Chapter 2 Polynomials

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Important Identities : -

- $(x + y)^2 = x^2 + 2xy + y^2$
- $(x - y)^2 = x^2 - 2xy + y^2$
- $(x + y)(x - y) = x^2 - y^2$
- $(x + a)(x + b) = x^2 + (a + b)x + ab$
- $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 = x^3 + y^3 + 3xy(x + y)$
- $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3 = x^3 + y^3 - 3xy(x - y)$
- $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
- $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
- $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
- $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
- *If* $x + y + z = 0$ *, then* $x^3 + y^3 + z^3 = 3xyz$

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1. *Classify the following as monomials, binomials and trinomials :*
   (a) x^3 *b)* $2y^2 - 4y + 3$ *c)* $t^2 - 4$
   d) $\sqrt{2}$ *e)* $x^3 + 4x^2 + 5x$ *f)* $u^7 + u^2 - 4$.
2. *Write the coefficients of* x^2 *in each of the following :*
   a) $3x^2 - 4y$ *b)* $\frac{2}{3}x + \frac{4}{3}x^2 + 7y$ *c)* $3x + 4y - 5z$
   d) $x^2 + 2xy + 3y^2$ *e)* $\frac{x}{7} - \frac{7}{z}$.
3. *Write the degree each of the following :*
   a) $5x^3 + 4x^2 + 7x$ *b)* $4 - y^2$ *c)* $5t - 3$ *d)* 3
4. *Classify the following as linear, quadratic and cubic polynomials :*
   a) $x^2 + x$ *b)* $x - x^3$ *c)* $y + y^2 + 4$
   d) $1 + x$ *e)* $3t$ *f)* r^2
5. *Find the value of the polynomial* $5x - 4x^2 + 3$ *at :*
   a) $x = 0$ *b)* $x = -1$ *c)* $x = 2$
6. *Find the value of each of the following polynomials at indicated value of variables :*
   a) $p(x) = 5x^2 - 3x + 7$ *at* $x = 1$ *b)* $p(y) = 3y^3 - 4y + 4$ *at* $y = 2$
   c) $p(t) = 4t^4 + 5y^3 - t^2 + 6$ *at* $t = a$
7. *Check whether – 2 and 2 are zeroes of the polynomial* $x + 2$.
8. *Find the zero of the polynomial* $p(x) = 2x + 1$.
9. *Verify whether 2 and - 2 are zero of the polynomial* $x^2 - 4$.

10. *Verify whether 2 and 0 are zero of the polynomial $x^2 - 2x$.*

11. *Find the value of the following :*
    a) $(3x^2 - 3x + 1)(x - 1)$ *when* $x = 3$
    b) $(3x^2 - 1)(4x^3 - 4x - 3)$ *when* $x = -1$.

12. *Evaluate the following for given values of the variables :*
    a) $x^4 - x^3 + x^2 - x + 1$ *for* $x = 2$
    b) $x^3 + x^2 + x + 1$ *for* $x = -1$.

13. *Find the remainder and quotient in each of the following :*
    a) *Divide* $x^4 - 1$ *by* $x - 1$.
    b) *Divide* $x^3 - 3x^2 + 5x - 8$ *by* $x - 2$.

14. *Find the remainder when* $4x^3 - 3x^2 + 2x - 4$ *is divided by :*
    a) $x - 1$ b) $x - 2$ c) $x + 1$
    d) $x - 4$ e) $x + 2$ f) $x + \frac{1}{2}$

15. *Using remainder theorem, find the remainder :*
    a) *Divide* $x^6 - 1$ *by* $x - 1$ b) *Divide* $x^3 + 1$ *by* $x + 1$.

16. *Find the remainder when* $x^4 + x^3 - 2x^2 + x + 1$ *is divided by* $x + 1$.

17. *Find the remainder when the polynomial* $p(x) = x^3 + 2x^2 - 2x + 1$ *is divided by* $x + 3$.

18. *Find the remainder when the polynomial* $p(x) = x^2 + 4x + 2$ *is divided by* $x + 2$.

19. *Find the remainder when* $3x^4 - x^3 + 3x^2 - 4x + 1$ *is divided by* $x - 3$.

20. *If* $x - 2$ *is a factor of each of the following polynomials, then find the value of a in each case :*
    a) $x^2 - 3x + 5a$ b) $x^3 - 2ax^2 + ax - 1$ c) $x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$.

21. *Factorise :* $6x^2 + 17x + 5$

22. *Factorise :* $x^3 - 23x^2 + 142x - 120$.

23. *Using a suitable identity, find the following products :*
    a) $(x + 5)(x - 3)$ b) $(4x + 3)(4x + 5)$ c) $(x + y)(x + y)$
    d) $\left(x^2 + \frac{1}{2}\right)\left(x^2 - \frac{1}{2}\right)$ e) $(3x + 4)(3x + 7)$ f) $(5a + 3)(5a + 2)$

24. *Expand using suitable formula :*
    a) $(2a + 3)^2$ b) $(3a - 5)^2$

25. *Factorise the following :*
    a) $x^2 + 6x + 9$ b) $24x^2 - 41x + 12$ c) $x^2 - x - 6$
    d) $16x^2 + 8x + 1$ e) $9x^2 - 16y^2$ f) $4x^3 - 4x$

g) $(x + 1)^2 - (x - 1)^2$ h) $9x^2 + 6x + 1 - 25y^2$ i) $25x^2 - 10x + 1 - 36y^2$

j) $x^3 + x - 3x^2 - 3$ k) $x^2 + y - xy - x$ l) $3ax - 6ay - 8by + 4bx$

m) $xy - ab + bx - ay$ n) $1 - x^2 - y^2 - 2xy$ o) $8 - 4a - 2a^3 + a^4$

p) $a^2 + b^2 + 2ab + 2bc + 2ca$ q) $x^3 + 64$ r) $25x^2 - 10x + 1$

s) $x^2 - 11x - 42$ t) $12x^2 - 10x + 2$ u) $125a^3 + \frac{b^3}{27}$

v) $a^4 - a$ w) $x^3 - 125$ x) $27x^3y^3 - 8z^3$

y) $8x^3 - (2x - y)^3$ z) $(a + b)^3 - (a - b)^3$

26. *factorise the following :*

i) $4(x - y)^2 - 12(x - y)(x + y) + 9(x + y)^2$ ii) $3(x + y)^2 - 5(x + y) + 2$

iii) $12(x^2 + 7x)^2 - 8(x^2 + 7x)(2x - 1) - 15(2x - 1)^2$ iv) $x^2 - 5x + 6$

v) $4x^2 + 9y^2 + z^2 + 12xy + 4xz + 6yz$ vi) $x^3 - x^2y + \frac{1}{3}xy^2 - \frac{1}{27}y^3$

vii) $\frac{1}{8}a^3 + \frac{1}{4}a^2b + \frac{1}{6}ab^2 + \frac{1}{27}b^3$

27. *Solve using appropriate formula :*

i) $(2a + 3)(2a - 3)$ ii) $(105)^2$ iii) $(49)^2$

iv) $(536)^2 - (136)^2$ v) *if* $4x = 7^2 - 3^2$, *then find the value of x.*

vi) 998×1002

28. *Simplify :* $(a + b)^3 + (a - b)^3 + 6a(a^2 - b^2)$.

29. *Show that if* $2(a^2 + b^2) = (a + b)^2$, *then* $a = b$.

30. *Expand each of the following : -*

i) $(x + 2y)^3$ ii) $(2x - 3y)^3$ iii) $(x^2 + 2y)^3$

iv) $\left(\frac{1}{3}x - \frac{2}{3}y\right)^2$ v) $\left(\frac{1}{3}x + \frac{5}{3}y\right)^2$

31. *Evaluate the following using suitable identities :*

i) $(98)^3$ ii) $(101)^3$ iii) $(999)^3$

32. *Show that if* $(a + b)$ *is not zero, then the equation :* $a(x - a) = 2ab - b(x - b)$ *has a solution* $x = a + b$.

33. *Foctorise each of the following :*

i) $a^4 - b^4$ ii) $a^4 - 16b^4$ iii) $a^2 - (b - c)^2$

iv) $x^2 + 7xy + 12y^2$ v) $x^2 + 2ax - b^2 - 2ab$ vi) $(x^2 + x)^2 + 4(x^2 + x) - 12$

vii) $5x^2 + 16x + 3$.