CBSE ANNUAL EXAMINATION
[Solutions With Detailed Explanations]

SECTION – A (Each question carries 1 Mark)

Q01. Find the distance of the plane $3x-4y+12z=3$ from the origin.

Sol. As the distance of plane $ax+by+cz+d=0$ from a point (x_1, y_1, z_1) is $\dfrac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}\,units$.

So, distance of origin (0, 0, 0) from the given plane $3x-4y+12z=3$ is

$$=\frac{|3(0)-4(0)+12(0)-3|}{\sqrt{(3)^2+(-4)^2+(12)^2}}=\frac{3}{\sqrt{169}}\,units=\frac{3}{13}\,units.$$

Q02. Find the scalar components of the vector $\overrightarrow{AB}$ with initial point A(2, 1) and terminal point B(–5, 7).

Sol. As $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=(-5\hat{i}+7\hat{j})-(2\hat{i}+\hat{j})$

$\Rightarrow \overrightarrow{AB}=-7\hat{i}+6\hat{j}$.

So, the scalar components of $\overrightarrow{AB}$ are –7, 6.

Q03. Find the principal value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$.

Sol. As range of $\tan^{-1}x$ is $\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$ and that of $\sec^{-1}x$ is $\left[0,\pi\right]-\left\{\dfrac{\pi}{2}\right\}$.

So, $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)=\tan^{-1}\left(\tan\dfrac{\pi}{3}\right)-\sec^{-1}\left(\sec\dfrac{2\pi}{3}\right)$

$\therefore \qquad =\dfrac{\pi}{3}-\dfrac{2\pi}{3}=-\dfrac{\pi}{3}$.

Q04. Given $\int e^x(\tan x+1)\sec x\,dx=e^x f(x)+c$. Write $f(x)$ satisfying this.

Sol. Let $I=\int e^x(\tan x+1)\sec x\,dx=e^x f(x)+c$

$\Rightarrow e^x f(x)+c=\int e^x(\tan x+1)\sec x\,dx$

$\Rightarrow \qquad =\int e^x(\sec x+\sec x\tan x)\,dx \qquad \Rightarrow \qquad =\int e^x\sec x\,dx+\int e^x\sec x\tan x\,dx$

$\Rightarrow \qquad =\sec x\int e^x dx-\int\left[\dfrac{d}{dx}\sec x\int e^x dx\right]dx+\int e^x\sec x\tan x\,dx$

(On applying **By parts** in first integral)

$\Rightarrow \qquad =e^x\sec x-\int e^x\sec x\tan x dx+\int e^x\sec x\tan x\,dx$

$\Rightarrow e^x f(x)+c=e^x\sec x+c$

On comparing both the sides, we get

$\therefore \quad f(x)=\sec x$.

Q05. Evaluate: $\displaystyle\int_0^2\sqrt{4-x^2}\,dx$.

Sol. Let $I=\displaystyle\int_0^2\sqrt{4-x^2}\,dx \qquad \Rightarrow I=\int_0^2\sqrt{(2)^2-x^2}\,dx=\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$\Rightarrow I=\left[\dfrac{2}{2}\times 0+2\sin^{-1}\left(\dfrac{2}{2}\right)\right]-\left[0+2\sin^{-1}(0)\right]=2\sin^{-1}\left(\sin\dfrac{\pi}{2}\right)=2\times\dfrac{\pi}{2}$

$\therefore \quad I=\pi$.

Q06. Let A be a square matrix of order 3×3. Write the value of |2A|, where $|A|=4$.

Sol. As $|kA|=k^n|A|$, where n is the order of matrix A and k is any non-zero scalar.

So, $|2A| = 2^3|A| = (8)(4) = 32$.

Q07. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A^T - B^T$.

Sol. As $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \Rightarrow B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

So, $A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$.

Q08. The binary operation $* : R \times R \to R$ is defined as $a*b = 2a + b$. Find $(2*3)*4$.

Sol. Since $* : R \times R \to R$ is defined as $a*b = 2a + b$.

So, $2*3 = 2(2) + 3 = 7$.

Then, $(2*3)*4 = 7*4 = 2(7) + 4 = 18$.

Q09. Write the value of $(\hat{k} \times \hat{i}).\hat{j} + \hat{i}.\hat{k}$.

Sol. Since $\hat{k} \times \hat{i} = \hat{j}$ and $i.\hat{k} = 0$

So, $(\hat{k} \times \hat{i}).\hat{j} + \hat{i}.\hat{k} = \hat{j}.\hat{j} + 0$

$\therefore \quad = 1 + 0 = 1$.

Q10. Find the value of $x + y$ from the following equation:

$$2\begin{pmatrix} 1 & 3 \\ 0 & x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}.$$

Sol. $2\begin{pmatrix} 1 & 3 \\ 0 & x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & 6 \\ 0 & 2x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 2+y & 6 \\ 0+1 & 2x+2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix} \Rightarrow \begin{pmatrix} 2+y & 6 \\ 1 & 2x+2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$

By equality of matrices, we have $2 + y = 5,\ 2x + 2 = 8 \quad \Rightarrow y = 3,\ x = 3$.

So, $x + y = 3 + 3 = 6$.

SECTION – B (Each question carries 4 Marks)

Q11. Using properties of determinants, show that: $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$.

Sol. Consider **LHS** and, let $\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

$= \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ [Applying $R_1 \to R_1 - (R_2 + R_3)$

$= 2\begin{vmatrix} 0 & -c & -b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ [Taking 2 common from R_1

$$= 2\begin{vmatrix} 0 & -c & -b \\ b & a & 0 \\ c & 0 & a \end{vmatrix}$$ [Applying $R_3 \to R_3 + R_1$ and $R_2 \to R_2 + R_1$

$$= 2\left\{0\begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix} - (-c)\begin{vmatrix} b & 0 \\ c & a \end{vmatrix} - b\begin{vmatrix} b & a \\ c & 0 \end{vmatrix}\right\}$$ [Expanding along R_1

$$= 2\{c(ab-0) - b(0-ac)\} = 2(2abc)$$

$= 4abc =$ **RHS**. [Hence Proved.

Q12. Evaluate: $\int_{-1}^{2} |x^3 - x|\,dx$. **OR** Evaluate: $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$.

Sol. Let $I = \int_{-1}^{2} |x^3 - x|\,dx$

$\Rightarrow \quad = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx$, where $f(x) = |x^3 - x|$

Now, $f(x) = \begin{cases} (x^3 - x), \ if \ -1 < x < 0 \\ -(x^3 - x), \ if \ 0 < x < 1 \\ (x^3 - x), \ if \ 1 < x < 2 \end{cases}$.

So, $I = \int_{-1}^{0} (x^3 - x)\,dx + \int_{0}^{1} (x - x^3)\,dx + \int_{1}^{2} (x^3 - x)\,dx$

$\Rightarrow \quad = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$

$\Rightarrow \quad = \left\{[0-0] - \left[\frac{1}{4} - \frac{1}{2}\right]\right\} + \left\{\left[\frac{1}{2} - \frac{1}{4}\right] - [0-0]\right\} + \left\{\left[\frac{16}{4} - \frac{4}{2}\right] - \left[\frac{1}{4} - \frac{1}{2}\right]\right\}$

$\Rightarrow \quad = \frac{1}{4} + \frac{1}{4} + 2 + \frac{1}{4} = \frac{11}{4}$.

OR

Let $I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx \quad ...(i)$

$= \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}dx$ [By using $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}dx \quad ...(ii)$

On adding equations (i) and (ii), we get

$2I = \int_0^{\pi} \frac{(x+\pi-x)\sin x}{1+\cos^2 x}dx \quad \Rightarrow I = \frac{\pi}{2}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$

Let $f(x) = \frac{\sin x}{1+\cos^2 x} \Rightarrow f(\pi - x) = \frac{\sin(\pi - x)}{1+\cos^2(\pi - x)} = \frac{\sin x}{1+\cos^2 x}$

i.e., $f(\pi - x) = f(x)$. So by using, $\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx, \ if \ f(2a-x) = f(x)$, we get

$I = \left(\frac{\pi}{2}\right) \times 2\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx = \pi\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx$

Put $\cos x = t \Rightarrow \sin x dx = -dt$. Also, when $x = 0 \Rightarrow t = 1$ and, when $x = \pi/2 \Rightarrow t = 0$.

So, $\mathrm{I}=\pi\int_1^0 \frac{-dt}{1+t^2}=\pi\int_0^1 \frac{dt}{1+t^2}$ $\quad\Rightarrow \mathrm{I}=\pi\left[\tan^{-1}t\right]_0^1=\pi\left[\tan^{-1}(1)-\tan^{-1}(0)\right]$

$\therefore\ \mathrm{I}=\pi\left(\frac{\pi}{4}-0\right)=\frac{\pi^2}{4}$ or, $\left(\frac{\pi}{2}\right)^2$.

Q13. A ladder $5m$ long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of $2cm/s$. How fast is its height on the wall decreasing when the foot of the ladder is $4m$ away from the wall?

Sol. Let y m be the height of the wall at which the ladder touches. Also, let the foot of the ladder be x m away from the wall. Then by Pythagoras theorem, we have

$$x^2+y^2=5^2 \Rightarrow y=\sqrt{25-x^2}$$

Then, the rate of change of height (i.e., y) with respect to time t is given by,

$$\frac{dy}{dt}=-\frac{x}{\sqrt{25-x^2}}\times\frac{dx}{dt}=-\frac{2x}{\sqrt{25-x^2}} \qquad \left[\text{As it is given that } \frac{dx}{dt}=2cm/s\right.$$

Now, when $x=4\,m$, we have : $\quad \frac{dy}{dt}=-\frac{2\times4}{\sqrt{25-4^2}}=-\frac{8}{3}\,cm/s.$

Hence, the height of the ladder on the wall is decreasing at the rate of $\frac{8}{3}\,cm/s$.

Q14. Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k},\ \vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p}.\vec{c}=18$.

Sol. Let $\vec{p}=x\hat{i}+y\hat{j}+z\hat{k}$. Since $\vec{p}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, so $\vec{p}.\vec{a}=0$ and $\vec{p}.\vec{b}=0$.

That means, $\vec{p}.\vec{a}=(x\hat{i}+y\hat{j}+z\hat{k}).(\hat{i}+4\hat{j}+2\hat{k})=0\Rightarrow x+4y+2z=0$ ...(*i*)

and, $\vec{p}.\vec{b}=(x\hat{i}+y\hat{j}+z\hat{k}).(3\hat{i}-2\hat{j}+7\hat{k})=0\Rightarrow 3x-2y+7z=0$ ...(*ii*)

Also, we have, $\vec{p}.\vec{c}=18\Rightarrow(x\hat{i}+y\hat{j}+z\hat{k}).(2\hat{i}-\hat{j}+4\hat{k})=18=0\Rightarrow 2x-y+4z=18$ ...(*iii*)

Solving (i) and (ii) by using Cross-multiplication, we have

$$\frac{x}{28+4}=\frac{y}{6-7}=\frac{z}{-2-12}=\lambda \qquad \Rightarrow x=32\lambda, y=-\lambda, z=-14\lambda.$$

Substituting these values in (iii), we get $2(32\lambda)-(-\lambda)+4(-14\lambda)=18\Rightarrow \lambda=2$.

So, $x=64, y=-2, z=-28$.

Hence, the required vector $\vec{p}$ is, $\vec{p}=64\hat{i}-2\hat{j}-28\hat{k}$.

Q15. If $x=\sqrt{a^{\sin^{-1}t}}$ and $y=\sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx}=-\frac{y}{x}$.

OR Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ with respect to x.

Sol. We have $x=\sqrt{a^{\sin^{-1}t}}$

Taking logarithm on both the sides, we get

$$\log x=\log\sqrt{a^{\sin^{-1}t}}$$

$$\Rightarrow \log x=\left(\frac{\log a}{2}\right)\sin^{-1}t$$

On differentiating w.r.t. t both the sides,

$$\frac{1}{x}\frac{dx}{dt}=\left(\frac{\log a}{2}\right)\frac{1}{\sqrt{1-t^2}}$$

$$\Rightarrow \frac{dx}{dt}=\left(\frac{\log a}{2}\right)\frac{x}{\sqrt{1-t^2}}$$

And, $y=\sqrt{a^{\cos^{-1}t}}$

Taking logarithm on both the sides, we get

$$\log y=\log\sqrt{a^{\cos^{-1}t}}$$

$$\Rightarrow \log y=\left(\frac{\log a}{2}\right)\cos^{-1}t$$

On differentiating w.r.t. t both the sides,

$$\frac{1}{y}\frac{dy}{dt}=\left(\frac{\log a}{2}\right)\left(-\frac{1}{\sqrt{1-t^2}}\right)$$

$$\Rightarrow \frac{dy}{dt}=\left(\frac{\log a}{2}\right)\left(-\frac{y}{\sqrt{1-t^2}}\right).$$

So, $\frac{dy}{dx}=\frac{dy}{dt}.\frac{dt}{dx}=\left(\frac{\log a}{2}\right)\left(-\frac{y}{\sqrt{1-t^2}}\right)\left(\frac{2}{\log a}\right)\frac{\sqrt{1-t^2}}{x}=-\frac{y}{x}$

$\therefore \quad \frac{dy}{dx}=-\frac{y}{x}$. [Hence Proved.

OR

Let $y=\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$

Put $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$ ...(*i*)

So, $y=\tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]=\tan^{-1}\left[\frac{\sec\theta-1}{\tan\theta}\right]$ $\Rightarrow y=\tan^{-1}\left[\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right]$

$\Rightarrow y=\tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$ $\Rightarrow y=\tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right]=\tan^{-1}\left(\tan\frac{\theta}{2}\right)=\frac{\theta}{2}$

$\Rightarrow y=\frac{1}{2}(\tan^{-1}x)$ [By (i)

On differentiating with respect to *x*, we have: $\frac{dy}{dx}=\frac{1}{2}\times\frac{1}{1+x^2}$

$\therefore \quad \frac{dy}{dx}=\frac{1}{2(1+x^2)}$.

Q16. Show that $f:\mathrm{N}\to\mathrm{N}$, given by $f(x)=\begin{cases}x+1, \text{ if } x \text{ is odd}\\ x-1, \text{ if } x \text{ is even}\end{cases}$ is both one-one and onto.

OR Consider the binary operations *: R × R→R and o: R × R→R defined as $a*b=|a-b|$ and $a\ o\ b=a$ for all $a, b\in\mathrm{R}$. Show that '*' is commutative but not associative, '*o*' is associative but not commutative.

Sol. Suppose $f(x_1)=f(x_2)$. If x_1 is odd and x_2 is even, then we will have $x_1+1=x_2-1$, i.e., $x_2-x_1=2$ which is impossible. Similarly, the possibility of x_1 being even and x_2 being odd is ruled out, using the same argument. Therefore, both x_1 and x_2 must be either odd or even. Suppose both x_1 and x_2 are odd. Then,

$$f(x_1)=f(x_2)\Rightarrow x_1+1=x_2+1\Rightarrow x_1=x_2.$$

Similarly if both x_1 and x_2 are even. Then,

$$f(x_1)=f(x_2)\Rightarrow x_1-1=x_2-1\Rightarrow x_1=x_2.$$

Thus, f is one-one.

Also, any odd number $2r+1$ in the co-domain N is the image of $2r+2$ in the domain N and any even number $2r$ in the co-domain N is the image of $2r-1$ in the domain N. Thus, f is onto.

OR

It is given that *: R ×R → R and *o*: R × R → R is defined as $a*b=|a-b|$ and $a\ o\ b=a$ for all $a,b\in\mathrm{R}$. For $a,b\in\mathrm{R}$, we have: $a*b=|a-b|\Rightarrow b*a=|b-a|=|-(a-b)|=|a-b|$.

So, $a*b=b*a$. Thus, the operation * is commutative.

It can be observed that,
(1*2)*3 = (|1–2|)*3 = 1*3 = |1–3| = 2. Also, 1*(2*3) = 1*(|2–3|) = 1*1 = |1–1| = 0.
$\therefore$ (1*2)*3 ≠ 1*(2*3) where 1, 2, 3 $\in$ R. Thus, the operation * is not associative.

Now, consider the operation o:
It can be observed that $1\ o\ 2 = 1$ and $2\ o\ 1 = 2$.
$\therefore\ 1\ o\ 2 \neq 2\ o\ 1$ where $1, 2 \in \mathrm{R}$.
$\therefore$ The operation o is not commutative.

Let $a, b, c \in \mathrm{R}$. Then, we have: $(a\ o\ b)\ o\ c = a\ o\ c = a$. Also, $a\ o\ (b\ o\ c) = a\ o\ b = a$.
$\therefore\ (a\ o\ b)\ o\ c = a\ o\ (b\ o\ c)$
Thus, the operation o is associative.

Q17. Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

Sol. Let the number of red cards drawn be denoted by X which is a random variable. Clearly, X can take the values 0, 1 or 2.

$\therefore$ P (X = 0) = P (two non-red cards) $= \dfrac{{}^{26}C_2}{{}^{52}C_2} = \dfrac{25}{102}$.

P (X = 1) = P (one red card and one non-red cards) $= \dfrac{{}^{26}C_1 \times {}^{26}C_1}{{}^{52}C_2} = \dfrac{52}{102}$.

$\therefore$ P (X = 2) = P (two red cards) $= \dfrac{{}^{26}C_2}{{}^{52}C_2} = \dfrac{25}{102}$.

Therefore, mean of X = E(X) $= \sum_{i=1}^{n} X_i P(X_i) == 0 \times \dfrac{25}{102} + 1 \times \dfrac{52}{102} + 2 \times \dfrac{25}{102} = 1$

Also, $\mathrm{Var}(X) = \sum_{i=1}^{n} X_i^2 P(X_i) - \left[E(X)\right]^2$

$$\Rightarrow \quad = \left[0^2 \times \frac{25}{102} + 1^2 \times \frac{52}{102} + 2^2 \times \frac{25}{102}\right] - (1)^2 = \frac{76}{51} - 1$$

$\therefore\ \mathrm{Var}(X) = \dfrac{25}{51}$.

Q18. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.

OR Find the particular solution of the differential equation: $x(x^2-1)\dfrac{dy}{dx} = 1;\ y = 0$ when $x = 2$.

Sol. Let C denote the family of circles in the second quadrant and touching the coordinate axes. Let (–h, h) be the coordinate of the centre of any member of this family. It is clear that the radius will be h.

So, equation representing this family is: $(x+h)^2 + (y-h)^2 = h^2$ …(i)

i.e., $x^2 + y^2 + 2hx - 2hy + h^2 = 0$ …(ii)

On differentiating (ii) w.r.t. x, we get $2x + 2y\dfrac{dy}{dx} + 2h - 2h\dfrac{dy}{dx} = 0$

$$\Rightarrow x + y\frac{dy}{dx} = h\left(\frac{dy}{dx} - 1\right) \qquad \Rightarrow h = \frac{x + yy'}{y' - 1}.$$

Substituting the value of h in equation (i), we get

$$\left(x + \frac{x+yy'}{y'-1}\right)^2 + \left(y - \frac{x+yy'}{y'-1}\right)^2 = \left(\frac{x+yy'}{y'-1}\right)^2$$

$\Rightarrow \left[xy' - x + x + yy'\right]^2 + \left[yy' - y - x - yy'\right]^2 = \left[x + yy'\right]^2 \qquad \Rightarrow (x+y)^2(y')^2 + (x+y)^2 = (x+yy')^2$

$\therefore\ (x+y)^2\left[(y')^2 + 1\right] = \left[x + yy'\right]^2$

i.e., $\therefore\ (x+y)^2\left[\left(\dfrac{dy}{dx}\right)^2 + 1\right] = \left[x + y\dfrac{dy}{dx}\right]^2$

This is the required differential equation representing the given family of circles.

OR We have, $x(x^2-1)\dfrac{dy}{dx}=1$

$$\Rightarrow dy=\frac{dx}{x(x^2-1)} \qquad \Rightarrow \int dy=\int\frac{dx}{x(x-1)(x+1)} \qquad \ldots(i)$$

Consider, $\dfrac{1}{x(x-1)(x+1)}=\dfrac{A}{x}+\dfrac{B}{x-1}+\dfrac{C}{x+1}$

$\Rightarrow\ 1=(A+B+C)x^2+(B-C)x-A$.

On equating the coefficients of like terms, we get: $A+B+C=0,\ B-C=0,\ -A=1$.

On solving these equations, we have: $A=-1,\ B=\dfrac{1}{2},\ C=\dfrac{1}{2}$.

So by (i) we have: $\displaystyle\int dy=-\int\frac{1}{x}dx+\frac{1}{2}\int\frac{1}{x-1}dx+\frac{1}{2}\int\frac{1}{x+1}dx$

$$\Rightarrow\ y=-\log|x|+\frac{1}{2}\log|x-1|+\frac{1}{2}\log|x+1|+k$$

$$\Rightarrow y=\frac{1}{2}\log\left|\frac{x^2-1}{x^2}\right|+k \qquad \ldots(ii)$$

Now since $y=0,$ when $x=2$. So, $0=\dfrac{1}{2}\log\left|\dfrac{4-1}{4}\right|+k\Rightarrow k=\dfrac{1}{2}\log\left(\dfrac{4}{3}\right)$

Substituting the value of k in equation (ii), we get

$$y=\frac{1}{2}\log\left|\frac{x^2-1}{x^2}\right|+\frac{1}{2}\log\left(\frac{4}{3}\right)$$

$$\therefore\quad y=\frac{1}{2}\log\left|\frac{4(x^2-1)}{3x^2}\right|.$$

This is the required particular solution of the given differential equation.

Q19. If $x=a\left(\cos t+\log\tan\dfrac{t}{2}\right)$, $y=a\sin t$, find $\dfrac{d^2y}{dt^2}$ and $\dfrac{d^2y}{dx^2}$.

Sol. Given $x=a\left(\cos t+\log\tan\dfrac{t}{2}\right)$ [On differentiating with respect to t both the sides

$$\frac{dx}{dt}=a\left(-\sin t+\frac{1}{\tan\frac{t}{2}}\times\sec^2\left(\frac{t}{2}\right)\times\frac{1}{2}\right)=a\left(-\sin t+\frac{\cos\frac{t}{2}}{\sin\frac{t}{2}}\times\frac{1}{\cos^2\frac{t}{2}}\times\frac{1}{2}\right)=a\left(-\sin t+\frac{1}{\sin t}\right)$$

$$\Rightarrow\frac{dx}{dt}=a\cos t\cot t \qquad \ldots(i)$$

Also, $y=a\sin t$ [On differentiating with respect to t both the sides

$$\Rightarrow\frac{dy}{dt}=a\cos t \qquad \ldots(ii)$$

Again differentiating with respect to t both the sides, we have

$$\frac{d^2y}{dt^2}=\frac{d}{dt}(a\cos t)=a(-\sin t)$$

$$\therefore\ \frac{d^2y}{dt^2}=-a\sin t.$$

Now, by (i) and (ii), we have

$$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=(a\cos t)\times\frac{1}{a\cos t\cot t} \qquad \Rightarrow\frac{dy}{dx}=\tan t$$

On differentiating with respect to x both the sides

$$\frac{d^2y}{dx^2}=\frac{d}{dx}(\tan t)=(\sec^2 t)\frac{dt}{dx} \qquad \Rightarrow \frac{d^2y}{dx^2}=\sec^2 t\times\frac{1}{a\cos t\cot t}$$

$$\therefore \quad \frac{d^2y}{dx^2}=\frac{\sec^3 t\ \tan t}{a}.$$

Q20. Find the coordinates of the point where the line through the points (3,–4,–5) and (2,–3, 1) crosses the plane $3x+2y+z+14=0$.

Sol. The equation of the straight line passing through the points (3,–4,–5) and (2,–3, 1) is:

$$\frac{x-3}{2-3}=\frac{y-(-4)}{-3-(-4)}=\frac{z-(-5)}{1-(-5)} \quad \Rightarrow \frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}=\lambda\ (say)$$

The coordinates of any random point on this line is $\text{P}(3-\lambda,\ \lambda-4,\ 6\lambda-5)$.

Consider that the line intersects the given plane $3x+2y+z+14=0$ at $\text{P}(3-\lambda,\ \lambda-4,\ 6\lambda-5)$.

So, $3(3-\lambda)+2(\lambda-4)+(6\lambda-5)+14=0 \qquad \Rightarrow \lambda=-2$.

Thus, the required **point of intersection** is $\text{P}\big(3-(-2),\ (-2)-4,\ 6(-2)-5\big)$ i.e., $\text{P}(5,-6,-17)$.

Q21. Find the particular solution of the following differential equation:

$$x\frac{dy}{dx}-y+x\sin\left(\frac{y}{x}\right)=0, \text{ given that when } x=2,\ y=\pi.$$

Sol. We have $x\frac{dy}{dx}-y+x\sin\left(\frac{y}{x}\right)=0 \qquad \Rightarrow \frac{dy}{dx}=\frac{y}{x}-\sin\left(\frac{y}{x}\right)$ ...(i)

It is evident that the given differential equation is homogeneous.

So, put $y=vx \ \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ (On differentiating w.r.t. x both sides)

Substituting these in equation (i), we have

$$v+x\frac{dv}{dx}=\frac{vx}{x}-\sin\left(\frac{vx}{x}\right) \quad \Rightarrow v+x\frac{dv}{dx}=v-\sin v \quad \Rightarrow x\frac{dv}{dx}=-\sin v$$

$$\Rightarrow x\frac{dv}{dx}=-\sin v \qquad \Rightarrow \int \operatorname{cosec} v\, dv=-\int\frac{dx}{x}$$

$$\Rightarrow \log\left|\operatorname{cosec} v-\cot v\right|=-\log|x|+\log|k| \qquad \Rightarrow \log\left|\operatorname{cosec} v-\cot v\right|=\log\left|\frac{k}{x}\right|$$

$$\Rightarrow \operatorname{cosec}\left(\frac{y}{x}\right)-\cot\left(\frac{y}{x}\right)=\frac{k}{x} \qquad \Rightarrow k\sin\left(\frac{y}{x}\right)=x\left[1-\cos\left(\frac{y}{x}\right)\right]$$

It is given that when $x=2,\ y=\pi$.

So, $k\sin\left(\frac{\pi}{2}\right)=2\left[1-\cos\left(\frac{\pi}{2}\right)\right] \qquad \Rightarrow k(1)=2\,[1-0] \qquad \Rightarrow k=2$.

Thus, $x\left[1-\cos\left(\frac{y}{x}\right)\right]=2\sin\left(\frac{y}{x}\right)$.

This is the required particular solution of the given differential equation.

Q22. Prove that: $\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$.

Sol. Let $\sin^{-1}\left(\frac{3}{5}\right)=x \Rightarrow \sin x=\frac{3}{5} \Rightarrow \tan x=\frac{3}{4} \Rightarrow x=\tan^{-1}\left(\frac{3}{4}\right)$.

$\therefore \ \sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{3}{4}\right)$ …(i)

Also, let $\cos^{-1}\left(\frac{12}{13}\right)=y \Rightarrow \cos y=\frac{12}{13} \Rightarrow \tan y=\frac{5}{12} \Rightarrow y=\tan^{-1}\left(\frac{5}{12}\right)$

$\therefore \cos^{-1}\left(\frac{12}{13}\right)=\tan^{-1}\left(\frac{5}{12}\right)$ ...(ii)

And, let $\sin^{-1}\left(\frac{56}{65}\right)=z \Rightarrow \sin z=\frac{56}{65} \Rightarrow \tan z=\frac{56}{33} \Rightarrow z=\tan^{-1}\left(\frac{56}{33}\right)$

$\therefore \sin^{-1}\left(\frac{56}{65}\right)=\tan^{-1}\left(\frac{56}{33}\right)$ ...(iii)

Now, we take **LHS**:

$$\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\left(\frac{3}{4}\right)$$

$$=\tan^{-1}\left[\frac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}.\frac{3}{4}}\right]=\tan^{-1}\left[\frac{20+36}{48-15}\right]$$

$$=\tan^{-1}\left(\frac{56}{33}\right)$$ [Using equation (iii)

$$=\sin^{-1}\left(\frac{56}{65}\right)=\textbf{RHS}.$$ [Hence Proved.

SECTION – C (Each question carries 6 Marks)

Q23. Evaluate: $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$. **OR** Evaluate: $\int \frac{x^2+1}{(x-1)^2(x+3)}dx$.

Sol. Let $I=\int \sin^{-1}x\left(\frac{x}{\sqrt{1-x^2}}\right)dx$

Integrating by parts taking $\sin^{-1}x$ as first function and $\frac{x}{\sqrt{1-x^2}}$ as second function.

$$I=\sin^{-1}x\int\left(\frac{x}{\sqrt{1-x^2}}\right)dx-\int\left[\frac{d}{dx}(\sin^{-1}x)\int\left(\frac{x}{\sqrt{1-x^2}}\right)dx\right]dx$$

$$\Rightarrow\ =-\frac{1}{2}\sin^{-1}x\int\left(\frac{-2x}{\sqrt{1-x^2}}\right)dx+\frac{1}{2}\int\left[\frac{1}{\sqrt{1-x^2}}\int\left(\frac{-2x}{\sqrt{1-x^2}}\right)dx\right]dx$$

$$\Rightarrow\ =-\frac{1}{2}\sin^{-1}x\left[2\sqrt{1-x^2}\right]+\frac{1}{2}\int\frac{1}{\sqrt{1-x^2}}\left[2\sqrt{1-x^2}\right]dx$$

$$\Rightarrow\ =-\sqrt{1-x^2}\sin^{-1}x+\int 1dx \qquad \Rightarrow I=-\sqrt{1-x^2}\sin^{-1}x+x+k.$$

OR Let $I=\int \frac{x^2+1}{(x-1)^2(x+3)}dx$ ...(i)

Consider $\frac{x^2+1}{(x-1)^2(x+3)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+3}$

$$\Rightarrow x^2+1=A(x-1)(x+3)+B(x+3)+C(x-1)^2.$$

On equating the coefficients of like terms, we obtain $A=\frac{3}{8}, B=\frac{1}{2}, C=\frac{5}{8}$.

So by (i), we have: $I=\frac{3}{8}\int\frac{1}{(x-1)}dx+\frac{1}{2}\int\frac{1}{(x-1)^2}dx+\frac{5}{8}\int\frac{1}{(x+3)}dx$

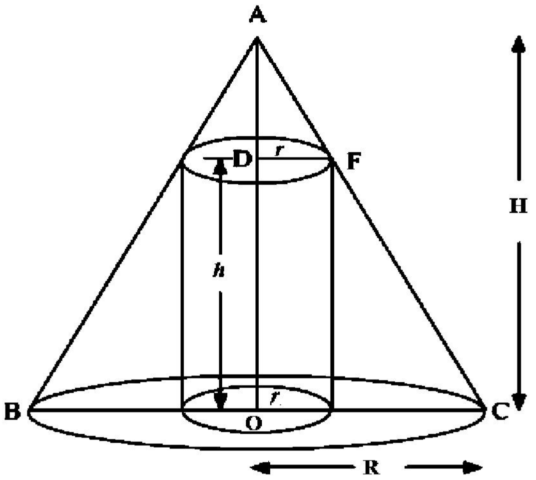
$$\Rightarrow\ I=\frac{3}{8}\log|x-1|+\frac{1}{2}\left(-\frac{1}{x-1}\right)+\frac{5}{8}\log|x+3|+k$$

$$\therefore\ \text{I} = \frac{3}{8}\log|x-1| + \frac{5}{8}\log|x+3| - \frac{1}{2(x-1)} + k.$$

Q24. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

OR An open box with a square base is to be made out of a given quantity of cardboard of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

Sol.



Let H and R be the height and radius of the base of the cone ABC respectively. Suppose the radius and height of the cylinder inscribed in the cone be r and h respectively

Now, DF = r. AD = H – h.
As ΔADF~ΔAOC

So, $\frac{\text{AD}}{\text{AO}} = \frac{\text{DF}}{\text{OC}} \Rightarrow \frac{\text{H}-h}{\text{H}} = \frac{r}{\text{R}}$

i.e., $h = \left(1 - \frac{r}{\text{R}}\right)\text{H}$.

Let S be the curved surface area of the cylinder.

So, $\text{S} = 2\pi rh \qquad \Rightarrow \text{S} = 2\pi r\left(1-\frac{r}{\text{R}}\right)\text{H} \qquad \Rightarrow \text{S} = 2\pi\text{H}\left(r - \frac{r^2}{\text{R}}\right)$

Now, differentiating with respect to r, we get: $\frac{d\text{S}}{dr} = 2\pi\text{H}\left(1 - \frac{2r}{\text{R}}\right)$

Again differentiating with respect to r, we get: $\frac{d^2\text{S}}{dr^2} = 2\pi\text{H}\left(-\frac{2}{\text{R}}\right) = -\frac{4\pi\text{H}}{\text{R}}$.

For the points of local maxima or minima, $\frac{d\text{S}}{dr} = 0$

i.e., $2\pi\text{H}\left(1-\frac{2r}{\text{R}}\right) = 0 \Rightarrow r = \frac{\text{R}}{2}$

Now, $\left.\frac{d^2\text{S}}{dr^2}\right]_{at\ r=\frac{\text{R}}{2}} = -\frac{4\pi\text{H}}{\text{R}} < 0$.

So, curved surface area S, of the cylinder is maximum at $r = \text{R}/2$.
Hence, the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

OR

Let the length, breadth and height of the open box be x, x and h units respectively.

$\therefore$ Area of cardboard used in the open box $= x^2 + 4xh \Rightarrow c^2 = x^2 + 4xh \qquad \Rightarrow h = \frac{c^2 - x^2}{4x}$.

Suppose V be the volume of the open box.

$\therefore\ \text{V} = x^2 h = x^2\left(\frac{c^2-x^2}{4x}\right) \qquad \Rightarrow \text{V} = \frac{1}{4}(c^2x - x^3)$

On differentiating w.r.t. x, we have: $\frac{d\text{V}}{dx} = \frac{1}{4}(c^2 - 3x^2)$

Again differentiating w.r.t. x, we have: $\frac{d^2\text{V}}{dx^2} = -\frac{3}{2}x$

For the points of local maxima or minima, $\frac{d\text{V}}{dx} = 0$

i.e., $\frac{1}{4}(c^2 - 3x^2) = 0$ $\Rightarrow x = \frac{c}{\sqrt{3}}$ [Rejecting $x = -\frac{c}{\sqrt{3}}$ as, x can't be negative.

Now, $\left.\frac{d^2V}{dx^2}\right]_{at\ x=\frac{c}{\sqrt{3}}} = -\left(\frac{3}{2}\right)\left(\frac{c}{\sqrt{3}}\right) < 0$

Hence, volume of the open box is maximum when $x = \frac{c}{\sqrt{3}}$ units.

Now, maximum volume of the open box is, $V = \frac{1}{4}\left(c^2\left(\frac{c}{\sqrt{3}}\right) - \left(\frac{c}{\sqrt{3}}\right)^3\right) = \frac{1}{4}\left(\frac{c^3}{\sqrt{3}} - \frac{c^3}{3\sqrt{3}}\right)$

i.e., $V = \frac{c^3}{4\sqrt{3}}\left(1 - \frac{1}{3}\right)$ $\Rightarrow V = \frac{c^3}{6\sqrt{3}}$ cubic units. [Hence Proved.

Q25. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3, or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3, or 4 with the die?

Sol. Let E_1 be the event that the outcome on the die is 5 or 6 and E_2 be the event that the outcome on the die is 1, 2, 3, or 4. So, $P(E_1) = 2/6 = 1/3$, $P(E_2) = 4/6 = 2/3$.
Let E be the event of getting exactly one head.
$P(E|E_1)$ = Probability of getting exactly one head by tossing the coin three times if she gets 5 or 6 = 3/8.
$P(E|E_2)$ = Probability of getting exactly one head in a single throw of coin if she gets 1, 2, 3, or 4 = 1/2.

Observe that the probability that the girl threw 1, 2, 3, or 4 with the die, if she obtained exactly one head, is given by $P(E_2|E)$.
Using Bayes' theorem, we have

$$P(E_2|E) = \frac{P(E_2)P(E|E_2)}{P(E_1)P(E|E_1) + P(E_2)P(E|E_2)}$$

$$\Rightarrow \quad = \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}} = \frac{8}{11}.$$

Q26. A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contains at last 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C while Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹5 per kg to purchase Food I and ₹7 per kg to purchase Food II. Determine the minimum cost of such a mixture. Formulate the above as a LPP and solve it graphically.

Sol. Let the dietician mix x kg of food I and y kg of food II to make the mixture.
To minimize, Z = ₹$(5x + 7y)$
Subject to the constraints:
$2x + y \geq 8$ …(i)
$x + 2y \geq 10$ …(ii)
and $x, y \geq 0$

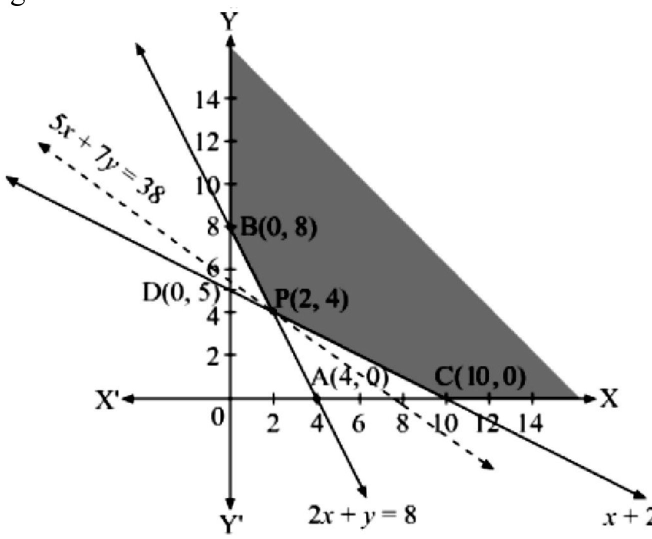
Considering the equations corresponding to the inequations (i) and (ii),

$2x + y = 8$

x	0	4
Y	8	0

$x + 2y = 10$

x	0	10
Y	5	0



Take the testing points as (0, 0) for (i), we have: $2(0)+(0) \geq 8 \Rightarrow 0 \geq 8$, which is false.
Take the testing points as (0, 0) for (ii), we have: $(0)+2(0) \geq 10 \Rightarrow 0 \geq 10$, which is false.
The shaded region as shown in the given figure is the feasible region, which is **unbounded**.
The coordinates of the corner points of the feasible region are B(0, 8), P(2, 4) and C(10, 0).

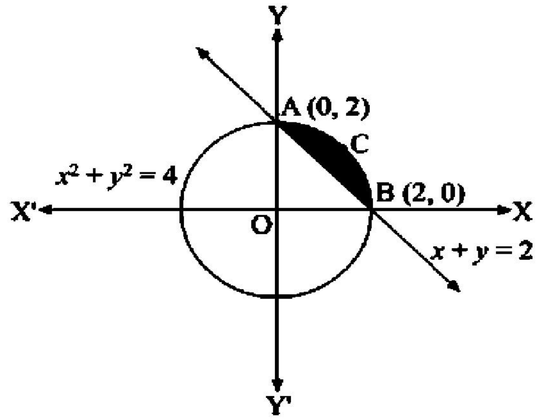
So, Value of Z at B(0, 8) = ₹(5× 0 + 7× 8) = ₹56
Value of Z at P(2, 4) = ₹(5× 2 + 7× 4) = ₹38
Value of Z at C(10, 0) = ₹(5× 10 + 7× 0) = ₹50
Thus, the minimum value of Z is ₹38.

Since the feasible region is unbounded, we need to verify whether Z = ₹38 is minimum value of given objective function or not.
For this, draw a graph of $5x + 7y < 38$.
We observe that the open half plane determined by $5x + 7y < 38$ has no points in common with the feasible region. So, Z is minimum for $x = 2$ and $y = 4$ and the minimum value of Z is ₹38.

Thus, the minimum cost of the mixture is ₹38 for 2kg of food I and 4kg of food II.

Q27. Find area of the region $\left\{(x,y): x^2+y^2 \le 4,\ x+y \ge 2\right\}$.

Sol.



We have $\left\{(x,y): x^2+y^2 \le 4,\ x+y \ge 2\right\}$.

Consider $x^2+y^2=4$ ...(i), $x+y=2$ ...(ii).

On solving curves (i) and (ii) simultaneously to get the point of intersection, we have

$$x^2+(2-x)^2=4 \Rightarrow 2x^2-4x=0 \Rightarrow 2x(x-2)=0$$
$$\therefore\ x=0,\ x=2 \Rightarrow y=2,\ y=0.$$

So, we have (0, 2) and (2, 0) as the point of intersections of given two curves.

$\therefore$ Required area = Area (OACBO) – Area (OABO)

$$\Rightarrow \quad = \int_0^2 y_c\,dx - \int_0^2 y_l\,dx$$

$$\Rightarrow \quad = \int_0^2 \sqrt{2^2-x^2}\,dx - \int_0^2 (2-x)\,dx \quad \Rightarrow \quad = \left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2 - \left[\frac{(2-x)^2}{2\times(-1)}\right]_0^2$$

$$\Rightarrow \quad = \left[\left(0+2\sin^{-1}\left(\frac{2}{2}\right)\right)-\left(0+2\sin^{-1}(0)\right)\right]+\frac{1}{2}\left[0-4\right]$$

$$\Rightarrow \quad = (\pi-2)\text{ sq.units}.$$

Q28. Find the coordinates of the foot of perpendicular and the length of the perpendicular drawn from the point P(5, 4, 2) to the line $\vec{r} = -\hat{i}+3\hat{j}+\hat{k}+\lambda(2\hat{i}+3\hat{j}-\hat{k})$. Also find the image of P in this line.

Sol. The given point is P(5, 4, 2) and the given line AB (*say*) is $\vec{r} = -\hat{i}+3\hat{j}+\hat{k}+\lambda(2\hat{i}+3\hat{j}-\hat{k})$.

Cartesian equation of line is: $\dfrac{x+1}{2}=\dfrac{y-3}{3}=\dfrac{z-1}{-1}=\lambda$.

So, the coordinates of any random point Q on this line is $Q(2\lambda-1,\ 3\lambda+3,\ 1-\lambda)$.

Let Q be the foot of the perpendicular on the given line for some value of λ.

Direction Ratios of PQ are $2\lambda-1-5,\ 3\lambda+3-4,\ 1-\lambda-2$ i.e., $2\lambda-6, 3\lambda-1, -1-\lambda$.

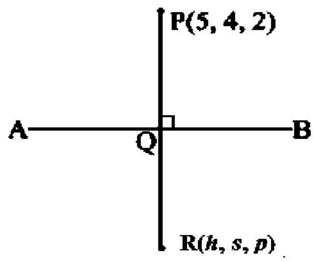
As PQ is perpendicular to given line.

So, $2(2\lambda-6)+3(3\lambda-1)-1(-1-\lambda)=0 \Rightarrow \lambda=1$.

$\therefore$ Coordinates of the **Foot of perpendicular** on the line is $Q(1,6,0)$.

And, **Length of perpendicular** is $PQ=\sqrt{(1-5)^2+(6-4)^2+(0-2)^2}$

$$\Rightarrow PQ = 2\sqrt{6}\text{ units}.$$



Also, let the image of P in the line be R(*h*, *s*, *p*). Then Q will be the mid-point of PQ.

So, $Q(1,6,0)=Q\left(\dfrac{5+h}{2},\dfrac{4+s}{2},\dfrac{2+p}{2}\right) \quad \Rightarrow h=-3,\ s=8,\ p=-2$.

Hence, the **Image of point P** in the given line is R(–3, 8, –2).

Q29. Using matrices, solve the following system of linear equations:

$3x+4y+7z=4;\ 2x-y+3z=-3;\ x+2y-3z=8$.

Sol. The given system of equations is: $3x+4y+7z=4;$

$2x-y+3z=-3;$

$x+2y-3z=8$

By using matrix method: let $A=\begin{bmatrix}3 & 4 & 7\\ 2 & -1 & 3\\ 1 & 2 & -3\end{bmatrix}$, $B=\begin{bmatrix}4\\ -3\\ 8\end{bmatrix}$ and, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$.

Since $AX=B \qquad \Rightarrow X=A^{-1}B \qquad$ ...(i)

Now, $|A|=\begin{vmatrix}3 & 4 & 7\\ 2 & -1 & 3\\ 1 & 2 & -3\end{vmatrix}=3(3-6)-4(-6-3)+7(4+1)$

$\therefore\ |A|=62\neq 0 \qquad \Rightarrow$ A is **non-singular** and hence, it is invertible i.e., A^{-1} exists.

Consider C_{ij} be the cofactors of element a_{ij} in matrix A, we have

$C_{11}=-3,\qquad C_{12}=9,\qquad C_{13}=5$

$C_{21}=26,\qquad C_{22}=-16,\qquad C_{23}=-2$

$C_{31}=19,\qquad C_{32}=5,\qquad C_{33}=-11$

So, $adj A=\begin{bmatrix}-3 & 9 & 5\\ 26 & -16 & -2\\ 19 & 5 & -11\end{bmatrix}^T=\begin{bmatrix}-3 & 26 & 19\\ 9 & -16 & 5\\ 5 & -2 & -11\end{bmatrix}$

$\therefore\ A^{-1}=\dfrac{1}{|A|}(adj A)=\dfrac{1}{62}\begin{bmatrix}-3 & 26 & 19\\ 9 & -16 & 5\\ 5 & -2 & -11\end{bmatrix}$

Now by (i), we have $X=A^{-1}B$

So, $X=\dfrac{1}{62}\begin{bmatrix}-3 & 26 & 19\\ 9 & -16 & 5\\ 5 & -2 & -11\end{bmatrix}\begin{bmatrix}4\\ -3\\ 8\end{bmatrix}$

$\Rightarrow \quad =\dfrac{1}{62}\begin{bmatrix}-12-78+152\\ 36+48+40\\ 20+6-88\end{bmatrix}$

$\Rightarrow \quad =\dfrac{1}{62}\begin{bmatrix}62\\ 124\\ -62\end{bmatrix}$

i.e., $\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1\\ 2\\ -1\end{bmatrix}$

By equality of matrices, we get

$x=1,\ y=2,\ z=-1$

Hence, $x=1,\ y=2,\ z=-1$ is the required solution.