(d) *There is no overall choice. However,* ***internal choice*** *has been provided in* ***04 questions of four marks each*** *and* ***02 questions of six marks each****. You have to attempt only one of the alternatives in all such questions.*

(e) *Use of calculators in not permitted. You may ask for logarithmic tables, if required.*

SECTION A

Question numbers 1 to 6 carry 1 mark each.

Q01. A matrix A of order 3 has determinant value 7. What is the value of |3A|?

Q02. If R = {(x, y) : x + 2y = 8} is a relation on N, write the range of R.

Q03. If $f(x) = \int_0^x t \sin t \, dt$, then write the value of $f'(x)$.

Q04. What is the principal value of $\sin^{-1} \cos \frac{\pi}{9}$?

Q05. Radius of a circle is increasing at the rate of 0.7cm/s. Find the rate of increase of its circumference?

Q06. Find the angle between x-axis and the vector $\hat{i} + \hat{j} + \hat{k}$.

SECTION B

Question numbers 7 to 19 carry 4 marks each.

Q07. Solve the differential equation : $dy/dx + y \cot x = 2x + x^2 \cot x$, $x = \pi/4$, $y = 1$.

Q08. Find the equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance of $\frac{2}{\sqrt{3}}$ units from the point (3, 1, –1).

Q09. Evaluate : $\int \cos 2\theta \log\left(\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right) d\theta$.

Q10. Obtain the equation of tangent to the curves $x = a\sin^3\theta$, $y = b\cos^3\theta$ at $\theta = \frac{\pi}{4}$.

Q11. Given that for the function $f(x) = x^3 - bx^2 + a\,x$, $x \in [1,3]$, Rolle's Theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of a and b. **OR** Show that $f(x) = |3x + 2|$ is not differentiable at x = –2/3.

Q12. Prove that : $\cos^{-1}(x) + \cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3 - 3x^2}}{2}\right) = \frac{\pi}{3}$. **OR** Solve for x : $\tan^{-1} x + 2\cot^{-1} x = \frac{2\pi}{3}$.

Q13. If $y = \tan^{-1}\left(\frac{x}{a}\right) + \log\sqrt{\frac{x-a}{x+a}}$; prove that $\frac{dy}{dx} = \frac{2ax^2}{x^4 - a^4}$

OR If $y = x\log\left(\frac{x}{a + bx}\right)$ then, prove that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

Q14. On the set R–{–1}, a binary operation is defined by a * b = a + b + ab for all a, b ∈ R–{–1}. Prove that * holds both commutative & associative properties on R–{–1}. Find the identity element and prove that every element of R–{–1} is invertible.

Q15. For any two vectors $\vec{a}$ and $\vec{b}$, show that: $(1+|\vec{a}|^2)(1+|\vec{b}|^2) = (1-\vec{a}.\vec{b})^2 + |\vec{a} + \vec{b} + \vec{a}\times\vec{b}|^2$.

OR The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

Q16. Evaluate : $\int_0^{\pi} \frac{1}{5 + 4\cos x} dx$. **OR** Evaluate : $\int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x} dx$.

Q17. **A)** Show that the points (2, 3, 4), (–1,–2, 1) and (5, 8, 7) are collinear.

B) Evaluate the integral of $\int \tan^2\frac{x}{2} dx$.

C) Let $A = [a_{ij}]$ be a square matrix of order 3 and C_{ij} denotes the cofactor of $[a_{ij}]$ in A. If |A| = 5, then write the value of $a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33}$.

D) Find the value of μ where it is given that $\mu = \hat{i}.(\hat{j}\times\hat{k}) + \hat{j}.(\hat{k}\times\hat{i}) + \hat{k}.(\hat{i}\times\hat{j})$.

Q18. A class has 15 students whose ages are 14,17,15,14,21,17,19,20,16,18,20,17,16,19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find the mean of X.

Q19. Using properties of determinants, prove that : $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = -2(x^3 + y^3)$.

SECTION C

Question numbers 20 to 26 carry 6 marks each.

Q20. Using integration, find the area of the region : $\{(x,y) : 9x^2 + y^2 \le 36,\ 3x + y \ge 6\}$.

What is the importance of integration in life.

Q21. Prove that the image of (3,–2, 1) in the plane $3x - y + 4z = 2$ lies on the plane $x + y + z + 4 = 0$.
OR Find the distance of the point (–1, –5, –10) from the point of intersection of the plane $\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 5$ and the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$.

Q22. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

Q23. Evaluate : $\int_0^1 \sin^{-1}(x\sqrt{1-x} - \sqrt{x-x^3})\, dx$.

Q24. Assume that the chances of a patient having a heart-attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart-attack by 30% and prescription of certain drug reduces its chances by 25%. At a time, a patient can chose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart-attack. Find the probability that the patient followed a course of meditation and yoga.
Interpret the result & state which of the above stated methods is more beneficial for the patient.
OR There are three coins. One is two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed, and it shows heads. What is the probability that it was the two-headed coin?

Q25. An amount of ₹600 crores is spent by the government in three schemes. Scheme A is for saving girl child from the cruel parents who don't want girl child and get the abortion before her birth. Scheme B is for saving of newlywed girls from death due to dowry. Scheme C is planning for good health for senior citizen. Now twice the amount spent on Scheme C together with amount spent on Scheme A is ₹700 crores. And three times the amount spent on Scheme A together with amount spent on Scheme B and Scheme C is ₹1200 crores. Find the amount spent on each schemes using matrices. **What is the importance of saving girl child from the cruel parents who don't want girl child and get the abortion before her birth?**

Q26. A manufacturing company makes two types of teaching aids A and B of Mathematics for class XII. Each type of A requires 9 labour hours of fabricating and 1 hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of ₹80 on each piece of type A and ₹120 on each piece of type B. How many pieces of type A and type B should be manufactured per week to get a maximum profit? Make it as an LPP and solve graphically. What is the maximum profit per week?

Q01. Use $|kA| = k^n A$, where n is order of A. So, $|3A| = 189$. Q02. {1, 2, 3}

Q03. x sin x Q04. $\sin^{-1}\cos\frac{\pi}{9} = \frac{\pi}{2} - \cos^{-1}\cos\frac{\pi}{9} = \frac{7\pi}{18}$ Q05. 4.4cm/s

Q06. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ Q07. $y = x^2 + \left(\frac{16-\pi^2}{16\sqrt{2}}\right)\text{cosec } x$

Q08. The equation of a plane passing through the line of intersection of the planes x + 2y + 3z = 2 and x – y + z = 3 is $x+2y+3z-2+\lambda(x-y+z-3)=0$ i.e., $x(1+\lambda)+y(2-\lambda)+z(3+\lambda)-2-3\lambda=0$ …(i)

Since (i) is at a distance of $\frac{2}{\sqrt{3}}$ units from the point (3, 1,–1) therefore,

$$\frac{|3(1+\lambda)+1(2-\lambda)-1(3+\lambda)-2-3\lambda|}{\sqrt{(1+\lambda)^2+(2-\lambda)^2+(3+\lambda)^2}} = \frac{2}{\sqrt{3}} \Rightarrow \lambda = -\frac{7}{2}$$

Substituting the value of λ in (i), we get the required equation of plane : $5x-11y+z=17$.

Q09. $\frac{\sin 2\theta}{2}\log\tan\left(\frac{\pi}{4}+\theta\right) - \frac{1}{2}\log\sec 2\theta + k$ Q10. $2\sqrt{2}x + 2\sqrt{2}y = a + b$

Q11. $a = 11, b = 6$ Q12. Put $x = \cos\theta$ OR $\sqrt{3}$

Q13. We have $y = \tan^{-1}\left(\frac{x}{a}\right) + \log\sqrt{\frac{x-a}{x+a}}$ $\Rightarrow y = \tan^{-1}\frac{x}{a} + \frac{1}{2}[\log(x-a) - \log(x+a)]$

On differentiating w.r.t. x both sides, we get : $\frac{dy}{dx} = \frac{1}{1+\frac{x^2}{a^2}}.\frac{1}{a} + \frac{1}{2}\left[\frac{1}{x-a} - \frac{1}{x+a}\right]$

$$\Rightarrow \frac{dy}{dx} = \frac{a}{a^2+x^2} + \frac{1}{2}\left[\frac{2a}{x^2-a^2}\right] = \frac{a}{x^2+a^2} + \frac{a}{x^2-a^2} \qquad \therefore \frac{dy}{dx} = \frac{2ax^2}{x^4-a^4}.$$

OR $y = x\left[\log x - \log(a+bx)\right] \Rightarrow y' = \frac{a}{a+bx} + \frac{y}{x}$ ...(i) $\Rightarrow xy' = \frac{ax}{a+bx} + y$

$$\Rightarrow xy'' + y' = \frac{(a+bx).a - ax(b)}{(a+bx)^2} + y' \Rightarrow xy'' = \frac{a^2}{(a+bx)^2} = \left(y' - \frac{y}{x}\right)^2 \text{ [By (i)]} \quad \therefore x^3y'' = (xy'-y)^2$$

Q14. Identity Element : 0 is the identity element for * defined on R–{–1}.

Also, inverse of an element a is : $-\frac{a}{a+1} \in R-\{-1\}$

Q15. OR Value of $\lambda = 1$, Unit vector $= \frac{3\hat{i}+6\hat{j}-2\hat{k}}{7}$ Q16. $\frac{\pi}{3}$ OR $\frac{\pi}{2}$

Q17. A) Let A(2, 3, 4), B(–1,–2, 1) & C(5, 8, 7). The d.r.'s of AB : 3, 5, 3; the d.r.'s of BC : 6, 10, 6. Since $\frac{3}{6} = \frac{5}{10} = \frac{3}{6}$, i.e., the d.r.'s of AB and BC are proportional so, AB and BC are parallel.

But B is a common point so, A, B and C must be collinear.

B) $2\tan\left(\frac{x}{2}\right) - x + k$ C) 5 D) 3

Q18. The probability distribution of the random variable X is :

X	14	15	16	17	18	19	20	21
P(X)	2/15	1/15	2/15	3/15	1/15	2/15	3/15	1/15

The mean of random variable $X = \sum X\,P(X) = \frac{263}{15}$. Q20. $3(\pi-2)$ sq.units

Q21. Find the image (0,–1,–3) of given point in 1st plane and then show that it satisfies the 2nd plane.

OR The point of intersection is (2,–1, 2) so, required distance is 13units.

Q22. $\left(\frac{4\pi R^3}{3\sqrt{3}}\right)$ cubic units Q23. $\frac{\pi}{4}-1$

Q24. Let A : the patient follows a course of meditation and yoga, B : he takes a certain drug.
Then, P(A) = 1/2, P(B) = 1/2. Let E : the patient suffers a heart-attack.
Also $P(E|A) = (70/100).(40/100)$, $P(E|B) = (75/100).(40/100)$.

By Bayes' Theorem, we get : $P(A|E) = \frac{P(E|A)P(A)}{P(E|A)P(A)+P(E|B)P(B)} = \frac{14}{29}$.

Interpretation of result : It is evident that if a patient follows a course of meditation and yoga, then he is less likely to get heart-attack. [Since P(B|E) = 15/29.] So, clearly a course of meditation and yoga is more beneficial as compared to the intake of drugs.

OR Let E_1 : choosing first (two headed) coin, E_2 : choosing second (biased) coin, E_3 : choosing third coin. Also, let A : the coin showing heads.

$\therefore\ P(E_1) = P(E_2) = P(E_3) = \frac{1}{3},\ P(A|E_1) = 1,\ P(A|E_2) = \frac{75}{100},\ P(A|E_3) = \frac{60}{100}$.

By Bayes' Theorem, $P(E_1|A) = \frac{P(A|E_1)P(E_1)}{P(A|E_1)P(E_1)+P(A|E_2)P(E_2)+P(A|E_3)P(E_3)}$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times\frac{3}{4}+\frac{1}{3}\times\frac{3}{5}} = \frac{20}{47}.$$

Q25. ₹300crores, ₹100crores and ₹200crores respectively for Scheme A, B and C. Importance of saving the girl child : (i) In our country, male population is more than female population.
(ii) It is essential for a human being to save the life of all.

Q26. Let the number of pieces of type A and type B manufactured per week be x and y respectively.
To maximize : Z = ₹ (80x + 120y)
Subject to constraints : $x \ge 0,\ y \ge 0,\ 9x+12y \le 180 \Rightarrow 3x+4y \le 60$ and, $x+3y \le 30$.
Maximum profit of ₹1680 is obtained when 12 pieces of type A and 6 pieces of type B are manufactured by the company per week.