

**2013-14 - MATHEMATICS MODEL PAPER- 2**

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**SECTION – A(10 X 1 = 10 MARKS)**

1. Check whether  $f$  is one to one and onto where  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined as  $f(x) = x^4$ .
2. Find the value of  $\tan^{-1} \sqrt{3} - \sec^{-1}(-2)$
3. Construct a  $3 \times 2$  matrix whose elements are given by  $a_{ij} = \frac{1}{2}|i - 3j|$
4. Find  $K$  if the matrix  $A = \begin{pmatrix} 3k & 2 \\ 3 & 2k \end{pmatrix}$  is singular.
5. If  $\begin{vmatrix} \cos \alpha & \sin \beta \\ \sin \alpha & \cos \beta \end{vmatrix} = \frac{1}{2}$ , where  $\alpha, \beta$  are acute find  $\alpha + \beta$ .
6. If  $\vec{a}$  and  $\vec{b}$  include an angle of 120 degrees and their magnitude are 2 and  $\sqrt{3}$  then find the value of  $\vec{a} \cdot \vec{b}$ .
7. Find the value of  $\left[ (\hat{i} + \hat{j}) \quad (\hat{j} + \hat{k}) \quad (\hat{k} + \hat{i}) \right]$ .
8. Find the angle between the planes  $\vec{r} \cdot (2\vec{i} - \vec{j} - \vec{k}) = 15$  and  $\vec{r} \cdot (-\vec{i} + \vec{j} - 3\vec{k}) = 3$ .
9. Evaluate  $\int \tan^2 x \, dx$
10. Evaluate  $\int \frac{2x}{x^4 + 4} dx$

**SECTION – B(12 X 4 = 48 MARKS)**

11. Let  $*$  be a binary operation on the set  $\mathbb{Q}$  of rational numbers as follows: (i)  $a * b = a + b - ab$  where  $a, b \in \mathbb{R} - \{1\}$ , (ii)  $a * b = a + b + 1$ , Check whether (i) these are valid binary operation (ii) commutative and (iii) associative.
12. Prove that  $\tan^{-1} x + \tan^{-1} \frac{2x}{1-x^2} = \tan^{-1} \frac{3x-x^3}{1-3x^2}$  (OR) Solve for  $x$  if  $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} \frac{8}{31}$
13. Prove that  $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

14. Find the value of  $a$  and  $b$  such that the function defined by

$$f(x) = \begin{cases} x^2 + ax + b & \text{if } 0 \leq x < 2 \\ 3x + 2 & \text{if } 2 \leq x \leq 4 \\ 2ax + 5b & \text{if } 4 < x \leq 8 \end{cases} \quad \text{is continuous on } [0,8]$$

15. Differentiate  $y = x^{\sin x} + (\sin x)^{\cos x}$  w.r.t.  $x$

16. The total cost  $C(x)$  in Rupees, associated with the production of  $x$  units of an item is given by  $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$ . Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output. (OR)

Show that the function  $f$  given by  $f(x) = \tan^{-1}(\sin x + \cos x)$ ,  $x > 0$  is always a strictly increasing function in  $\left(0, \frac{\pi}{4}\right)$

17. Evaluate  $\int (6x+5)\sqrt{6+x-x^2} dx$  (OR) Evaluate  $\int_0^{\pi} \frac{x}{1+\sin x} dx$

18. Solve  $(1+e^{2x})dy + (1+y^2)e^x dx = 0$  given that  $x=0, y=1$ .

19. Obtain the differential equation from the equation  $y = e^x(\cos x + \sin x)$ .

20. If  $\vec{a}, \vec{b}$  and  $\vec{c}$  are unit vectors such that  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ , find the value of  $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ . (OR)

Find the value of  $\lambda$  if the points  $A(-1, 4, -3)$ ,  $B(3, \lambda, -5)$ ,  $C(-3, 8, -5)$  and  $D(-3, 2, 1)$  are coplanar.

21. Find the equation of the plane through the points  $(2, 1, 0)$ ,  $(3, -2, -2)$  and  $(3, 1, 7)$ .

22. Out of a group of 8 highly qualified doctors in a hospital, 6 are very kind and cooperative with their patients and so are very popular, while the other two remain reserved. For a health camp, three doctors are selected at random. Find the probability distribution of the number of very popular doctors. What values are expected from the doctors.

**SECTION – B (7 X 6 = 42 MARKS)**

23. A window is in the form of a rectangle surmounted by a semi-circle. If the perimeter of the window is 30 meters, find the dimensions so that the greatest possible amount of light may be admitted.

24. For keeping fit  $X$  people believe in morning walk,  $Y$  people believe in yoga and  $Z$  people join Gym. Total no. of people are 70. Further 20% 30% and 40% people are suffering from any disease who believe in morning walk, yoga and GYM respectively. Total no. of such people is 21. If morning walk cost nothing, Yoga cost Rs 500/month and GYM cost Rs 400/month and total expenditure is Rs 23000. (i) Formulate a matrix problem. (ii) Calculate the no. of each type of people. (iii) Why exercise is important for health?

25. An NGO is helping the poor people of earthquake hit village by providing medicines. In order to do this they set up a plant to prepare two medicines A and B. There is sufficient raw material available to make 20000 bottles of medicine A and 40000 bottles of medicine B but

there are 45000 bottles into which either of the medicine can be put. Further it takes 3 hours to prepare enough material to fill 1000 bottles of medicine A and takes 1 hour to prepare enough material to fill 1000 bottles of medicine B and there are 66 hours available for the operation. If the bottle of medicine A is used for 8 patients and bottle of medicine B is used for 7 patients. How the NGO should plan his production to cover maximum patients? How can you help others in case of natural disaster?

26. Evaluate  $\int (x^2 \tan^{-1} x) dx$

27. Find the length of the foot of perpendicular drawn from the point  $(2, -1, 5)$  to the line  $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$ . Also find its image.

(OR)

Find the equation of the planes bisecting the angle between the planes  $x+2y+2z-3=0$ ,  $3x+4y+12z+1=0$  and specify the plane which bisect the acute angles.

28. Draw a rough sketch of  $\{(x, y) : x^2 + y^2 \leq 1, 2x + 2y \geq 1\}$ . Find the area of the region enclosed.

29. A drunkard man takes a step forward with probability 0.6 and takes a step backward with probability 0.4. He takes 9 steps in all. Find the probability that he is just one step away from the initial point.

(OR)

A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike, and 0.32 that the construction job will be completed on time if there is a strike. Determine the probability that the construction job will be completed on time.

WISH YOU ALL THE BEST.

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