TARGET MATHEMATICS
THE EXCELLENCE KEY
AGYAT GUPTA (M.Sc., M.Phil.)

22 YEAR OF QUALITY EDUCATION

GENERAL INSTRUCTIONS :-

CODE:- AG-12

1. All questions are compulsory.
2. The question paper consists of 34 questions divided into four sections A,B,C and D. Section – A comprises of 8 question of 1 mark each. Section – B comprises of 6 questions of 2 marks each. Section – C comprises of 10 questions of 3 marks each and Section – D comprises of 10 questions of 4 marks each.
3. Question numbers 1 to 8 in Sections – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four mark each. You have to attempt only one If the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 6 printed pages.

MATHEMATICS ***CLASS X*** ***(SA-1)***

Time : 3 to $3\frac{1}{4}$ Hours Maximum Marks : 90

SUMMATIVE ASSESSMENT –I (2013)

SECTION A

Q.1 The product of the HCF and LCM of the smallest prime number and the smallest composite number is :

(A) 2 (B) 4 (C) 6 (D) 8 Ans. D

Q.2 The quadratic polynomial $p(x)$ with -81 and 3 as product and one of the zeroes respectively is :

(A) $x^2+24x-81$ (B) $x^2-24x-81$

(C) $x^2-24x+81$ (D) $x^2+24x+81$ Ans. A

Q.3 Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio :

(A) 2 : 3 (B) 4 : 9 (C) 81 : 16 (D) 16 : 81

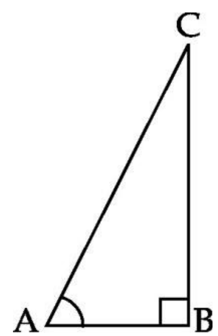
Ans. D

Q.4 The mean of 5 observations $x,\ x+2,\ x+4,\ x+6$ and $x+8$ is 11, then the value of x is :

(A) 4 (B) 7 (C) 11 (D) 6

Ans. B

Q.5 In the figure given below, ΔABC is right angled at B and $\tan A=\frac{4}{3}$. If AC = 15 cm the length of BC is :



(A) 4 cm (B) 3 cm (C) 12 cm (D) 9 cm Ans. C

Q.6 If $\sin(\theta+36°)=\cos\theta$, where θ and $\theta+36°$ are acute angles, then value of θ is :

	(A) $36°$ (B) $54°$ (C) $27°$ (D) $90°$ Ans. C
Q.7	The decimal expansion of $\frac{33}{2^2 \times 5}$ will terminate after : (A) One decimal place (B) Two decimal places (C) Three decimal places (D) More than three decimal places Ans. B
Q.8	If $x = a,\ y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then the values of a and b are respectively : (A) 3 and 5 (B) 5 and 3 (C) 3 and 1 (D) -1 and -3 Ans. C
	SECTION B
Q.9	Prove that $15 + 17\sqrt{3}$ is an irrational number Ans. Let $15 + 17\sqrt{3}$ be a rational number $\therefore 15 + 17\sqrt{3} = \frac{p}{q}$ $17\sqrt{3} = \frac{p}{q} - 15$ $\sqrt{3} = \frac{p - 15q}{17q}$ Since p and q are integers $\frac{p - 15q}{17}$ is a rational number $\therefore \sqrt{3}$ is rational But we know that $\sqrt{3}$ is irrational (a) Our assumption is wrong (b) $15 + 17\sqrt{3}$ irrational
Q.10	For what value of k will be following system of linear equations has no solution? $(k+1)x + y = 1;\ 3x + (k-1)y = 2k + 5$. Ans. For no solution $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ i.e. $\frac{k+1}{3} = \frac{1}{k-1} \neq \frac{1}{2k+5}$ $(k+1)(k-1) = 3$ $k^2 - 1 = 3$ $k^2 = 4$ $\therefore \quad k = \pm 2$
Q.11	If the areas of two similar triangles are equal, prove that they are congruent. Ans. $\frac{ar(ABC)}{ar(DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$ – [Ratio theorem] $ar(ABC) = ar(DEF) \Rightarrow \frac{ar(ABC)}{ar(DEF)} = 1$ $\Rightarrow AB = DE,\ BC = EF,\ AC = DF$ By SSS $\Delta ABC \cong \Delta DEF$
Q.12	Find modal height of the following frequency distribution :

Height in cms	135-140	140-145	145-150	150-155	155-160
No. of persons	4	9	12	10	7

Ans. Modal class $= 145 - 150$

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	Mode $= 145 + \frac{12-9}{24-10-9} \times 5$ $=145+3=148$
Q.13	Form a quadratic equation with rational coefficients, one of whose roots is $\frac{2-\sqrt{3}}{5}$. Ans. $25x^2 - 20x + 1$ **OR** Show that 5^n can never end with the digit zero for any natural number n. Ans. If a number 4^n, for any natural number n ends with digit 0, then it is divisible by 5. The prime factorization of 4^n must contain the prime factor 5. Fundamental theorem of arithmetic guarantees that there are no other prime in factorisation of 4^n. Hence 4^n can never end with the digit zero for $n \in N$.
Q.14	If $\sin(A+B) = \frac{\sqrt{3}}{2}$, $\cos(A-B) = \frac{\sqrt{3}}{2}$, $0 < A+B \le 90°$, A >B, find A and B. Ans. $\sin(A+B) = \frac{\sqrt{3}}{2} \Rightarrow A+B = 60°$ $\cos(A-B) = \frac{\sqrt{3}}{2}$ $A-B=30°$ solving to get $A=45°, B=15°$
	SECTION C
Q.15	Write the denominator of $\frac{91}{1250}$ in the form of $2^m 5^n$, where m, n are non-negative integers. Also write its decimal expansion without actual division. Ans. $1250 = 2^1 \times 5^4$ $\therefore \frac{91}{1250} = \frac{91 \times 2^3}{2^1 \times 5^4 \times 2^3} = \frac{728}{2^4 \times 5^4} = \frac{728}{(2\times5)^4}$ $=0.0728$ OR Use Euclid division lemma to show that cube of any positive integer is either of the form 9 m, 9m + 1, or 9m + 8 Ans. By Euclid division lemma $a = bq + r, 0 \le r < b$ ____ (1) putting $b=3$ in (1) we have $a = 3q + r, 0 \le r < 3$ when $r=0$ $a = 3q$ $a^3 = 9(3q^3) = 9m$ when $r=1$ $a = 3q+1$ $a^3 = 27q^3 + 27q^2 + 9q + 1 = 9(m) + 1$ When $r=2$ $a = 3q+2$ $a^3 = 27q^3 + 54q^2 + 18q + 8 = 9m + 8$ Hence cube of any positive integer is either of the form 9 m, 9m+1 or 9 m+8
Q.16	If $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ and $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1$,

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	prove that $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$. **Ans.**
	$\frac{x^2}{a^2}\cos^2\theta+\frac{y^2}{b^2}\sin^2\theta+\frac{2xy}{ab}\sin\theta\cos\theta=1$
	$\frac{x^2}{a^2}\sin^2\theta+\frac{y^2}{b^2}\cos^2\theta-\frac{2xy}{ab}\sin\theta\cos\theta=1$
	(1) + (2)
	$\frac{x^2}{a^2}(\cos^2\theta+\sin^2\theta)+\frac{y^2}{b^2}(\sin^2\theta+\cos^2\theta)=2$
	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$
Q.17	Solve the following system of linear equations graphically : $2(x-1)=y$ and $x+3y=15$ .Also find the coordinates of points where lines meet the y-axis. **Ans.** Correct graph of each lines
	Solution (3, 4)
	Pt where it meet y-axis on graph $(0,\ -2)$ $(0, 5)$
Q.18	Show that $\frac{\sin\theta-2\sin^3\theta}{2\cos^2\theta-\cos\theta}=\tan\theta$.
	Ans. LHS $=\frac{\sin\theta\ (1-2\sin^2\theta)}{\cos\theta\ (2\cos^2\theta-1)}$
	$=\frac{\sin\theta\ (1-2\sin^2\theta)}{\cos\theta\ [2(1-\sin^2\theta)-1]}$

	$=\frac{\sin\theta\ (1-2\sin^2\theta)}{\cos\theta\ (2-2\sin^2\theta-1)}$
	$=\frac{\sin\theta\ (1-2\sin^2\theta)}{\cos\theta\ (1-2\sin^2\theta)}$
	$=\tan\theta=\text{RHS}$
	OR
	If $\tan A=2$. Evaluate $\sec A\sin A+\tan^2 A-\operatorname{cosec} A$. **Ans.**
	hypotenuse $=\sqrt{5}$
	$\sec A=\sqrt{5},\ \sin A=\frac{2}{\sqrt{5}},\ \operatorname{cosec} A=\frac{\sqrt{5}}{2}$
	$\left(\sqrt{5}\times\frac{2}{\sqrt{5}}\right)+(2)^2-\frac{\sqrt{5}}{2}$
	$2+4-\frac{\sqrt{5}}{2}\Rightarrow\frac{12-\sqrt{5}}{2}$
Q.19	Change the given distribution to more than type distribution and draw its ogive.

Classes	20-25	25-30	30-35	35-40	40-45	45-50
Frequency	3	6	12	9	6	4

Ans. More than type distribution :

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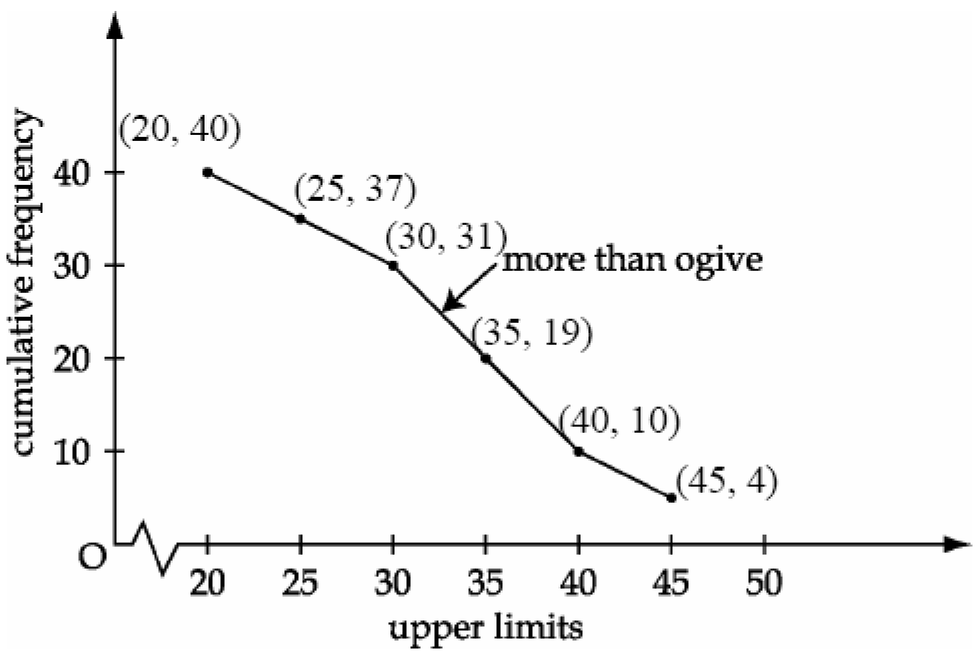
	c.f.
More than 20	40
More than 25	37
More than 30	31
More than 35	19
More than 40	10
More than 45	4



Q.20 Solve for u and v by changing into linear equations

$$2(3u - v) = 5uv;\ 2(u + 3v) = 5uv$$

Ans. $\frac{6u}{uv} - \frac{2v}{uv} = \frac{5uv}{uv}$

$\frac{2u}{uv} + \frac{6v}{uv} = \frac{5uv}{uv}$

$\frac{6}{v} - \frac{2}{u} = 5$; $\frac{2}{v} + \frac{6}{u} = 5$

Let $\frac{1}{v} = a$ and $\frac{1}{\mu} = b$

$6a - 2b = 5$

$2a + 6b = 5 \times 3$

$6a + 18b = 15$

$b = \frac{1}{2}$ $a = 1$

$v = 1$ $u = 2$

OR

Solve for x and y: $\frac{5}{x-1} + \frac{1}{y-2} = 2$ & $\frac{6}{x-1} - \frac{3}{y-2} = 1$.

Ans x = 4, y =5 .

Q.21 The given distribution shows the number of runs scored by some top batsmen of the world in one - day international cricket matches. Find the mode of the data.

Runs Scored	3000-4000	4000-5000	5000-6000	6000-7000
No. of batsmen	4	18	9	7
Runs Scored	7000-8000	8000-9000	9000-10000	10000-11000
No. of batsmen	6	3	1	1

Ans. Modal class = 4000 – 5000

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$l = 4000, f_1 = 18, f_0 = 4, f_2 = 9$

$h = 1000.$

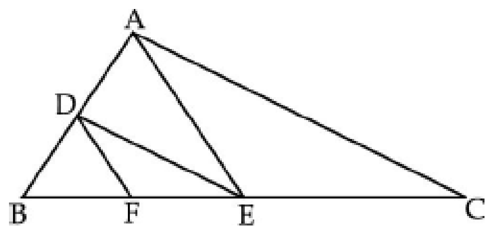
$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 4000 + \frac{18-4}{36-13} \times 1000$$

$$= 4000 + 608.7$$

$$= 4608.7$$

Q.22 In given figure DE||AC, DF||AE. If the lengths of BF and FE in centimeters and 4 and 5 respectively, then find the length of EC.



ANS:

In ΔABC, DE||AC $\Rightarrow \frac{BD}{DA} = \frac{BE}{EC}$ (1)

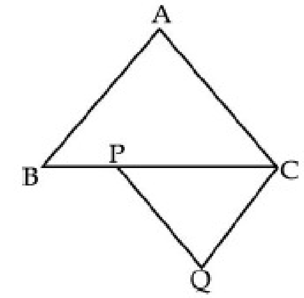
In ΔABE, DF||AE $\Rightarrow \frac{BD}{DA} = \frac{BF}{FE}$ (2)

From (1), (2) $\frac{BE}{EC} = \frac{BF}{FE}$

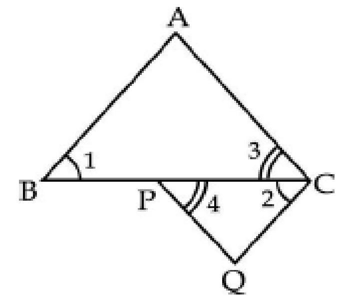
$$\frac{9}{EC} = \frac{4}{5}$$

$$EC = \frac{9 \times 5}{4} = 11.25 \text{ cm}$$

Q.23 n the given figure, AB || CQ and AC || PQ. If BP $= \frac{1}{3}$BC, find the ratio of the areas of ΔABC and ΔQCP.



ANS:



AB||CQ, BC is transversal

$\Rightarrow \angle 1 = \angle 2$

Also, AC||PQ, BC is transversal

$\Rightarrow \angle 3 = \angle 4$

$\therefore \Delta ABC \sim \Delta QCP$ [AA similarity]

$$\Rightarrow \frac{ar(ABC)}{ar(QCP)} = \frac{BC^2}{CP^2}$$

$$= \frac{(BC)^2}{\left(\frac{2}{3}BC\right)^2} \qquad \left[\text{as } BP = \frac{1}{3}BC \Rightarrow CP = \frac{2}{3}BC\right]$$

$$= \frac{9\,BC^2}{4\,BC^2}$$

$\therefore$ the required ratio is 9 : 4

OR

In figure, PQR and SQR are two triangles on the same base QR. If PS

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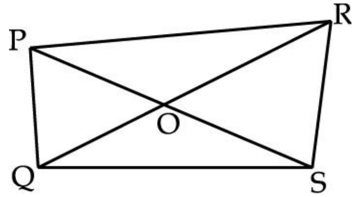
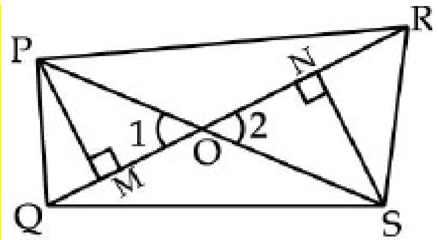
intersects QR at O, show that $\frac{ar(\Delta PQR)}{ar(\Delta SQR)} = \frac{PO}{SO}$



Ans.



5

Draw PM ⊥ QR, SN ⊥ QR

In ΔPOM, ΔSON

$\angle M = \angle N = 90°$

$\angle 1 = \angle 2$ [Vertically opposite angles]

$\therefore$ $\Delta POM \sim \Delta SON$ [AA similarity]

$\therefore$ $\frac{PM}{SN} = \frac{PO}{SO}$ (1)

Now $\frac{ar(PQR)}{ar(SQR)} = \frac{\frac{1}{2} \times QR \times PM}{\frac{1}{2} \times QR \times SN}$

$= \frac{PN}{SN}$ (2)

From (1), (2) $\frac{ar(PQR)}{ar(SQR)} = \frac{PO}{SO}$

Hence Proved

Q.24 If the zeros of the polynomial $f(x) = x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find a & b. Ans. $a = 1, b = \pm\sqrt{2}$

SECTION D

Q.25 Obtain all other zeroes of $2x^4 - 6x^3 + 3x^2 + 3x - 2$, if two of it's zeroes are $1/\sqrt{2}$ and $-1/\sqrt{2}$. Ans.

6

Two zeroes are $\frac{1}{\sqrt{2}}$ and $\frac{-1}{\sqrt{2}}$

One factor is $\left(x - \frac{1}{\sqrt{2}}\right)\left(x + \frac{1}{\sqrt{2}}\right)$

i.e. $2x^2 - 1$

$(2x^4 - 6x^3 + 3x^2 + 3x - 2) \div (2x^2 - 1)$

$= (x^2 - 3x + 2)$

Another factor is $x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x = +1$ and $+2$

Hence other zeroes are 1 and 2.

Q.26 If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, then the sides are divided in the same ratio. (Prove it)

Q.27 If two sides and a median bisecting third side of a triangle are respectively proportional to the corresponding sides and the median of another triangle, then prove that the two triangles are similar.

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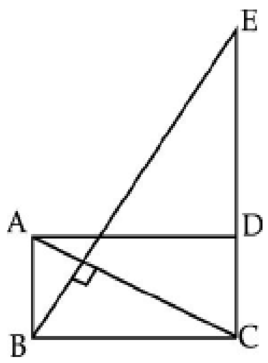
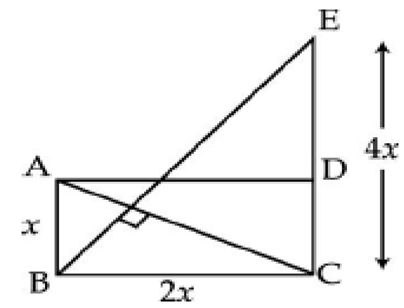
	OR In given figure ABCD is a rectangle, in which BC = 2AB. A point E lies CD produced such that CE = 2BC. Find AC : BE.  **SOL:**



$$\frac{AB}{BC} = \frac{x}{2x} = \frac{1}{2}$$

$$\frac{BC}{CE} = \frac{y}{2y} = \frac{1}{2} \qquad \text{where } y = 2x$$

In ΔABC and ΔBCE

$$\frac{AB}{BC} = \frac{BC}{CE} = \frac{1}{2}$$

and $\angle B = \angle C = 90°$

$\therefore \quad \Delta ABC \sim \Delta BCE$ [By SAS similarity]

$$\therefore \quad \frac{AB}{BC} = \frac{BC}{CE} = \frac{AC}{BE}$$

$$\Rightarrow \quad \frac{AC}{BE} = \frac{1}{2} \quad \text{or} \quad AC : BE = 1 : 2$$

Q.28	8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days.8 Find the time taken by one man alone and that by one boy alone to finish that work . Ans. One man alone can finish the work in 140 days and one boy alone can finish the work in 280 days.
Q.29	If the remainder on division of $x^3 + 2x^2 + kx + 3$ by x - 3 is 21,find the quotient and the value of *k*. Hence, find the zeroes of the cubic polynomial . $x^3 + 2x^2 + kx - 18$. ANS K = - 9 AND ZEROS 3 -2 , -

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	3 .
Q.30	In a morning walk three persons step off together, their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that they can cover the distance in complete steps? **Sol.** Required minimum distance each should walk so, that they can cover the distance in complete step is the L.C.M. of 80 cm, 85 cm and 90 cm $80 = 2^4 \times 5$ $85 = 5 + 17$ $90 = 2 \times 3^2 \times 5$ $\therefore$ LCM = $2^4 \times 3^2 \times 5^1 \times 17^1$ LCM = $16 \times 9 \times 5 \times 17$ LCM = 12240 cm, = 122 m 40 cm.
Q.31	Prove that: $\sec^2\theta - \dfrac{\sin^2\theta - 2\sin^4\theta}{2\cos^4\theta - \cos^2\theta} = 1$

$$\text{LHS} = \frac{1}{\cos^2\theta} - \frac{\sin^2\theta\,(1-2\sin^2\theta)}{\cos^2\theta\,(2\cos^2\theta-1)}$$

$$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta\,(1-2\sin^2\theta)}{\cos^2\theta\,(2(1-\sin^2\theta)-1)}$$

$$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta\,(1-2\sin^2\theta)}{\cos^2\theta\,(1-2\sin^2\theta)}$$

$$= \frac{1-\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\cos^2\theta} = 1 = \text{RHS.}$$

Q.32	Prove that: $(1+\tan A\tan B)^2 + (\tan A - \tan B)^2 = \sec^2 A.\sec^2 B$ .ANS: LHS $(1+\tan A\tan B)^2 + (\tan A - \tan B)^2$ $\Rightarrow 1 + \tan^2 A\tan^2 B + 2\tan A\tan B + \tan^2 A + \tan^2 B - 2\tan A\tan B$ $\Rightarrow 1 + \tan^2 A\tan^2 B + \tan^2 A + \tan^2 B$ $\Rightarrow \sec^2 A + \tan^2 B(1+\tan^2 A)$ $\Rightarrow \sec^2 A + (\sec^2 B - 1)\sec^2 A$ $\Rightarrow \sec^2 A + \sec^2 A\sec^2 B - \sec^2 A$ $\Rightarrow \sec^2 A\sec^2 B$ RHS **OR** Prove that:

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$$\frac{\sin A}{\sec A+\tan A-1}+\frac{\cos A}{\cos ecA+\cot A-1}=1$$ .ANS:

$$LHS=\frac{\sin A}{\sec A+\tan A-1}+\frac{\cos A}{\operatorname{cosec} A+\cot A-1}$$

$$=\frac{\sin A\cos A}{1+\sin A-\cos A}+\frac{\sin A\cos A}{1+\cos A-\sin A}$$

$$=\frac{2\sin A\cos A}{[1+(\sin A-\cos A)][1-(\sin A-\cos A)]}$$

$$=\frac{2\sin A\cos A}{2\sin A\cos A}$$

$$=1=RHS$$

Q.33 The median of the distribution given below is 14.4. Find the values of x and y, If the sum of frequency is 20.

C.I.	0-6	6-12	12-18	18-24	24-30
F	4	X	5	Y	1

ANS

CI	f	cf
writing		
0 - 6	4	4
6 - 12	x	$4+x$
12 - 18	5	$9+x$
18 - 24	y	$9+x+y$
24 - 30	1	$10+x+y$

formula

$$\text{median}=l+\left[\frac{\frac{n}{2}-cf}{f}\right]\times h$$

$l=12, f=5, cf=4+x, h=6$ & $x+y=10$

to find $x=4$

& $y=6$

Q.34 The mean of the following distribution is 62.8 and the sum of all frequencies is 50. Computer the missing frequencies f_1 and f_2 .

Classes	0-20	20-40	40-60	60-80	80-100	100-120	Total
Frequency	5	f_1	10	f_2	7	8	50

Ans. We have

$$5+f_1+10+f_2+7+8=50$$

$$f_1+f_2=20$$

$$\therefore f_2=20-f1$$

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C.I	f_i	x_i	$f_i x_i$
$0-20$	5	10	50
$20-40$	f_1	30	$f_i x_i$
$40-60$	10	50	$30 f_1$
$60-80$	$20-f_1$	70	$1400-70 f_1$
$80-100$	7	90	630
$100-120$	8	110	880
	$\Sigma fi = 50$		$\Sigma f_i x_i = 3460 - 40 f_1$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$62.8 = \frac{3460 - 40 f_1}{50}$$

$$f_1 = 8, \ f_2 = 12$$

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USE SOFT WORDS AND HARD ARGUMENTS.

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