TARGET MATHEMATICS
THE EXCELLENCE KEY
AGYAT GUPTA (M.Sc., M.Phil.)

21st YEAR OF QUALITY EDUCATION

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General Instructions :

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A,B and C. Section – A comprises of 10 question of 1 mark each. Section – B comprises of 12 questions of 4 marks each and Section – C comprises of 7 questions of 6 marks each .
3. Question numbers 1 to 10 in Section – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one lf the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 5 printed pages.
7. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

PRE-BOARD EXAMINATION 2012 -13

Time : 3 Hours — अधिकतम समय : 3
Maximum Marks : 100 — अधिकतम अंक : 100
Total No. Of Pages :5 — कुल पृष्ठों की संख्या : 5

CLASS – XII **CBSE** **MATHEMATICS**

SECTION A

Q.1 Evaluate : मान ज्ञात कीजिए : $x+y+z$ **if** $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\-1\\0\end{bmatrix}$. **Ans. 0** Ans.= $\begin{bmatrix}0&-1&1\\2&0&-2\\5&-2&-3\end{bmatrix}$

Q.2 If the vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k},\ \vec{b}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{c}=3\hat{i}+\lambda\hat{j}+5\hat{k}$ are coplanar, find the value of λ.

यदि सदिष $\vec{a}=2\hat{i}-\hat{j}+\hat{k},\ \vec{b}=\hat{i}+2\hat{j}+3\hat{k}$ तथा $\vec{c}=3\hat{i}+\lambda\hat{j}+5\hat{k}$ एक समतलीय है, तो λ का मान ज्ञात कीजिए।

Ans 2

Q.3 Find the vector equations of a line which passes through the point (1, 2, 3) and is parallel to the line $\frac{-x-2}{1}=\frac{y+3}{7}=\frac{2z-6}{3}$

किसी रेखा के कार्तिक तथा सदिष समीकरण ज्ञात कीजिए जो बिंदु (1, 2, 3) से होकर जाती है तथा रेखा $\frac{-x-2}{1}=\frac{y+3}{7}=\frac{2z-6}{3}$ के समांतर है। |दे $(1+2j+3k)+\lambda(-2i+14j+3k)$

Q.4 A binary operation * on the set of rational number Q, is defined as a * b = a + b + ab. Check the operation * for associativity. Ans.* YES associative

एक परिमेय संख्या वाले समुच्चय Q पर की गई द्वि – अधारी संक्रिया को a*b = a + b + ab द्वारा व्यक्त करते है। जाँच कीजिए कि * साहचर्य प्रक्रिया है।

Q.5 Evaluate : मान ज्ञात कीजिए : $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$. Ans. =1

Q.6 Find a unit vector in the direction of the resultant of vectors $\hat{i}+2\hat{j}+3\hat{k},\ -\hat{i}+2\hat{j}+\hat{k}$ and $3\hat{i}+\hat{j}$.

सदिशों $\hat{i}+2\hat{j}+3\hat{k},\ -\hat{i}+2\hat{j}+\hat{k}$ और $3\hat{i}+\hat{j}$. की परिणामी दशा में एकांक संदिश ज्ञात कीजिए। Ans. $\frac{3i+5j+4k}{5\sqrt{2}}$

Q.7 If $\sin^{-1}x+\cot^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}$, then the value of x .

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	यदि $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, तब x का मान होगा Ans $\frac{1}{\sqrt{5}}$
Q.8	Find the projection of the vector $\hat{i} - 2\hat{j} + \hat{k}$ on the vector $4\hat{i} - 4\hat{j} + 7\hat{k}$. Ans. $\frac{19}{9}$ सदिश $\hat{i} - 2\hat{j} + \hat{k}$ की सदिश $4\hat{i} - 4\hat{j} + 7\hat{k}$ पर प्रलम्बता ज्ञात कीजिए।
Q.9	If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$. Ans. $\frac{5i + 2j + 2k}{3}$ यदि $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ तथा $\vec{b} = \hat{j} - \hat{k}$, तो सदिश $\vec{c}$ ज्ञात कीजिए यदि $\vec{a} \times \vec{c} = \vec{b}$ तथा $\vec{a}.\vec{c} = 3$ ।
Q.10	Evaluate : मान ज्ञात कीजिए : $\int_0^{\pi/4} \sqrt{1 - \sin 2x}\, dx$. Ans. $\sqrt{2} - 1$
	SECTION B
Q.11	Solve the differential equation $x\frac{dy}{dx} = y(\log y - \log x + 1)$. अवकल समीकरण $x\frac{dy}{dx} = y(\log y - \log x + 1)$ को हल कीजिए। Ans : $\log x = \log\left(\log\frac{y}{x}\right) + \log c$ or / अथवा Solve the differential equation $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$.Find its particular solution, given that y = 0 when x = 0 . अवकल समीकरण $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ को हल कीजिए। इस समीकरण का निश्चित हल ज्ञात कीजिए दिया है y = 0 जब x = 0 Ans $y(1+x^2) - \frac{4x^3}{3} = 0$
Q.12	If a,b,c are non-zero numbers,show that : यदि a,b,c शून्योत्तर संख्याएँ है तो दर्शाइए : $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(b+c)(c+a)(a+b)$.
Q.13	Evaluate : मान ज्ञात कीजिए $\int_0^{\pi} \frac{x\,dx}{1 - \cos\alpha \sin x}$. Ans $\frac{\pi}{\sin\alpha}(\pi - \alpha)$
Q.14	Find the distance of the point $(-1,-5,-10)$ from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$. In your daily life which is most popular application of tossing a coin?Ans 13*unit* **Ans:** In daily life the most popular application of toss is in cricket. Before starting the game, a coin is tossed and the team who wins the toss decides that it will do batting or bowling first. **There can be multiple answers to the value based questions. Students may have their own opinion about answering them, there is no specific solution. Marks would be given for all sensible answers.** समतल $x - y + z = 5$ एवं रेखा $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ के प्रतिच्छेद बिन्दु एवं बिन्दु $(-1,-5,-10)$ के बीच की दूरी है ।
Q.15	Consider a function f: $R^+ \to [-5,\infty)$ given by $f(x) = 9x^2 + 6x - 5$, where R^+ is the set of all non-negative real numbers. show that 'f' is invertible with $f^{-1}(x) = \frac{\sqrt{x+6} - 1}{3}$.

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	मान लीजिए कि फलन f: $R^+ \to [-5,\infty)$, $f(x) = 9x^2 + 6x - 5$, द्वारा परिभाषित है, जहाँ R^+ सभी वास्तविक धनोत्तर संख्याओं का समुच्चय है। दर्शाइए कि f का प्रतिलोम $f^{-1}(x) = \frac{\sqrt{x+6}-1}{3}$ से प्राप्त होता है।
Q.16	If $y = (\sin^{-1} x)^2$ prove that : यदि $y = (\sin^{-1} x)^2$ हो, तो सिद्ध कीजिए : $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$. Or अथवा If $x = a(\theta - \sin\theta)$ & $y = a(1-\cos\theta)$ find $\frac{d^2y}{dx^2} at \theta = \frac{\pi}{2}$. Ans $\frac{d^2y}{dx^2} = -\frac{1}{2}\text{cosec}^2\frac{\theta}{2}\times\frac{d\theta}{dx} = -\frac{1}{2}\text{cosec}^2\frac{\theta}{2}\times\frac{1}{a\sin\theta} \Rightarrow \left(\frac{d^2y}{dx^2}\right)_{\theta=\pi/2} = -\frac{1}{a}$ यदि $x = a(\theta - \sin\theta)$ और $y = a(1-\cos\theta), \theta = \frac{\pi}{2}$ पर $\frac{d^2y}{dx^2}$ ज्ञात कीजिए।
Q.17	The function f is given by दिया गया फलन f है। $f(x) = \begin{cases} \frac{1-\sin x}{\cos^2 x} & if \quad x < \frac{\pi}{2} \\ a & if \quad x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi - 2x)^2} & if \quad x > \frac{\pi}{2} \end{cases}$. Find the values of a and b if f is continuous at $x = \frac{\pi}{2}$. a तथा b का मान ज्ञात कीजिए यदि $x = \frac{\pi}{2}$ पर सतत् है। Ans $a = \frac{1}{2}; b = 4$
Q.18	Form the differential equation corresponding to $y^2 = a(b - x^2)$, where a and b are arbitrary constants. $y^2 = a(b - x^2)$ के अवकल समीकरण ज्ञात कीजिए, जहाँ a तथा b अचर स्थिरांक है। Ans $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\left(\frac{dy}{dx}\right) = 0$
Q.19	The mean and variance of a binomial distribution are 4 and $\frac{4}{3}$ respectively. Find P(x≥ 1). द्विपदीय वितरण में बहुलक तथा प्रसरण क्रमशः 4 तथा $\frac{4}{3}$ है। $P(x \geq 1)$ ज्ञात कीजिए। Ans. = $p = 2/3, q = 1/3, n = 6$ $p(x >,1) = 1 - \frac{1}{729} = \frac{728}{729}$
Q.20	Find the intervals in which the function f given by $f(x) = \sin x - \cos x$, $0 \leq x \leq 2\pi$ is (i) increasing, (ii) decreasing . Ans $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)\downarrow, \uparrow (0, \frac{3\pi}{4}) \& \left(\frac{7\pi}{4}, 2\pi\right)$ or Find the approximate value of f(5.001), where $f(x) = x^3 - 7x^2 + 15$. Ans -34·995 फलन f जो $f(x) = \sin x - \cos x$, से दिया है, x के (i) निम्निष्ठ (ii)उच्चिष्ठ मान ज्ञात कीजिए, जबकि $0 \leq x \leq 2\pi$ । या f(5.001) का सन्निकट मान ज्ञात कीजिए जब $f(x) = x^3 - 7x^2 + 15$ है।
Q.21	Evaluate: का मान ज्ञात कीजिए। $\int \frac{x\sin^{-1}x^2}{\sqrt{1-x^4}}dx$. Ans $\frac{1}{4}(\sin^{-1}x^2)^2 + c$

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	or <u>या</u> Evaluate: का मान ज्ञात कीजिए। $\int \frac{2x-1}{(x-1)(x+2)(x-3)}dx$ .Ans $-\frac{1}{6}\log(x-1)-\frac{1}{3}\log(x+2)+\frac{1}{2}\log(x-3)$
Q.22	Solve for x: x का मान ज्ञात कीजिए : $\sin^{-1}x+\sin^{-1}2x=\frac{\pi}{3}$. Ans $x=\sqrt{\frac{3}{28}}$
	SECTION C
Q.23	If $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$, find A^{-1} and use it to solve the system of equations:: x + y + 2z = 0; x + 2y –z =9; x – 3y + 3z = -14. Ans $x=1, y=3, z=-2; A^{-1}=\frac{-1}{11}\begin{bmatrix}3 & -4 & -5\\ -9 & 1 & 4\\ -5 & 3 & 1\end{bmatrix}$ यदि $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$ तो A^{-1} का मान ज्ञात कीजिए तथा इसका उपयोग समीकरण रू x + y + 2z = 0; x + 2y –z =9; x – 3y + 3z = -14. को हल करने में कीजिए।
Q.24	Evaluate :मान ज्ञात कीजिए : $\int \frac{dx}{x^4+7x^2+25}$. What are qualities of black colour. Ans $=\frac{1}{10\sqrt{17}}\tan^{-1}\left(\frac{x^2-5}{x\sqrt{17}}\right)-\frac{1}{20\sqrt{3}}\log\left[\frac{x^2-\sqrt{3}x+5}{x^2+\sqrt{3}x+5}\right]$ **ANS** black colour exhibits following qualities (a)it shows darkness. (b) it is morbid. **There can be multiple answers to the value based questions. Students may have their own opinion about answering them, there is no specific solution. Marks would be given for all sensible answers.**
Q.25	Find the point on the line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{6}$ at a distance 3 from the point (1,- 2,3) . रेखा $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{6}$ पर वह बिन्दु ज्ञात कीजिए जो बिन्दु (1,-2,3) से 3 की दूरी पर हो।Ans $(\frac{1}{7},-\frac{23}{7},\frac{3}{7})\&\left(\frac{13}{7},-\frac{5}{7},\frac{39}{7}\right)$ or अथवा Find the vector and cartesian form of the equation of the plane containing the lines $\vec{r}=(i+2j+3k)+\lambda(7i+3j+2k)$and // to line $\vec{r}=(3i-j+3k)+\lambda(2i+2j+7k)$. उस तल की सदिश तथा कार्तीय रूप में समीकरण ज्ञात कीजिए, जिस पर $\vec{r}=(i+2j+3k)+\lambda(7i+3j+2k)$ तथा // $\vec{r}=(3i-j+3k)+\lambda(2i-2j+7k)$ रेखाएँ हैं। Ans $17x-45y+8z+49=0$ & $\vec{r}\bullet(17i-45j+8k)+49=0$
Q.26	Draw the rough sketch of $y^2=x+1$ and $y^2=-x+1$ and find the area enclosed by the two curves. $y^2=x+1$ तथा $y^2=-x+1$ का खाका खींचिए तथा दोनो वक्रों से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए। Ans Ans Required Area $=2\left[\int_0^{-1}\sqrt{(x+1)}dx+\int_0^{1}\sqrt{1-x}dx\right]=\frac{8}{3}unit^2$
Q.27	Prove that the height and the radius of the base of an open cylinder of given surface area and maximum volume are equal. सिद्ध कीजिए कि एक खुले बेलन की ऊँचाई तथा त्रिज्या समान होगी जिसका पृष्ठ क्षेत्रफल तथा अधिकतम आयतन दिया हो।

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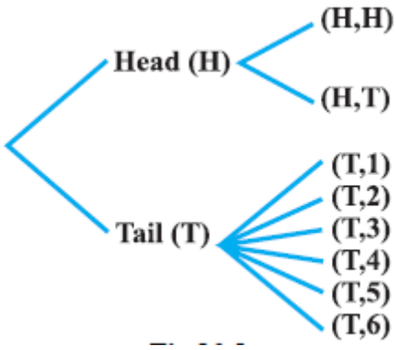
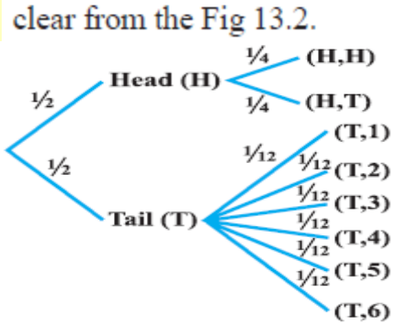
	or अथवा Show that the semi-vertical angle or a cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$. दर्शाइए कि एक शंकु का अर्द्धशीर्ष कोण $\tan^{-1}\sqrt{2}$ होगा जिसकी तिर्यक ऊँचाइ तथा अधिकतम आयतन दिया है।
Q.28	A producer has 30 and 17 units of labour and capital respectively which he can use to produce two types of goods X and Y. To produce one ulnit of X, 3 units of capital and 2 units of labour are required and to produce one unit of Y, 3 units of labour and 1 unit of capital is required. If X and Y are priced at ₹ 100 and ₹ 120 respectively, how should the producer use his resources to maximize the total revenue ? From the LPP and solve it. एक उत्पादक के पास क्रमशः 30 तथा 17 इकाइयॉ श्रम तथा पूँजी की हैं जिससे वह दो प्रकार के उत्पाद X तथा Y उत्पादित करता है। X की एक इकाई का उत्पादन करने के लिए 3 इकाई पूँजी की तथा 2 इकाई श्रम की आवश्यकता होती है और Y की एक इकाई उत्पादित करने के लिए 3 इकाई श्रम की तथा 1 इकाई पूँजी की आवश्यकता होती है। यदि X और Y की कीमत क्रमशः 100 ₹ और 120 ₹ है, तो उत्पादक कुल आगम को अधिकतम करने के लिए अपने संसाधनों का किस प्रकार उपयोग करेगा ? रैखिक प्रोग्रामन समस्या (LPP) बनाकर हल कीजिए।Ans z = 100 x + 120 y ; $x, y \geq 0; 2x+3y \leq 30, 3x+y \leq 17, P:(0,10); (\frac{17}{3},0)(0,0)(3,8)\max at(3,8) = 1260$
Q.29	Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one tail'. Give the points to be kept in mind while driving a vehicle on road.  **Solution** The outcomes of the experiment can be represented in following diagrammatic manner called the 'tree diagram'.The sample space of the experiment may be described as S = {(H,H), (H,T), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)} where (H, H) denotes that both the tosses result into head and (T, *i*) denote the first toss result into a tail and the number *i* appeared on the die for *i* = 1,2,3,4,5,6. Thus, the probabilities assigned to the 8 elementary events (H, H), (H, T), (T, 1), (T, 2), (T, 3) (T, 4), (T, 5), (T, 6) are $\frac{1}{4},\frac{1}{4},\frac{1}{12},\frac{1}{12},\frac{1}{12},\frac{1}{12},\frac{1}{12},\frac{1}{12}$ respectively which is clear from the Fig 13.2.  Let F be the event that 'there is at least one tail' and E be the event 'the die shows a number greater than 4'. Then

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Let F be the event that 'there is at least one tail' and E be the event 'the die shows a number greater than 4'. Then

$$F = \{(H,T), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)\}$$

$$E = \{(T,5), (T,6)\} \text{ and } E \cap F = \{(T,5), (T,6)\}$$

Now $P(F) = P(\{(H,T)\}) + P(\{(T,1)\}) + P(\{(T,2)\}) + P(\{(T,3)\}) + P(\{(T,4)\}) + P(\{(T,5)\}) + P(\{(T,6)\})$

$$= \frac{1}{4}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}=\frac{3}{4}$$

and $P(E \cap F) = P(\{(T,5)\}) + P(\{(T,6)\}) = \frac{1}{12}+\frac{1}{12}=\frac{1}{6}$

Hence $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{6}}{\frac{3}{4}} = \frac{2}{9}$

ANS While driving a vehicle on road (a) we should always move on the hand side (b) traffic rules should be followed strictly. (c) we should drive the vehicle at normal speed. (d) we should obey the traffic signals. **There can be multiple answers to the value based questions. Students may have their own opinion about answering them, there is no specific solution. Marks would be given for all sensible answers.**

OR

The probability of a shooter hitting a target is 3 / 4 . How much minimum number of times must he/she fire so that the probability of hitting the target at least once is more than 0.99? Suggest necessary preparations to be made before going on a trip.

Solution Let the shooter fire n times. Obviously, n fires are n Bernoulli trials. In each trial, p = probability of hitting the target = $\frac{3}{4}$ and q = probability of not hitting the target = $\frac{1}{4}$. Then $P(X = x) = {}^nC_x\, q^{n-x}\, p^x = {}^nC_x \left(\frac{1}{4}\right)^{n-x}\left(\frac{3}{4}\right)^x = {}^nC_x \frac{3^x}{4^n}$.

Now, given that,

P(hitting the target at least once) > 0.99

i.e. $P(x \geq 1) > 0.99$

Therefore, $1 - P(x = 0) > 0.99$

or $1 - {}^nC_0 \frac{1}{4^n} > 0.99$

or ${}^nC_0 \frac{1}{4^n} < 0.01$ i.e. $\frac{1}{4^n} < 0.01$

or $4^n > \frac{1}{0.01} = 100$

The minimum value of n to satisfy the inequality (1) is 4.

Thus, the shooter must fire 4 times.

ANS preparations before going to trip are: (a) plan the trip (what to do and where to go)
(b) do not take too much of cash. Use debit cards and credit cards.
(c) check the weather forecast of the place.
(d) check the journey tickets for the detail and keep them with you. **There can be multiple**

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	answers to the value based questions. Students may have their own opinion about answering them, there is no specific solution. Marks would be given for all sensible answers.
	BELIEVE ONE WHO HAS PROVED IT. BELIEVE AN EXPERT.

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