[Model Test–02/Cl–XII(CBSE'13)/23rd Oct'12] MODEL TEST [FM–100/Time–3 hrs.]

GENERAL INSTRUCTIONS:

i) All questions are compulsory.

ii) The question paper consists of **29** questions divided into Three sections A, B and C. Section A comprises of **10** questions of **one mark** each, Section B comprises of **12** questions of **four marks** each, and Section C comprises of **7** questions of **six** marks each.

iii) All questions in section A are to be answered in one word, one sentence or as per the exact requirements of the question.

iv) There is no overall choice. However, internal choice has been provided in **4 questions of four marks each** and **2 questions of six marks each**. You have to attempt only one of the alternatives in all such questions.

iv) Use of calculators is not permitted.

SECTIONS – A (**10 questions of 1 mark each**)

1. Find the value of $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$. [1]

2. If f :R→ R be defined by $f(x)=(2012-x^{2013})^{\frac{1}{2013}}$, then find fof(x). [1]

3. If $2A+B=\begin{pmatrix}2 & 3\\ 5 & 1\end{pmatrix}$ *and* $A+B=\begin{pmatrix}-6 & 0\\ 7 & 1\end{pmatrix}$, find A. (A & B are two 2×2 matrices.) [1]

4. Without expanding prove that $\begin{vmatrix}x+y & y+z & z+x\\ z & x & y\\ 1 & 1 & 1\end{vmatrix}=0$. [1]

5. A ={a_{ij}} is a 2×2 matrix, whose elements are given by $a_{ij}=\frac{i}{j}$, Write the matrix A. [1]

6. Find the slope of the tangent to the curve y =3x^2+4x at the point whose abscissa is (–2) . [1]

7. Evaluate: $\int\frac{\cos x}{\sin(x-a)}dx$. [1]

8. If the vectors $a=2\hat{i}-\hat{j}+\hat{k}$, $\hat{b}=\hat{i}+2\hat{j}+3\hat{k}$ & $\hat{c}=3\hat{i}+\lambda\hat{j}+5\hat{k}$ are coplanar, find the value of λ. [1]

9. Find a unit vector in the direction of the vector $a=\hat{i}+\hat{j}+2\hat{k}$. [1]

10. Find the coordinates of the point where the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4}$ meets the plane x+y+4z = 6. [1]

SECTIONS – B (**12 questions of 4 marks each**)

11. If the function f : R→R is given by $f(x)=\frac{x+3}{2}$ and g : R→R is given by g(x) = 2x – 3, find (i) fog and (ii) gof. Is f $^{-1}$ = g ? [4]

12. Evaluate : $\int\frac{\sin 2x}{a^2\sin^2+b^2\cos^2 x}dx$. [4]

13. Given $A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$, find adjoint of A. Hence find A^{-1}. [4]

OR, If $A=\begin{bmatrix}2 & -3\\ 3 & 4\end{bmatrix}$ show that A^2 – 6A+17 I = 0. Hence find A^{-1}.

14. Evaluate: $\int e^x \frac{1+\sin x}{1+\cos x} dx$ [4]

15. For which value of λ is the function defined by $f(x)=\begin{cases}\lambda(x^2-2x) & if\ x\le 0\\ 4x+1 & if\ x>0\end{cases}$ continuous at x= 0 ? What about continuity at x = 1 ? [4]

16. Find the point on the curve y =x^3 – 11x + 5 at which the tangent is y = x – 11. [4]

17. Prove that, $2\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}.\tan\frac{\theta}{2}\right]=\cos^{-1}\frac{a.\cos\theta+b}{a+b.\cos\theta}$. [4]

OR, Prove that, $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]=\sqrt{\frac{1+x^2}{2+x^2}}$

18. Using properties of definite integral, prove that $\int_0^{\pi}\frac{x.\tan x}{\sec x\cos ec\, x}dx=\frac{\pi^2}{4}$ [4]

19. If $y=(\log x)^x+x^{\cos x}$, find $\frac{dy}{dx}$. [4]

20. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ *and* $c\hat{i}+c\hat{j}+b\hat{k}$ be coplanar, show that c^2 = ab. [4]

OR, Find the projection of $\vec{b}+\vec{c}$ *on* $\vec{a}$, where $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}$, $\hat{b}=\hat{i}+2\hat{j}-2\hat{k}$ *and* $\hat{c}=2\hat{i}-\hat{j}+4\hat{k}$.

21. Find the coordinates of the foot of the perpendicular drawn from the point A (1, 8, 4) to the line joining the points B (0, −1, 3) and C (2, −3, −1). [4]

OR, Find the equation of the plane which is perpendicular to the plane 5x + 3y + 6z + 8 = 0 and which contains the line of intersection of the planes x + 2y + 3z – 4 = 0 and 2x + y – z + 5 = 0.

22. There are two bags I and II. Bag I contains 2 white and 3 red balls and Bag II contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag II. [4]

<u>SECTIONS</u> – C (seven questions each of six marks)

23. Prove that, $\begin{vmatrix}ab & c & c^2\\ bc & a & a^2\\ ca & b & b^2\end{vmatrix}=(a-b)(b-c)(c-a)(ab+bc+ca)$ [6]

24. Find the point on the curve x^2 = 8y which is nearest to the point (2, 4). [6]

OR, Prove that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

25. Find the area of the region enclosed between the two circles $x^2+y^2=1$ and $(x-1)^2+y^2=1$. [6]

26. Solve the differential equation $(xdy-ydx)+\sin\left(\frac{y}{x}\right)=(ydx+xdy)\,x\,\cos\left(\frac{y}{x}\right)$. [6]

27. Find the equation of the plane passing through the points (–1, –1, 2) and perpendicular to each of the planes whose equations are 2x + 3y – 3z = 2 and 5x – 4y + z = 6. [6]

OR, Find the equation of the plane passing through the points (3, 4, 1) and (0, 1, 0) and parallel to the line $\frac{x+3}{2}=\frac{y-3}{7}=\frac{z-2}{5}$.

28. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car & a truck are 0.01, 0.03 &0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver? [6]

29. A factory owner purchases two types of machines A & B for his factory. The requirements and the limitations for the machines are as follows :

Machine	Area occupied	Labour force	Daily output (in units)
A	1000 m^2	12 men	60
B	1200 m^2	8 men	40

He has maximum area of 9000 m^2 available, and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximize the daily output ? [6]

"Successful persons can do well,
because they think they can."

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