OPG's PLEASURE TEST SERIES 03

Time : 180 Minutes **[2013-14]** **Max. Marks : 100**

Mathematics Class XI

General Instructions :

(i) The question paper consists of three parts A, B and C. Each question of each part is compulsory.

(ii) Section A (Very Short Answer Type) consists of 06 questions of 1 mark each.

(iii) Section B (Short Answer Type) consists of 13 questions of 4 marks each.

(iv) Section C (Long Answer Type) consists of 7 questions of 6 marks each.

(v) There is no overall choice. However internal choices in **four questions** of section B and **two questions** of section C have been provided.

(vi) Use of calculator is not permitted.

SECTION – A

(This section contains 06 questions of **one mark** each.**)**

Q01. Write the negation of the following statement :

"The square root of every positive number is positive."

Q02. Write all the possible subsets of $\{\phi, \{\phi\}\}$

Q03. Is the relation R = {(2, 4), (3, 6), (2, 8), (1, 0)} a function? Justify your answer.

Q04. Solve : $\frac{|2-x|}{x-2} \geq 0$.

Q05. What is the value of $8\cos^3\frac{\pi}{9} - 6\cos\frac{\pi}{9}$.

Q06. If |z| = 2 and arg(z) = $\frac{\pi}{4}$ then find z.

SECTION – B

(This section contains 13 questions of **four marks** each.**)**

Q07. Let A and B be two finite sets such that n(A) = m and n(B) = p. If the ratio of number of elements of power sets of A and B is 64 : 1 and n(A) + n(B) = 32, find the value of m and p.

OR Using properties of sets, prove that $A \cup (B - A) = (A \cup B)$.

Q08. Find the ratio in which the line joining the points (2, 4, 5) and (3, 5,–4) is divided by XY-plane. Also find the coordinates of this point.

Q09. Evaluate the sum of the series : $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots$ upto n terms.

OR Evaluate the sum of the series : $1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \ldots$ upto 40 terms.

Q10. Solve : $3\tan\theta + \cot\theta = 5\,\text{cosec}\,\theta$.

Q11. Solve for x : $\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}$, $\frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$ and express on the number line.

Q12. Find the length of the perpendicular drawn from the point (4,–7) to the line joining the origin and the point of intersection of the lines 2x – 3y + 14 = 0 and 5x + 4y – 7 = 0.

Q13. The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and 100m long is supported by vertical wires attached to the cable, the longest wire being 30m and the shortest being 6m. Find the length of a supporting wire attached to the roadway 18m from the middle.

OR Find the eccentricity of an ellipse if the distance between its foci is same as the length of its latus-rectum.

Q14. Evaluate : $\lim_{x\to 0} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$.

Q15. Using principle of mathematical induction, prove that $10^n + 3.4^{n+2} + 5$ is divisible by 9.

Q16. If $f(x)$ satisfies the equation $x^2 f(x) + f(1-x) = 2x - x^4$ then find the value of $f(1/3)$.

Q17. Find the number of arrangements of the letters of the word REPUBLIC. How many arrangements start with a vowel? Do you know when and why is the Republic Day celebrated? What is its importance for an Indian citizen?

Q18. For what real values of θ, $\dfrac{1-i\sin\theta}{1+2i\sin\theta}$ is purely real?

Q19. Find the mean deviation about the median for the following data :

x_i	3	6	9	12	13	15	21	22
f_i	3	4	5	2	4	5	4	3

OR For a group of 200 students, the mean and standard deviations of scores were found to be 40 and 15 respectively. Later on it was discovered that the score of 43 was wrongly taken as 34. Find the correct mean and standard deviation.

SECTION – C

(This section contains 7 questions of **six marks** each.**)**

Q20. If A, B, C are three events associated with a random experiment, prove that :

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C).$$

OR In a relay race, there are five teams : A, B, C, D and E.

(a) What is the probability that A, B and C finish first, second and third, respectively.

(b) What is the probability A, B and C are first three to finish (in any order) (Assume that all finishing orders are equally likely).

Q21. A small firm manufactures sweaters of two types : Type A and Type B. The combined number of sweaters of both typess that it can handle per day is at most 24. Sweater of Type A takes one hour to be made and Sweater of Type B takes half an hour. The maximum number of hours available per day is 16. Formulate this information in the form of inequalities and draw a graph representing the solution of these inequations.

The firm decides to sell the sweaters of Type B at very nominal prices to the poor people to help them be safe from the cold. Also in order to cover up the looses, the management of the firm urges to sell sweaters of Type A at a bit higher price. What do you conclude from their decision? Justify your answer.

Q22. Find the sum of the sequence 5, 5.5, 5.55, 5.555, … to 30 terms.

Q23. Show that the area of the triangle the equations of whose sides are $y = m_1x + c_1$, $y = m_2x + c_2$ and $x = 0$ is given as $\dfrac{1}{2} \times \left(\dfrac{[c_1 - c_2]^2}{m_2 - m_1} \right)$ sq.units.

Q24. Differentiate $f(x) = \dfrac{4\sin x - 2x - x\cos x}{2 + \cos x}$ with respect to x.

Q25. If $\sin\theta = -\dfrac{4}{5}$, $\pi < \theta < \dfrac{3\pi}{2}$ then find the remaining trigonometric functions. Hence find $\sin 2\theta$.

OR In any triangle ABC, prove that : $(b^2 - c^2)\cot A + (c^2 - a^2)\cot B + (a^2 - b^2)\cot C = 0$.

Q26. (i) Evaluate : ${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$.

(ii) If three consecutive coefficients in the expansion of $(1 + x)^n$ are in the ratio 6 : 33 : 110. Determine the value of n and r.

Q02. $\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}$ **Q03.** No, because the element 2 has two images in R as (2, 4) and $(2, 8) \in R$. **Q04.** $(2, \infty)$ **Q05.** 1

Q06. $\sqrt{2} + \sqrt{2}\,i$ **Q07.** $m = 19,\ p = 13$ **Q08.** 5:4, $\left(\frac{23}{9}, \frac{41}{9}, 0\right)$

Q09. $S_n = \frac{n}{2n+1}$ OR $S_n = \frac{n(n+3)}{4}$, $S_{40} = 430$ **Q10.** $\theta = n\pi;\ 2n\pi \pm \frac{\pi}{3}, n \in Z$

Q11. $x > 3$ **Q12.** 1 Unit

Q13. 9.11m (Approx.) OR $\frac{\sqrt{5}-1}{2}$ **Q14.** 0

Q16. 8/9 **Q17.** 8!, ${}^3C_1 \times 7!$ **Q18.** $\theta = n\pi, n \in Z$

Q19. 4.97 OR 40.045, 14.995

Q20. NCERT Page 407 Example 16 OR (a) 1/60 (b) 6/60

Q21. Inequations will be : $x + y \le 24, 2x + y \le 32, x \ge 0, y \ge 0$. Now solve graphically.

Q22. $\frac{50}{81}\left(269 + (0.1)^{30}\right)$ **Q23.** NCERT Page 230 Example 23

Q24. $\frac{\cos x(4 - \cos x)}{(2 + \cos x)^2}$ **Q25.** $\cos\theta = -\frac{3}{5}, \tan\theta = \frac{4}{3}, \operatorname{cosec}\theta = -\frac{5}{4}, \sec\theta = -\frac{5}{3}$, $\cot\theta = \frac{3}{4}$ and $\sin 2\theta = \frac{24}{25}$.

Q26. (i) 0 (ii) n = 12, r = 2.

❖ Dear Student/Teacher,

I would urge you for a little favour. Please notify me about any error(s) you notice in this (or other Maths) work. It would be beneficial for all the future learners of Maths like us. Any constructive criticism will be well acknowledged. Please find below my contact info when you decide to offer me your valuable suggestions. I'm looking forward for a response.

Also I would wish if you inform your friends/students about my efforts for Maths so that they may also benefit.

Let's *learn* Maths with smile :-)

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