Sample Paper – 2014
Class – XII
Subject – Mathematics

Instructions: Question Number 1 to 10 carry 1 marks each, Question Number 11 to 22 carry 4 marks each, Question Number 23 to 29 carry 6 mark each.

Section A

1. Give an example of a relation. Which is transitive but neither reflexive nor symmetric
2. Find the value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$.
3. Using the property of determinants and without expanding prove that: $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$
4. Find area of the triangle with vertices at the points : (-2, -3), (3, 2), (-1, -8).
5. Evaluate: $\int (x^3 - 1)^{\frac{1}{3}} x^5 \, dx$.
6. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ externally in the ratio 1 : 2.
7. Find the intercepts cut off by the plane 2x + y - z = 5 on the coordinate axis.
8. Find the value of $\hat{i}.\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{i}\times\hat{k}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right)$..
9. Evaluate $\int \sqrt{(1-x)^2 + 1}\, dx$.
10. Construct a 2×2 matrix, whose elements are given by : $a_{ij} = \frac{1}{2}\left|-3i + j\right|$.

Section B

11. Consider f : $R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse of f given by $f^{-1}(y) = \sqrt{y - 4}$, where R_+ is the set of all non-negative real numbers.

12. If $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$, then find the value of x. OR Prove that : $\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

13. If x = a (cost + t sin t) and y = a (sin t – t cos t), find $\frac{d^2y}{dx^2}$.

14. Determine if *f* defined by : $f(x)\begin{cases} x^2\sin\frac{1}{x}, \text{if } x\neq 0 \\ 0, \quad \text{if } x=0 \end{cases}$ is a continuous function at x=0?

15. Using propertis of determinant prove that: $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix}=(1+pxyz)(x-y)(y-z)(z-x)$.

16. Using differentials, find the approximate value of $(0.999)^{\frac{1}{10}}$ corrected up to 3 places of decimal.

OR

Verify Mean Value Theorem, if $f(x)=x^2-4x-3$ in the interval [a, b], where a = 1 and b = 4.

17. Evaluate: $\int\frac{1}{\cos(x+a)\cos(x+b)}dx$.

18. Solve the Differential equation : $x^2\,dy+(xy+y^2)dx=0$; y = 1 when x = 1.

19. Solve the differential equation: $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right]\frac{dx}{dy}=1\ (x\neq 0)$.

OR

Form the differential equation of the family of circles touching the y-axis at origin.

20. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

OR

For any three vectors $\vec{a},\vec{b},\vec{c}$, show that $\left[\vec{a}+\vec{b}\quad \vec{b}+\vec{c}\quad \vec{c}+\vec{a}\right]=2\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]$.

21. Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes $\vec{r}.(\hat{i}-\hat{j}+2\hat{k})=5$ and $\vec{r}.(3\hat{i}+\hat{j}+\hat{k})=6$.

22. The random variable X has a probability distribution P(X) of the following form,

: $P(X)=\begin{cases} k, \text{ if } x=0 \\ 2k, \text{ if } x=1 \\ 3k, \text{ if } x=2 \\ 0, \text{ otherwise} \end{cases}$ where *k* is some number

Determine the value of k. (b) Find P (X < 2), $P(X \le 2), P(X \ge 2)$

Section C

23. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with its vertex at one end of the major axis.

24. Evaluate the definite integral: $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)\,dx$ OR Evaluate $\int_1^3 (3x^2+4)dx$. as a limit of sum.

25. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$.

26. Solve system of linear equations using matrix method, x – y + z = 4 ,2x + y – 3z = 0 ,x + y + z = 2.

27. An aero plane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

28. Find the equation of the perpendicular drawn from the point P(2,4,-1) to the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$.

OR

Find the distance of the point (-2,3,-4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parelle to the plane 4x+12y-3z+1=0.

29. Assume that the chances of a patient having a heart attack are 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack. By 30% and prescription of certain drug reduces its chances by 25%. At a time n patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?