[Model Test–01/Cl–XII(CBSE)/22nd June’12] MODEL TEST [FM–45 /Time– 75 min.]

1. a) If $\begin{vmatrix} -5 & 5 & 10 \\ 5 & -5 & x \\ 0 & 10 & 5 \end{vmatrix} = 0$, find the value of x. [1]

b) Write the co-factor of a_3 in $\begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix}$. [1]

c) Evaluate : $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$. [1]

d) Write the adjoint of the following matrix: $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$ [1]

e) If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to [a] [1]
a) I b) I – A c) A d) 3A

f) If $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ is such that $A^2 = I$, then [b] [1]
a) $1 + a^2 + bc = 0$ b) $1 - a^2 - bc = 0$ c) $1 - a^2 + bc = 0$ d) $1 + a^2 - bc = 0$

g) If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & -\cos\alpha \end{bmatrix}$, then for what value of α is A an identity matrix? [1]

h) Find the value of x and y if : $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$. [1]

2. a) If $A^{-1} = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$, determine A. [Ans: $\frac{1}{7}\cdot\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$] [2]

b) Evaluate without expanding at any stage : $\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix}$. [2]

c) If $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$, show that $A^2 - 6A + 17I = O$. Hence find A^{-1}. [2]

3.a) If the area of a triangle joining the three points (1, 1), $\left(4, \frac{t}{2}\right)$ and (3, 3t) be 5 sq units, using determinant find the value of t. [3]

b) If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = x^2 - 2x - 3$, find $f(A)$. [3]

c) Express the matrix $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. [3]

Confidence

d) Using the properties of determinants, solve for x: $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$. [3]

4. a) Given $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}$, find adjoint of A. Hence find A^{-1}. [4]

b) Using properties of determinants, P. T. : $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = (a^3+b^3+c^3-3abc)$. [4]

5. Find the inverse of the following matrix using elementary operations $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$ [6]

OR, Using matrices solve the following system of equations: $\begin{aligned} x+2y+3z &= 6 \\ 3x+2y-2z &= 3 \\ 2x-y+z &= 2 \end{aligned}$

6. a) Write the principal value of $\cot^{-1}\left(-\sqrt{3}\right)$. [1]

b) Solve for x: $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ [4]

OR, Prove that, $\tan^{-1}x + \cot^{-1}(x+1) = \tan^{-1}(x^2+x+1)$.

"Learning is a Treasure,
which accompanies its owner everywhere."