Guess Paper – 2014
Class – XII
Subject – Mathematics

General Instructions :

(i) All questions are compulsory.

*(ii) The question paper consists of 29 questions divided into **3** sections **A, B** and **C**. Section A comprises of 10 questions of 1 mark each, Section B comprises of 12 questions of 4 marks each , Section C comprises of 7 questions of **6** marks each.*

(iii) All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

Section – A

Question numbers 1 to 10 carry 1 mark each.

1. For what value of k, the matrix $\begin{pmatrix} 2k+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2k-3 \end{pmatrix}$ is skew symmetric?

2. Find the value of x, if $\begin{pmatrix} 5x+y & -y \\ 2y-x & 3 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ -3 & 3 \end{pmatrix}$

3. Write the value of the determinant:

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

4. Evaluate: $\int \frac{sec^2\left(\log[\![x]\!]\right)}{x}\, dx$

5. If $f: R \to R$ is a bijection given by $f(x) = x^3 + 3$, $\quad find\ \ f^{-1}(x)$

6. Find $|\vec{x}|$, $\quad if\ for\ a\ unit\ vector\ \vec{a}, (\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 12$

7. Write the vector equation of the line : $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$

8. Find the value of $sin^{-1}\left(-\frac{1}{2}\right) + cos^{-1}\left(-\frac{1}{2}\right)$

9. Find the unit vector perpendicular to both the vectors $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{b} = 6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$

10. Evaluate: $\int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}\, dx$

Section – B

Question numbers 11 to 22 carry 4 marks each.

11. Consider $f: R_+ \to [4, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6}-1}{3}\right)$

12. Solve for x: $2tan^{-1}(sinx) = tan^{-1}\left(2\sec x\right), \quad x \neq \frac{\pi}{2}$

13. Show that : $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix} = (a^3 + b^3)^2$

14. If $x = a\left(\cos t + t \sin t\right)$ and $y = a\left(\sin t - t \cos t\right)$, find $\frac{d^2y}{dx^2}$.

(or)

If x sin (a + y) + sin a cos (a + y) = 0, prove that $\frac{dy}{dx} = \frac{sin^2(a+y)}{\sin a}$

15. Show that the function $g(x) = |x - 2|, \quad x \in R,$ is continuous but not differentiable at x = 2.

16. Find the intervals in which the following function is increasing or decreasing:

$$f(x) = 8 + 36x + 3x^2 - 2x^3$$

(or)

Prove that $y = \frac{4\sin\theta}{(2 + \cos\theta)} - \theta$ is an increasing function in $\left[0, \frac{\pi}{2}\right]$.

17. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx$

(or)

Evaluate: $\int \frac{5x + 3}{\sqrt{x^2 + 4x + 10}}\, dx$

18. A speaks truth in 60 % of the cases and B in 90 % of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact? Which values A is lacking and should improve upon?

19. Solve the following differential equation:

$(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1 + x^2}$, given y = 0 when x = 1.

20. Show that the differential equation $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$ is homogeneous, and solve it.

21. Three vectors $\vec{a}, \quad \vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity

$\mu = \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}, \quad$ if $|\vec{a}| = 1, \quad |\vec{b}| = 4$ and $|\vec{c}| = 2.$

22. Find the shortest distance between the lines whose vector equations are

$\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$ and
$\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$

(or)

Find the vector equation of the plane through the points $(2, 1, -1)$ and $(-1, 3, 4)$ and perpendicular to the plane $x - 2y + 4z = 10$

Section – C

Question numbers 23 to 29 carry 6 marks each.

23. Evaluate: $\int \frac{2x}{(1 + x^2)(3 + x^2)}\, dx$

24. Find the equation of the plane through the points A (1, 1, 0), B (1, 2, 1) and C (-2, 2, -1) and hence find the distance between the plane and the line $\frac{x - 6}{3} = \frac{y - 3}{-1} = \frac{z + 2}{1}$

25. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and found to be both diamonds. Find the probability of the lost card being a diamond.

26. Using integration find the area of region bounded by the triangle whose vertices are (-1, 0), (1, 3) and (3, 2)

(or)

Find the area of the region $\{(x, y): y^2 \leq 4x, \quad 4x^2 + 4y^2 \leq 9\}$ using method of integration

27. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

(or)

Find the equation of the tangent to the parabola $y^2 = 8x$ which is parallel to line $4x - y + 3 = 0$

28. A company manufactures two types of stickers A : "SAVE ENVIRONMENT" and

B : "BE COURTEOUS". Type A requires 5 minutes each for cutting and 10 minutes each for assembling. Type B requires 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours available fo r assembling in a day. He earns a profit of ₹ 50 on each type A and ₹ 60 on each type B. How many stickers of each type should the company manufacture in a day to maximize profit? Give your views about "SAVE ENVIRONMENT" and "BE COURTEOUS".

29. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value which the management of the colony must include for awards.

Paper Submitted by:

Name B SURESH
Email sureshvidhya2005@rediffmail.com
Phone No. 9443019183