SAMPLE TEST PAPER – 05

Max. Marks: 100 **Time Allowed: 3 Hours**

SECTION – A

Q01. If $f:\mathrm{R}\to\mathrm{R}$, $g:\mathrm{R}\to\mathrm{R}$ be defined by $f(x)=\frac{3x-7}{8}$, $g(x)=\frac{8x+7}{3}$, then find $fog(7)$.

Q02. Evaluate: $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$.

Q03. Evaluate: $\begin{vmatrix} a+ib & c+ip \\ -c+ip & a-ib \end{vmatrix}$.

Q04. Evaluate: $\int\frac{x^2}{1+x^3}dx$.

Q05. Find the cofactor of a_{12} in $\begin{vmatrix} 2 & 3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

Q06. Evaluate $\int_0^{3/2}[x]dx$, where $[x]$ represents a greatest integer function.

Q07. Write a unit vector parallel to $-\vec{a}$ if it is given that $\vec{a}=3\hat{i}-2\hat{j}+6\hat{k}$.

Q08. Find the tangent of the angle between $\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\hat{k}$.

Q09. If '*' is a binary operation defined on R and if $a*b=\frac{ab}{2}$, write the value for $(4*2)*6$.

Q10. For a vector equiangular with the coordinate axis, write its direction cosines.

SECTION – B

Q11. Prove that: $\tan^{-1}\left(x+\sqrt{1+x^2}\right)=\frac{\pi}{4}+\frac{1}{2}\tan^{-1}x,\ x\in\mathrm{R}$.

OR Simplify the expression: $\sin(\cot^{-1}\cos\tan^{-1}x)$.

Q12. Define symmetric matrix and skew-symmetric matrix. Let $\mathrm{A}=\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$. Express A as the sum of two matrices such that one is symmetric matrix while the other is skew-symmetric matrix.

Q13. Determine the value of *k* for which $f(x)$ is continuous at $x=2$ such that $f(x)=\begin{cases} 2x+1, & if\ x<2 \\ k, & if\ x=2 \\ 3x-1, & if\ x>2 \end{cases}$.

Q14. Find the equation of normal to curve $x=\sin 3t,\ y=\cos 2t$ at $t=\frac{\pi}{4}$.

Q15. Differentiate $e^{\tan x}$ with respect to 'x' by using first principle of derivatives.

Q16. Evaluate the integral: $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$.

Q17. Solve: $\left(x^2-y^2\right)dx+2xydy=0,\ y(1)=1$.

Q18. Find the particular solution of the differential equation: $(2y+x)dy-(2y-x)dx=0,\ y(1)=1$.

OR Solve the differential equation given as: $\cos^2 x\frac{dy}{dx}+y=\tan x$.

Q19. Find a point on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of $3\sqrt{2}\ units$ from the point $(1,2,3)$.

Q20. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$ then, find a vector $\vec{c}$ such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$.

OR Show that if the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar vectors then, $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are also coplanar vectors.

Q21. Consider $f:\mathrm{R}_+\to[-5,\infty)$ given by $f(x)=9x^2+6x-5$. Show that f is an invertible function. Hence find f^{-1}.

Q22. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of the number of successes.

OR Find $\mathrm{P}(|x-4|\le 2)$ if x follows a Binomial Distribution with the mean 4 and variance 2.

SECTION – C

Q23. Using integration, find area of the region bounded by $y^2=4x$ and $4x^2+4y^2=9$.

Q24. Using properties of determinant, prove that: $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}=4(abc)^2$.

Q25. Show that the height of a cylinder which can be inscribed in a cone of height h is $\frac{1}{3}h$.

OR Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with its vertex at one end of major axis.

Q26. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 bus drivers. The probability of an accident involving a scooter, a car and a bus are respectively 0.01, 0.03 and 0.15. One of the insured persons meets with an accident. What is the probability that he is a scooter driver? Explain the importance of public transport system over private vehicles in two points.

Q27. Find the equation of the plane which passes through the points (3, 4, 1) and (0, 1, 0) and is parallel to the line: $\frac{x+3}{2}=\frac{y-3}{7}=\frac{z-2}{5}$.

OR Find the equation of a plane which is perpendicular to the planes $2x+3y-3z-2=0$ and $5x-4y+z-6=0$ and passes through the point (–1,–1, 2).

Q28. If a young man drives his bike at 25kmph, he has to spend ₹2 per kilometer on petrol. If he drives it at a faster speed of 40kmph, the petrol cost increases to ₹5 per kilometer. He has ₹100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express the given situation as linear programming problem and then, solve it. Why should a student prefer to use bicycle?

Q29. Evaluate: $\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}\,dx$.

All the very best and God bless you in your examinations!

With lots of Love & Blessings for your bright future!

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