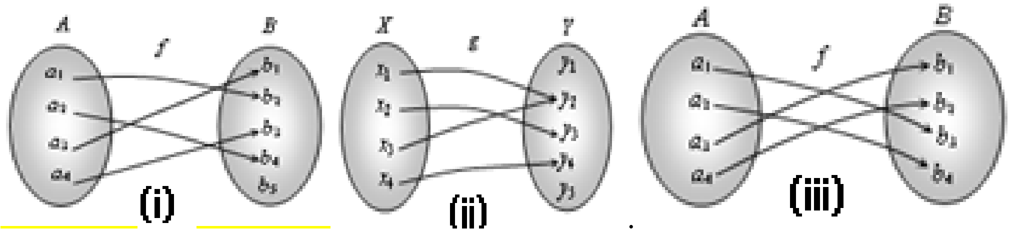
AGYAT GUPTA (TARGET MATHEMATICS)
Resi.: D-79 Vasant Vihar ; Office : 89-Laxmi bai colony
Ph.: 410685®,2630601(O)Mobile : 9425109601; 9425110860
PREMIER INSTITUTE for X , XI & XII .

CLASS XII

RELATION ,FUNCTION & BINARY OPERATION

Q.1 Write the name of function.



Q.2 If $\left(\frac{x}{2}+1, y-\frac{1}{2}\right)=\left(2,\frac{1}{2}\right)$, find the value of x and y .

Q.3 Let A = {-1, 0, 1, 2}, B = {-4, -2, 0, 2} and $f, g : A \rightarrow B$ be functions defined by $f(x) = x^2 - x$, $x \in$ A and $g(x) = 2\left|x-\frac{1}{2}\right|-1$, $x \in$ A are f and g equal. Justify your answer.

Q.4 Given that the ordered pairs (a , 7) and (-4,b) belongs to relation $\{(x, y): 2y-3x=8\}$. Find the value of a and b .

Q.5 Let relation $R=\{(x, y)\in w\times w: y=2x-4\}$. If (a , - 2) and $(4, b^2)$ belong to relation R, find the value of a and b .

Q.6 If f : R→R be defined as $f(x)=\frac{3x+7}{9}$, then find $f^{-1}(x)$.

Q.7 A relation R is defined on the set Z of integer as follows : ${}^xR_y \Leftrightarrow x^2+y^2=25$ .Express R and R^{-1} as the set of ordered pairs and hence find their respective domains and range .

Q.8 Let R be the relation on the set N of natural number defined by $R=\{(x, y): x+3y=12 \;\&\; x, y\in N$. (i) Write R in the roster form. (ii) Find domain of R (ii) Find range of R.

Q.9 Find the total number of function from set A to set B if A = { 2 , 3 , 4 }& set B= { a , b } .

Q.10 Find the total number of relation from set A to A if A = { 2 , 3 , 4 } .

Q.11 If $f(x)=|x| and\ g(x)=|5x-2|$. find fog & gof .

Q.12 Show that each of the relation R in the set A of all triangles as R = $\{(T_1,T_2): T_1 \sim T_2\}$ is an equivalence relation .Consider three right triangles T_1 with sides 3 , 4 , 5 ; T_2 with sides 6 , 8 , 10 & T_3 with sides , 5 , 12 , 13 . Which triangle among $T_1, T \;\&\; T_3$ are similar .

Q.13 Let R_+ be the set of all non-negative real numbers. Let $f: R_+ \rightarrow [4,\infty): f(x)=x^2+4$. Show that f is invertible and find f^{-1}.

Q.14 Let A {2, 3, 4, 5, 6, 7, 8, 9 } .Let R be the relation on A defined by $\{(x, y): x\in A \;\&\; x$ divides y)}. Find (i) R (ii)Domain of R (iii) Range of R (iv) R^{-1} state whether or not R^{-1}, is (a) Reflexive (b) symmetric (c) transitive .

Q.15 Show that each of the relation R in the set A = { $x\in Z: 0\le x\le 12$},given by
(i)R= {(a,b): $|a-b|$.is a multiple of 4}.
(ii)R= {(a,b):a = b} is an equivalence relation .Find the set of all elements to 1 in each cases.

Q.16 Is sine function onto in the set of real numbers ? Give reasons.

Q.17 Let $N\times N$ be the set of ordered pairs of natural numbers . Also let R be the relation in $N\times N$, defined by $(a,b)R(c,d)\Leftrightarrow a+d=b+c$ .Show that R is an equivalence relation .

Q.18 Consider $f: R_+ \rightarrow [-5,\infty)$ given by $f(x)=9x^2+6x-5$. Show that f is invertible with $f^{-1}(y)=\left[\frac{\sqrt{y+6}-1}{3}\right]$.

Q.19 Discuss whether the relation defined by R = { (x , y) : x is father of y } is Reflexive , symmetric & transitive are not .

Q.20 Show that the signum function $f: R\rightarrow R$ defined by

$$f(n)=\begin{cases}1, & if \quad x>0\\ 0, & if \quad x=0\\ -1, & if \quad x<0\end{cases}$$

is neither one-one nor onto.

Q.21 Let $N\times N$ be the set of ordered pairs of natural numbers . Also let R be the relation in $N\times N$, defined by $(a,b)R(c,d)\Leftrightarrow ad=bc$ .Show that R is an equivalence relation .

Target Mathematics by- AGYAT GUPTA ; Resi.: D-79 Vasant Vihar ; Office : 89-Laxmi bai colony
Ph. :2337615; 4010685®, 2630601(O) Mobile : 9425109601; 9425110860;9425772164(P)

Q.22	Prove that the function $f:R-\{3\}\to R-\{1\}$ given by $f(x)=\dfrac{x-2}{x-3}$ is bijection.
Q.23	Discuss whether the relation defined by R = { (x , y) : x is wife of y } is Reflexive , symmetric & transitive are not .
Q.24	Prove that the greatest Integer function $f:R\to R$ given by $f(x)=\lvert x\rvert$ is neither one –one nor on to .
Q.25	Show that the relation R in the set R of real no. defined as $R=\{(a,b):1+ab>0,\forall a,b\in R\}$ is Reflexive & symmetric but not transitive.
Q.26	Prove that the greatest Integer function $f:R\to R$ given by $f(x)=[x]$ is neither one –one nor on to where [x] denotes the greatest integer .
Q.27	Let N denote the set of all natural number and R be a relation on $N\times N$ defined by $(a,b)R(c,d)\Leftrightarrow ad(b+c)=bc(a+d)$. Cheque whether R is an equlivance relation .
Q.28	Show that the relation R in the set R of real no. defined as $R\{(a,b):a\le b^2\}$ is neither Reflexive, nor symmetric nor transitive.
Q.29	Find the total number of bijective function from set A to A if A = {1 , 2 , 3 , 4 } .
Q.30	Show that the relation R in the set R of real no. defined as $R\{(a,b):a\le b^3\}$ is neither Reflexive, nor symmetric nor transitive.
Q.31	Let $f:N\to N$ be defined by $f(n)=\begin{cases}\dfrac{n+1}{2}, & \text{if n is odd}\\ \dfrac{n}{2}, & \text{if n is even}\end{cases}$ for all $n\in N$. Find whether the function f is bijective.
Q.32	Consider the set A = { a , b , c} give an example of a relation R on A which is (i) Reflexive & symmetric but not transitive (ii) transitive. & symmetric but not Reflexive (iii) Reflexive & transitive but not symmetric .
Q.33	Show that $f:\{-1,1\}\to R;$ given by $f(x)=\dfrac{x}{x+2},x\neq-2$ is one – one. Find the inverse of the function $f:\{-1,1\}\to R.$
Q.34	Consider $f:R_+\to[15,\infty)$ given by $f(x)=4x^2+12x+15$. Show that *f* is invertible with $f^{-1}(y)=\left[\dfrac{\sqrt{y-6}-3}{2}\right]$.
Q.35	Let * be a binary operation on N given by a*b = HCF (a, b), a, b ε N. Write the value of 22*4.

Q.36	Let * be a binary operation on R. If a * b= a + b + ab ; a, b∈ R. Find x such that (2 * x) * 3 = 7 .
Q.37	Prove that the function $f:R-\{2\}\to R-\{1\}$ given by $f(x)=\dfrac{x-1}{x-2}$ is bijection.
Q.38	
Q.39	Find the total number of one one function from set A to A if A = { a , b , c } .
Q.40	If the binary operation *, defined on Q, is defined as a * b = 2a + b – ab, for all a, b∈ Q, find the value of 3 * 4.
Q.41	Let X be non empty set . P(X) be its power set . let * be an binary operation defined on elements of P(X) by , $A*B=A\cup B\forall A,B\in P(X)$ then (i) Prove that * is a binary operation in P(X). (ii) Is * commutative ? (iii)Is associative (iv) Find the identity element of in P(X) w.r.t. * . (v) Find all the invertible of P(x) . (vi) If ⊗ is another binary operation defined on P(X) as A⊗B = $A\cap B$ then verify that ⊗ distributive over * .
Q.42	Let $f(x)=[x]$ *and* $g(x)=\lvert x\rvert$. *Find* (i) $(gof)\left(\dfrac{-11}{3}\right)-(fog)\left(\dfrac{-11}{3}\right)$ (ii) $(gof)\left(\dfrac{5}{3}\right)-(fog)\left(\dfrac{5}{3}\right)$. (iii) $(f+3g)(-1)$ (iv) $(gof)(\sqrt{5})$.
Q.43	Prove that a relation on a set A is symmetric if and only if R = R^{-1}.
Q.44	If R is an equivalence relation on a set A , then R^{-1} **is also equivalence relation on A .**
Q.45	Let $f(x)=\lvert x\rvert$ and $g(x)=[x]$. Evaluate $(fog)\left(\dfrac{-5}{3}\right)-(gof)\left(\dfrac{-5}{3}\right)$.
Q.46	Let $f:R\to R$ & $g:R\to R$ be defined by $f(x)=2x+3$ & $g(x)=x^2+7$. then find the value of x such that $f\{g(x)\}=25$.
Q.47	Let relation $R=\{(x,y)\in w\times w: y=2x-4\}$. If (a , - 2) and $(4,b^2)$ belong to relation R, find the value of a and b .
Q.48	If $f(x)=\dfrac{4x+3}{6x-4},x\neq\dfrac{2}{3}$. Prove that $fof=x$ .What is the inverse of f .
Q.49	If $f:R-\left\{\dfrac{7}{5}\right\}\to R-\left\{\dfrac{3}{5}\right\}$ be defined as $f(x)=\dfrac{3x+4}{5x-7}$ & $g:R-\left\{\dfrac{3}{5}\right\}\to R-\left\{\dfrac{7}{5}\right\}$ be defined as $g(x)=\dfrac{7x+4}{5x-3}$.Prove that $gof=I_A$ & $(fog)=I_B$ where $B=R-\left\{\dfrac{3}{5}\right\}$ & $A=R-\left\{\dfrac{7}{5}\right\}$.Find

	also $g^{-1}, f^{-1} \& (gof)^{-1}$.
Q.50	Let X be non empty set . P(X) be its power set . let * be an binary operation defined on elements of P(X) by , $A * B = A \cap B \forall A, B \in P(X)$ then (i) Prove that * is a binary operation in P(X). (ii) Is * commutative ? (iii)Is associative (iv) Find the identity element of in P(X) w.r.t. * . (v) Find all the invertible of P(x) . (vi) If $\otimes$ is another binary operation defined on P(X) as A $\otimes$ B = $A \cup B$ then verify that $\otimes$ distributive over * .
Q.51	let $A = N \times N$ and * be an binary operation defined by $(a,b)*(c,d) = (ac, bd) \forall a,b,c,d \in N$ on the set R , then (i) Prove that * is a binary operation on N (ii) Is * commutative ? (iii)Is associative (iv) Find the identity element for * on $N \times N$ if any .
Q.52	If f(x) and g (x) be two invertible function defined as $f(x) = \frac{2x+1}{3x-5}$ be defined as $g(x) = \frac{3x+3}{7x-2}$. Prove that $(gof)^{-1} = f^{-1}og^{-1}$.
Q.53	let * be an binary operation defined $a * b = a + b + ab$ on the set R – {-1} , then (i) Prove that * is a binary operation R – {-1} (ii) Is * commutative ? (iii)Is associative (iv) Find the identity element R – {-1} w.r.t. * . and also prove that every element of R – {-1}is invertible .
Q.54	If * be the binary operation on the set Q of rational numbers defined by a * b = $\frac{ab}{4}$.For binary operation examine the commutative and associative property. Also find identity element and inverse .
Q.55	If $f, g : Q \to Q$ be two invertible function defined as $f(x) = 2x$ be defined as $g(x) = x + 2$ Prove that $(gof)^{-1} = f^{-1}og^{-1}$.
Q.56	Let A ={ 1,2,3,4} B = { 3, 5 , 7 , 9 } C = { 7 , 23 , 47 , 79 } and $f : A \to B$, $g : B \to C$ be defined as f(x) = 2x + 1 and g(x) $= x^2 - 2$. Express $(gof)^{-1}$ & $f^{-1}og^{-1}$ as the set of ordered pair and verify that $(gof)^{-1} = f^{-1}og^{-1}$.
Q.57	Let $f : \{2,3,4,5\} \to \{3,4,5,9\}$ and $g : \{3,4,5,9\} \to \{7,11,15\}$ be functions defined as f(2)=3,f(3)= 4,f(4)= 5, f(5) = 5 and g(3) = g(4) = 7 and g(5) = g(9) = 11. Find gof. Also find the domain and the range of gof.
Q.58	Let $f : \{1,3,4\} \to \{1,2,5\}$ & $g : \{1,2,5\} \to \{1,3\}$ be given by $f = \{(1,2),(3,5),(4,1)\}$
	$g = \{(1,3)(2,3)(5,1)\}$ write down of gof.
Q.59	Let * be a binary operation defined by a * b = 2a + b – 3. Find 3*4.
Q.60	Find the inverse element of the binary relation $a \otimes b = a + b - 4$.
Q.61	Let $f : R \to R$ be defined by $f(x) = x^2 + 1$, then (i) $f^{-1}(17)$ (ii) $f^{-1}(-5)$. (iii) $f^{-1}(10,37)$
Q.62	Define a binary operation * one the set {0,1,2,3,4,5} as a * b = $\begin{cases} a+b, a+b<6 \\ a+b-6, a+b\geq 6 \end{cases}$.Show that zero is the identity for this operation and each element a of the set is invertible with 6 – a being the inverse of a.
Q.63	let $A = N \times N$ and * be an binary operation defined by $(a,b)*(c,d) = (a+c, b+d) \forall a,b,c,d \in N$ on the set R , then (i) Prove that * is a binary operation on N (ii) Is * commutative ? (iii)Is associative (iv) Find the identity element for * on $N \times N$ if any .
Q.64	Determine the nature of the following functions for even and odd: (i) $f(x) = x\left(\frac{a^x - 1}{a^x + 1}\right)$ (ii) $f(x) = \log(x + \sqrt{x^2 + 1})$.
Q.65	Let * be a binary operation on N. If a * b= lcm of a & b ; a, b$\in$ N. Find (2 * 4) * 6 .
Q.66	Let * be a binary operation on Z. If a * b= a + b + 1 ; a, b$\in$ Z. Find the identity element with respect to operation * on Z .
Q.67	Let $f : R \to R$ be defined by $f(x) = x^2 + 5x + 9$, Find $f^{-1}(9)$.
Q.68	Let f , g be the function f = { (1 , 5) , (2 , 6) , (3 , 4)} ; g = {(4 , 7) (5,8) , (6 , 9)}.What is the range of f & g . Find gof .
Q.69	Let * be a binary operation on $N \times N$. If (a,b) * (c,d) = (ad + bc , bd) ; (a,b) ,(c,d) $\in N \times N$ .Prove that (i) * is closed to binary operation on $N \times N$ (ii) * is commutative on $N \times N$ (iii) * is associative on $N \times N$ (iii) Find the identity element with respect to operation * on $N \times N$ if any .
Q.70	Let * be the binary operation such that Q x Q = Q defined by a * b = a + b - ab; a,b$\in$ Q – {1}. Show that * is (i) associative, (ii) commutative.
Q.71	Examine which of the following is a binary operation (i) $a \otimes b = \frac{a+b}{2} \forall a, b \in N$ (ii) $a \otimes b = \frac{a+b}{2} \forall a, b \in Q$.
Q.72	Consider $f : \{1,2,3\} \to \{a,b,c\}$ and $g : \{a,b,c\} \to \{apple, ball, cat\}$ defined as

Target Mathematics by- AGYAT GUPTA ; Resi.: D-79 Vasant Vihar ; Office : 89-Laxmi bai colony
Ph. :2337615; 4010685®, 2630601(O) Mobile : 9425109601; 9425110860;9425772164(P)

	$f(1)=a; f(2)=b, f(3)=c; g(a)=apple; g(b)=ball; g(c)=cat$. Show that f, g and gof are invertible. Find out $f^{-1}, g^{-1} and (gof)^{-1}=f^{-1}og^{-1}$.
Q.73	If $y=f(x)=\frac{3x-5}{5x-3}$, show that $x=f(y)$.
Q.74	Let * be a binary operation defined by a * b=3a + 4b - 2. Find 4 * 5.
Q.75	Let * be a binary operation on Z. Find 2 * (-4) if a * b = 4ab; a, b∈ Z.
	DIFFERENTIAL EQUATION
Q.1	Solve the differential equation $x\frac{dy}{dx}=y(\log y-\log x+1)$.
Q.2	Show that the differential equation $2ye^{x/y}dx+\left(y-2xe^{x/y}\right)dy=0$ is homogeneous and find its particular solution given that x=0, when y = 1.
Q.3	From the differential equation of the family of curves y = e^x (A cos x + B sin x), where A and B are arbitrary constants.
Q.4	Find the particular solution of the differential equation $(xdy-ydx)y.\sin\left(\frac{y}{x}\right)=(ydx+xdy)x\cos\frac{y}{x}$, given that $y=\pi$ when x = 3.
Q.5	Solve the differential equation $(1+y^2)(1+\log x)dx+xdy=0$ given that when $x=1, y=1$.
Q.6	Solve : $ydx-(x+2y^2)dy=0$.
Q.7	Solve the initial value problems: $\sqrt{1-y^2}dx=(\sin^{-1}y-x)dy, y(0)=0$.
Q.8	Solve : $\frac{dy}{dx}=\frac{(x-y)+3}{2(x-y)+5}$.
Q.9	Solve : $\frac{dy}{dx}=e^{x+y}+e^y x^3$.
Q.10	Show that $Ax^2+By^2=1$ is a solution of the differential equation $x\left[y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right]=y\frac{dy}{dx}$.
Q.11	Solve the differential equation: $(x^2+1)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$.

Q.12	Solve the differential equation: $\frac{d^2x}{dy^2}=y\sin^2 y$.
Q.13	Solve the differential equation $2xy+y^2-2x^2\frac{dy}{dx}=0; y(1)=2$.
Q.14	Show that $y=ae^{2x}+be^{-x}$ is a solution of the differential equation $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0$.
Q.15	Show that $xy=ae^x+be^{-x}+x^2$ is a solution of the differential equation $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}-xy+x^2-2=0$.
Q.16	Show that the function $y=(A+Bx)e^{3x}$ is a solution of the equation $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=0$.
Q.17	Solve the following differential equation: $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$.
Q.18	Solve the following differential equation :$(1+y+x^2y)dx+(x+x^3)dy=0$, where y = 0 when x = 1.
Q.19	Prove that $y^2=a(b^2-x^2)$ is a solution of differential equation $x\left[y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right]-y\frac{dy}{dx}=0$.
Q.20	The slope of the tangent to a curve at any point (x, y)on it is given by $\frac{y}{x}-\cot\frac{y}{x}.\cos\frac{y}{x},(x\rangle 0, y\rangle 0)$ and the curve passes through the point $(1,\pi/4)$. Find the equation of the curve .
Q.21	Solve: $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$ subject to the initial condition y (0) = 0.
Q.22	Find the particular solution of the differential equation $\frac{dy}{dx}+y\cot x=2x+x^2\cot x, x\neq 0,$ given that y=0, when x=$\frac{\pi}{2}$.
Q.23	Obtain the differential equation of the family of ellipse of the family of ellipse having foci on y-axis and centre at the origin.

Target Mathematics by- AGYAT GUPTA ; Resi.: D-79 Vasant Vihar ; Office : 89-Laxmi bai colony
Ph. :2337615; 4010685®, 2630601(O) Mobile : 9425109601; 9425110860;9425772164(P)

Q.24	Solve the differential equation : $x\frac{d^2y}{dx^2}=1$ given that $y=1, \frac{dy}{dx}=0, when\ x=1.$
Q.25	Solve the differential equation $x\frac{dy}{dx}=y(\log y-\log x+1)$.
Q.26	Solve the differential equation : $(x^2-1)\frac{dy}{dx}+xy=\frac{1}{x^2-1}$.
Q.27	Solve the differential equation : $\frac{dy}{dx}=(3x+y+1)^2$.
Q.28	Solve the differential equation, $\frac{dy}{dx}+y\sec^2 x=\tan x\sec^2 x; y(0)=1$
Q.29	Form the differential equation of the family of curve $y=ae^x+be^{2x}+ce^{3x}$; where a, b, c are some arbitrary constants.
Q.30	Solve the differential equation: $(1+y^2)dx=(\tan^{-1}y-x)dy, y(0)=0.$
Q.31	Solve the following differential equation: $x^2\frac{dy}{dx}=y^2+2xy$ Given that y=1, when x=1.
Q.32	Solve : $(x^3-3xy^2)dx=(y^3-3x^2y)dy.$
Q.33	Solve the differential equation: $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x, x\rangle 0.$
Q.34	Write the order and degree of the differential equation, $y = x\frac{dy}{dx}+a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$.
Q.35	Write the I.F. of the differential equation $(1+x^2)\frac{dy}{dx}+y=e^{\tan^{-1}x}$.
Q.36	Write down the general sol of the differential equation $\frac{d^2y}{dx^2}=0$.
Q.37	Write the I.F. of the differential equation $(2x-10y^3)dy+ydx=0$.
Q.38	Write the order and degree of the differential equation, $\frac{d^4y}{dx^4}=y+\left(\frac{dy}{dx}\right)^4$.
Q.39	Find the particular solution of the differential equation $\frac{dy}{dx}+y\cot x=2x+x^2\cot x, x\neq 0,$ given that y = 0, when x = $\frac{\pi}{2}$.
Q.40	Find the particular solution of the differential equation $(xdy-ydx)y.\sin\left(\frac{y}{x}\right)=(ydx+xdy)x\cos\frac{y}{x}$, given that $y=\pi$ when x = 3 .
Q.41	Find the differential equation of all the lines in the xy-plane.
Q.42	Find the order and degree of the differential equation $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$.
Q.43	The general solution of the differential equation $(1+y^2)\,dx+(1+x^2)\,dy=0$.
Q.44	Order and degree of the differential equation $\frac{d^2y}{dx^2}=\left\{y+\left(\frac{dy}{dx}\right)^2\right\}^{1/4}$.
Q.45	The order and degree of the differential equation $\frac{d^2y}{dx^2}+\sqrt{1+\left(\frac{dy}{dx}\right)^3}=0$.
Q.46	Solve the differential equation: $\frac{d^2x}{dy^2}=y\sin^2 y$.
Q.47	Degree of the given differential equation $\left(\frac{d^2y}{dx^2}\right)^3=\left(1+\frac{dy}{dx}\right)^{1/2}$.
Q.48	The order and degree of the differential equation $\frac{d^2y}{dx^2}=\left\{y+\left(\frac{dy}{dx}\right)^2\right\}^{1/5}$.
Q.49	Solve : $cox(1+\cos y)dx-\sin y(1+\sin x)dy=0$.
Q.50	Integrating factor of the linear diff eq. $\frac{dy}{dx}+\frac{2x}{x^2+1}y=\frac{1}{(x^2+1)^2}$.
Q.51	Solve the differential equation: $x\frac{d^2y}{dx^2}=1$ given that $y=1, \frac{dy}{dx}=0, when\ x=1.$

Target Mathematics by- AGYAT GUPTA ; Resi.: D-79 Vasant Vihar ; Office : 89-Laxmi bai colony
Ph. :2337615; 4010685®, 2630601(O) Mobile : 9425109601; 9425110860;9425772164(P)

Q.52	Show the differential equation representing one parameter family of curves $(x^2 - y^2) = c(x^2 + y^2)^2$ is $\left(x^3 - 3xy^2\right)dx = \left(y^3 - 3x^2y\right)dy$.
Q.53	Solve the differential equation: $x \log x \dfrac{dy}{dx} + y = \dfrac{2}{x} \log x, x>0.$
Q.54	From the differential equation corresponding to $(x-a)^2 + (y-b)^2 = r^2$ by eliminating a and b .
Q.55	Show that the differential equation whose solution is $y = 2(x^2 - 1) + Ce^{-x^2}$ is $\dfrac{dy}{dx} + 2xy = 4x^3.$
Q.56	Solve the differential equation, $\dfrac{dy}{dx} + y\sec^2 x = \tan x \sec^2 x; y(0) = 1.$
Q.57	Solve : $\dfrac{dy}{dx} = e^{x-y} + x^2 e^{-y}.$
Q.58	Solve : $(1+x)(1+y^2)dx + (1+y)(1+x^2)dy = 0.$
Q.59	The normal lines to a given curve at each point pass through (2,0) . The curve passes through (2 , 3). Formulate the differential equation and hence find out the equation of the curve .
Q.60	Solve : $\dfrac{dy}{dx} = -\dfrac{x + y\cos x}{1 + \sin x}.$
Q.61	Solve : $\cos^2 x \dfrac{dy}{dx} + y = \tan x.$
Q.62	Solve the initial value problem: $x\dfrac{dy}{dx}\sin\left(\dfrac{y}{x}\right) + x - y\sin\left(\dfrac{y}{x}\right) = 0, y(1) = \dfrac{\pi}{2}.$
Q.63	Solve : $\dfrac{dy}{dx} = \cos(x+y) + \sin(x+y).$
Q.64	Solve $\dfrac{dy}{dx} = \dfrac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}.$
Q.65	Solve the differential equation $x^2dy + y(x+y)dx = 0$, given that y = 1when x = 1 .
Q.66	The slope of the tangent to a curve at a point (x , y) which passes through origin on it , is given by $\dfrac{x^4 + 2xy - 1}{1 + x^2}$. Find the equation of curve.

CONTINIUTY

Q.1	Find the value of a and b such that the function f defined by f(x) = $\begin{cases} 5 & if \quad x \le 2 \\ ax + b & if \quad 2 < x < 10 \\ 21 & if \quad x \ge 10 \end{cases}$.
Q.2	Discuss the continuity and differentiability of $f(x) = \begin{cases} 1 - x & x < 1 \\ (1-x)(2-x) & 1 \le x \le 2 \\ 3 - x & x > 2 \end{cases}$. at x = 1 & x = 2 .
Q.3	If $f(x) = \begin{cases} \dfrac{x-4}{\lvert x-4\rvert} + a & x<4 \\ a + b & x = 4 \\ \dfrac{x-4}{\lvert x-4\rvert} + b & x>4 \end{cases}$ Determine the values of a and b so that $f(x)$ is continuous at x=4 .
Q.4	Find all the points of discontinuity of the function f (x) = $[x^2]$on [1 , 2)where [ ]denotes the greatest integer function.
Q.5	The function $f(x)$ is defined as follows: $f(x) = \begin{cases} x^2 + ax + b & ,0 \le x<2 \\ 3x + 2 & ,2 \le x \le 4 \\ 2ax + 5b, & ,4<x \le 8 \end{cases}$. If is continuous on $[0,8]$, find the values of a and b .
Q.6	Determine the values of a, b, c for which the function $f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x}, & for x<0 \\ c & , for x = 0 \\ \dfrac{\sqrt{x + bx^2} - \sqrt{x}}{bx^{3/2}}, & for x>0 \end{cases}$ is continuous at x = 0 .
Q.7	Find all the point of discontinuity of the function f defined by $f(x) = \begin{cases} x + 2 & x \le 1 \\ x - 2 & 1<x<2 \\ 0 & x \ge 2 \end{cases}$.
Q.8	Show that the functions $f(x) = \lvert x + 2\rvert$ is a continuous at every $x \in R$ but fails to be

Target Mathematics by- AGYAT GUPTA ; Resi.: D-79 Vasant Vihar ; Office : 89-Laxmi bai colony
Ph. :2337615; 4010685®, 2630601(O) Mobile : 9425109601; 9425110860;9425772164(P)

differentiable at x = -2.

Q.9 Show that the function $f(x)=\begin{cases}\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} & if\ \ x\neq 0\\ 0 & if\ \ x=0\end{cases}$ is discontinuous at x = 0 .

Q.10 Find the values of a and b if f(x) = $\begin{cases}2ax+b, x<2\\ 19, x=2\\ 3a-2bx, x>2\end{cases}$ *is* continuous at x = 2.

Q.11 The function f is given by $f(x)=\begin{cases}\dfrac{1-\sin x}{\cos^2 x} & if x\langle\dfrac{\pi}{2}\\ a & if x=\dfrac{\pi}{2}\\ \dfrac{b(1-\sin x)}{(\pi-2x)^2} & if x\rangle\dfrac{\pi}{2}\end{cases}$.

Q.12 Find all point of discontinuity of f , where f is defined as following : $f(x)=\begin{cases}|x|+3 & if x\leq -3\\ -2x & -3<x<3\\ 6x+2 & if x\geq 3\end{cases}$.

Q.13 Is f (x) = $|x-1|+|x-2|$ continuous and differentiable at x = 1, 2 .

Q.14 Show that f(x) = $|x-3|, \forall x\in R$, is continuous but not differentiable at x = 3.

Q.15 Examine the continuity of the function f defined by f(x) = $\begin{cases}x\sin\dfrac{1}{x}, x\neq 0\\ 0, x=0\end{cases}$ at x = 0.

Q.16 **If f (x) =** $\begin{cases}\dfrac{x-5}{|x-5|}+a, if x\rangle 5\\ a+b, if x=5\\ \dfrac{x-5}{|x-5|}+b, if x<5\end{cases}$ **is a continuous functions. Find a, b .**

Q.17 **Let** $f(x)=\begin{cases}\dfrac{1-\cos 4x}{x^2}, if x\langle 0\\ a, if x=0\\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, if x\rangle 0\end{cases}$ **Determine the value of a so that** $f(x)$ **is continuous at x = 0 .**

Q.18 Show that $\lim_{x\to 2}\dfrac{|x-2|}{x-2}$ does not exist.

Q.19 **If** $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x} & if & x\neq\dfrac{\pi}{2}\\ 3 & if & x=\dfrac{\pi}{2}\end{cases}$ **find the value of** k **if** f **is continuous at** $x=\dfrac{\pi}{2}$.

Q.20 If the function $f(x)$ defined by $f(x)=\begin{cases}\dfrac{\log(1+ax)-\log(1-bx)}{x}, if\ x\neq 0\\ k, if\ x=0\end{cases}$. is continuous at x = 0 , find k .

Q.21 **Let** $f(x)=\begin{cases}\dfrac{1-\sin^3 x}{\cos^2 x} & if x\langle\dfrac{\pi}{2}\\ a & if x=\dfrac{\pi}{2}\\ \dfrac{b(1-\sin x)}{(\pi-2x)^2} & if x\rangle\dfrac{\pi}{2}\end{cases}$ **If f(x) is continuous at** $x=\dfrac{\pi}{2}$**, find a and b.**

LINEAR PROGRAMMING

Q.1 Minimize and maximize of Z = x + 2y subject to $x+2y\geq 100; 2x-y\leq 0; 2x+y\leq 200; x, y\geq 0$.

Q.2 There is a factory located at each of the two places P & Q . From these locations , a certain commodity is delivered to each of the three depots situated at A , B & C. The weekly requirements of the depots 5 , 5 & 4 units of commodity while the production capacity of the factories at P & Q are respectively 8 & 6 units .The cost of transportation per unit is given below. Formulate the above L.P.P. mathematically to determine how many units should be transported from each factory to each depot in order that the transportation cost is minimum.

TO	COST (in Rs)		
	A	B	C
P	16	10	15
Q	10	12	10

Q.3 Solve the following linear programming problem graphically: Minimise Z = 200 x + 500 $y$subject to the constraints:
$x+2y\geq 10$; $3x+4y\leq 24$; $x\geq 0, y\geq 0$

Target Mathematics by- AGYAT GUPTA ; Resi.: D-79 Vasant Vihar ; Office : 89-Laxmi bai colony
Ph. :2337615; 4010685®, 2630601(O) Mobile : 9425109601; 9425110860;9425772164(P)

Q.4	Solve the following problem graphically:Minimise and Maximise $Z = 3x + 9y$ .subject to the constraints: $x + 3y \le 60$; $x + y \ge 10$; $x \le y$; $x \ge 0, y \ge 0$
Q.5	A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further, no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximise the total profit of the society?;.
Q.6	A toy company manufactures two types of dolls , A & B . Market tests and available recourses have indicated that the combined production level should not exceeds 1200 dolls per week and the demand for dolls of type B is at most half of that for doll of type A. Further the production level of dolls of type A can exceeds three times the production of dolls of other type by at most 600 units . If the company makes profit of ₹ 12 and ₹ 16 per doll respectively on doll A and B ,how many each should be produce weekly in order to maximum profit ?
Q.7	An aero plane can carry a maximum of 200 passengers. A profit of Rs. 1000 is made on each first class ticket and a profit of Rs. 600 on each economy class ticket .The air line reserves at least 20 sets for first class. However, at least 4 times as many passengers prefer to travel in economy class ticket. Form a L . P .P to determine how many tickets of each class must be sold to maximize profit for the air lines.
Q.8	David wants to invest at most Rs12,000 in Bonds A and B. According to the rule, he has to invest at least Rs2,000 in Bond A and at least Rs4,000 in Bond B. If the rate of interest in bonds A and B respectively are 8% and 10% per annum, formulate the problem as L.P.P. and solve it graphically for maximum interest. Also determine the maximum interest received in a year.
Q.9	A dealer wishes to purchase a number of fans and sewing machines. He has only Rs5,760 to invest and has space for at most 20 items. A fan and sewing machine cost Rs360 and Rs240 respectively. He can sell a fan at a profit of Rs22 and sewing machine at a profit of Rs18. Assuming that he can sell whatever he buys, how should he invest his money in order to maximize his profit? Translate the problem into LPP and solve it graphically.
Q.10	A farmer has a supply of chemical fertilizer of type A which contains 10% of nitrogen and 6% of phosphoric acid and of type B which contains 5% of nitrogen and 10% of phosphoric acid. After soil testing it is found that at least 7 kg of nitrogen and the same quantity of phosphoric acid is required for a good crop. The fertilizer of type A costs Rs5.00 per kg and the type B costs Rs8.00 per kg. Using linear programming find how many kgs of each type of the fertilizer should be bought to meet the requirement and the cost be minimum. Solve the problem graphically.
Q.11	A manufacturer produces two types of steel trunks. He has two machines, A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type requires 3 hours on machine A and two hours on machine B. Machines A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of Rs.30 per trunk on the first type of trunk and Rs.25 per trunk on the second type. Formulate a Linear Programming Problem to find out how many trunks of each type he must make each day to maximize his profit.
Q.12	A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs50 per kg to purchase Food I and Rs70 per kg to purchase Food II. Formulate this problem as a linear programming problem to minimize the cost of such a mixture.
Q.13	Two tailors A and B are paid Rs150 and Rs200 per day respectively. A can stich 6 shirts and 4 paints while B can stich 10 shirts and 4 paints per day. Form a linear programming problem to minimize the labour cost to produce at least 60 shirts and 32 paints. Solve the problem graphically. [
Q.14	Kellogg is a new cereal formed of a mixture of bran and rice that contains at least 88 grams of protein and at least 36 milligrams of iron. Knowing that bran contains 80 grams of protein and 40 milligrams of iron per kilogram, and that rice contains 100 grams of protein and 30 milligrams of iron per kilogram, find the minimum cost of producing this new cereal if bran costs Rs5 per kilogram and rice costs Rs4 per kilogram.

Target Mathematics by- AGYAT GUPTA ; Resi.: D-79 Vasant Vihar ; Office : 89-Laxmi bai colony
Ph. :2337615; 4010685®, 2630601(O) Mobile : 9425109601; 9425110860;9425772164(P)

Q.15 Exhibit graphically the solution set of the linear in equations $x + y \le 5; 4x + y \ge 4; x + 5y \ge 5; x \le 4; y \le 3$.

Q.16 Solve the following LPP by graphical method : Minimize ; Z = 20 x + 10y . Subject to $x + 2y \le 40; 3x + y \ge 30 : 4x + 3y \ge 60$ & $x, y \ge 0$.

Q.17 A company makes three different products A, B and C by combining steel and rubber. Product A requires 2 units of steel and 3 units of rubber and can be sold at a profit of Rs. 40 per unit. Product B requires 3 unit of steel and 3 unit of rubber and can be sold at a profit of Rs. 45 per unit . Product C requires 1 unit of steel 2 unit of rubber and can be sold at a profit of Rs 24 per unit . There are 100 units of steel and 120 units of rubber available per day, what should be daily production of each of the products, so that the combined profit is maximum .Formulate it as a linear programming problem mathematically.

Q.18 A farmer decides to plant up to 10 hectares with cabbages and potatoes. He decides to grow at least 2 but not more than 8 hectares of cabbages and at least 1 but not more than 6 hectares of potatoes. If he can make a profit of Rs. 1500 per hectare on cabbages and Rs. 2000 per hectare on potatoes, how should he plan his farming so as to get the maximum profit? From an LPP and solve it graphically.

Q.19 A company manufactures two types of toys A and B. Type A requires 5 minute each for cutting and 10 minute each for assembling. Type B requires 8 minute for cutting and 8 minutes each for assembling. These are 3 hours available for cutting and 4 hours available for assembling in a day. The profit is Rs. 50 each for type A and Rs 60 each for type B. How many toys of each type should the company manufacture in a day to maximize the profit ?

Q.20 Suppose every gram of wheat provides 0.1 gm of proteins and 0.25 gm of carbohydrates and the corresponding values for rice are 0.05 gm and 0.5 gm respectively. Wheat costs Rs 5 per kg and rice Rs 20. The minimum daily requirements of proteins and carbohydrates for an average child are 50 gm and 200 gm respectively. In what quantities should wheat and rice be mixed in the daily diet to provide the minimum daily requirements of proteins and carbohydrates, if both the items are to be taken up in each diet ?

Q.21 A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹ 300 and that on a chain is ₹ 190, find the number of rings and chains that should be manufactured per day , so as to earn the maximum profit. Make it as an L.P.P. and solve it graphically.

Q.22 A category agency has two kitchens to prepare food at two places A and B. From these places ‘ Mid-day Meal ‘ is to be supplied to three different schools situated at

P, Q, R. The monthly requirements of the schools are respectively 40,40 and 50 food packets. A packets contains lunch for 1000 students. Preparing capacity of kitchens A and B are 60 and 70 packets per month respectively. The transportation cost per packet from the kitchen to schools is given below :

Transportation cost per packet (in rupees)

To	From	
	A	B
P	5	4
Q	4	2
R	3	5

How many packets from each kitchen should be transported to school so that the cost of transportation is minimum ? Also find the minimum cost .

Q.23 Two godowns A and B have a grain capacity of 100 quintals and 50 quintals respectively. They supply to three shops D, E and F, whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from godowns to the shops are given below :

Transportaton cost per quintal (in Rs.)		
From / To	A	B
D	6.00	4.00
E	3.00	2.00
F	2.50	3.00

How should the supplies be transported in order that transportation cost is minimum ?

Q.24 An oil company requires 12,000; 20,000 and 15,000 barrels of high grade, medium grade and low grade oil respectively. Refinery A produces 100, 300 and 200 barrels per day of high, medium and low grade oil respectively whereas the Refinery B produces 200, 400 and 100 barrels per day respectively. If A costs ₹ 400 per day and B costs ₹ 300 per day to operate ,how many days shoud each be run to minimize cost while satisfying requirements .

Q.25 A firm deals with two kinds of fruit juices -pineapple and the orange juice. These are mixed and two types of mixtures are obtained which are sold as soft drinks A and B .One tin of A needs 4 liters of pineapple juice and 1 liter of orange juice. One tin of B needs 2 liters pineapple and 3 liters of orange juice .The firm has only 46 liters of pineapple juice and 24 liters of orange juice. Each tin of A and B is sold at a profit of Rs.4/- and Rs.3/- respectively .How many tins of A and B should the firm produce to maximize profit ? Formulate the linear programming problem and solve it graphically using is profit line approach.

Q.26 A firm makes two types of furniture: chairs and tables. The contribution to profit for each product as calculated by the accounting department is Rs 20 per chair and 30 per table. Both products are to be processed on three machines and .The time required in hours by each product and total available in hours per week on each machine are as follows:

Machine	Chair	Table	Available Time
M1	3	3	36
M2	5	2	50
M3	2	6	60

How should the manufacture schedule their production in order to maximize profit?

Q.27 A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine. If the grower wants to minimize the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

	kg per bag	
	Brand P	Brand Q
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

Q.28 An aero plane can carry a maximum of 250 passengers. A profit of Rs. 500 is made on each first class ticket and a profit of Rs. 350 on each economy class ticket .The air line reserves at least 25 sets for first class. However, at least 3 times as many passengers prefer to travel in economy class ticket. Form a L . P .P to determine how many tickets of each class must be sold to maximize profit for the air lines.

Q.29 If a young man rides his motor cycle at 25 km per hour , he has to spend Rs 2 per kilometer on petrol; if he rides at a faster speed of 40 km per hour, the petrol cost increases to Rs 5 per kilometer . He has Rs 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express this as a linear programming problem and then solve it .

Q.30 A factory owner wants to purchase two types of machines A and B, for his factory. The machine A requires an area of $1000\,m^2$ and 12 skilled men for running it and its daily output is 50 units, whereas the machine B requires $1200\,m^2$ area and 8 skilled man, and its daily output is 40 units. If an area of $7600\,m^2$ and 72 skilled men be available to operate the machines, how many machine of each type should be bought to maximize the daily output.

Q.31 A producer has 30 and 17 units of labour and capital respectively which he can use to produce two types of goods X and Y. To produce one ulnit of X, 3 units of capital and 2 units of labour are required and to produce one unit of Y, 3 units of labour and 1 unit of capital is required. If X and Y are priced at Rs. 100 and Rs. 120 respectively, how should the producer use his resources to maximize the total revenue ? From the LPP and solve it.

Q.32 A farmer decides to plant upto 10 hectares with cabbages and potatoes. He decides to grow at least 2 but not more than 8 hectares of cabbages and at least 1 but not more than 6 hectares of potatoes. If he can make a profit of ₹ 1500 per hectare on cabbages and ₹ 2000 per hectare on potatoes, how should he plan his farming so as to get the maximum profit? From an LPP and solve it graphically.

Q.33 Kellogg is a new cereal formed of a mixture of barn and rice that contain at least 88 gram of protein and 36 milligram of iron .knowing that barn contain 80 gram of protein and 40 milligram of iron per kg and that rice contain 100 gram of protein and 30milligram of iron per kg, find the minimum cost of producing this new cereal if bran cost ₹ 5 per kg and rice cost ₹ 4 per kg.

Q.34 A toy manufacturers produce two types of dolls ; a basic version doll A and deluxe version doll B. Each doll of type B takes twice as long to produce as one doll of type A . The company have time to make a maximum of 2000 , dolls of type A per day , the supply of plastic is sufficient to produce 1500 dolls per day and each type requires equal amount of it .The deluxe version i.e. type B requires a fancy dress of which there are only 600 per day available . If the company makes profit of ₹ 3 and ₹ 5 per doll respectively on doll A and B , how many of each should be produced weekly in order to maximize the profit ? Solve it by graphical method.

Q.35 A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair in ₹ 30 while by selling one table the profit is ₹ 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit? Formulate the problem as L.P.P. and solve it graphically.

****

Target Mathematics by- AGYAT GUPTA ; Resi.: D-79 Vasant Vihar ; Office : 89-Laxmi bai colony
Ph. :2337615; 4010685®, 2630601(O) Mobile : 9425109601; 9425110860;9425772164(P)